Regularization behaviour of a nonlocal Gurson-type model

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Abstract

The Gurson-type model considered here deals with a nonlocal stress state which implies nonlocal equilibrium conditions as well as nonlocal constitutive equations. However, it is assumed that the kinematic equations of the theory of simple materials are still valid. The nonlocal Gurson-type model has been implemented into the finite element program Abaqus using its user material facility. The regularization behaviour of the model is discussed by means of typical two dimensional localization problems.

Introduction

Continuum models, such as continuum damage models, which include softening fail if localization of deformation occurs. After the onset of localization, the numerical results become mesh dependent where the following well known tendencies become apparent: the finer the chosen mesh the smaller the localization zone and if the mesh design shows a pronounced mesh direction the localization zone follows this direction. The first trend mentioned above can be used to show that this mesh dependence is spurious because it leads to physically unreasonable results and to justify the formulation of continuum damage models within the framework of higher order theories like gradient enhanced theories, micropolar theories or nonlocal continuum theories, where only the nonlocal approach is considered here. This topic has been widely discussed in the literature for example in Pijaudier-Cabot and Bazant [1], de Borst et al. [2].

With the aim to simulate ductile damage and fracture a continuum damage model was derived by Gurson [3] within the framework of simple material. This model was used in Tvergaard and Needleman [4] to discuss simple localization problems where only the damage variable has been formulated in a nonlocal way. Furthermore, a formulation of the Gurson-model within a micropolar framework was derived in Gologanu et al. [5] as well a gradient enhanced version of the model based on the nonlocal improvement proposed in [4] has been discussed in Reusch [6].

On the basis of a micromechanical argumentation, a nonlocal Gurson-type model has been proposed in Mühlich [7], Mühlich and Kienzler [8]. First, a brief review of the general results based on the micromechanical argumention will be given. Then, these results will be applied to the Gurson-model and the numerical implementation will be described. Finally, first results for simple localization problems will be discussed.

Brief review of the used nonlocal approach

In order to derive the nonlocal Gurson-type model, a body with volume $\mathcal{B}$ under mechanical loading is considered. It is assumed that the body consists of a material whose microstructure can be caracterized by the existence of statistically distributed voids and particles surrounded by an elastic-plastic matrix material. Continuum theories for this kind of materials whithin the framework of simple materials can be derived in general by means of the theory of homogenization using the concept of the representative volume element, see for example Nemat-Nasser and Hori [9]. However, this concept is only justified if the so-called Hill-Mandel Lemma, see for example [9], is valid at almost every macroscopic point. If localization of deformation occurs, this condition is no longer fulfilled because of the fact that strong gradients of the macroscopic quantities can be observed, a originally regular distribution of microstructural defects gets lost during the deformation history, etc.. If the classical theory of homogenization is applied to solve such problems by means of continuum mechanics, a so-called homogenization error is introduced. In order to corrrect this error, the total strain rate of the theory of simple materials has been extended by a nonlocal term as follows

$$\dot{\mathcal{W}} = \int_{\mathcal{B}} \left(\Sigma_{ij}(\underline{X})\dot{E}_{ij}(\underline{X}) + \int_{\mathcal{B}} \dot{W}^{(K)}(\underline{X}, \underline{X}')dV' \right) dV \tag{1}$$

where $\underline{X}$ and $\underline{X}'$ are spacial coordinate vectors, $\Sigma_{ij}(\underline{X})$ and $\dot{E}_{ij}(\underline{X})$ are the stresses and strain rates at a point $\underline{X}$.

Based on qualitative arguments and numerical calculations considering one dimensional arrays of unit cells

$$\dot{W}^{(K)}(\underline{X}, \underline{X}') = \frac{\varphi(\underline{X}, \underline{X}')}{\int_{\mathcal{B}} \varphi(\underline{X}, \underline{X}')dV'} \left[\Sigma_{ij}(\underline{X}) - \Sigma_{ij}(\underline{X}')\right] \dot{E}_{ij}(\underline{X}) \tag{2}$$

has been found in [7] to be an acceptable choice to correct the homogenization error. Although the unit cell calculations were carried out assuming irregular void distributions under homogeneous macroscopic loading conditions and linear elastic matrix material, equation (2) is used here as a first approximation. With respect to the weight function $\varphi(\underline{X}, \underline{X}')$ in (2)

$$\varphi(\underline{X}, \underline{X}') = Exp\left(-A||\underline{X} - \underline{X}'||^B\right) \tag{3}$$

was chosen, where A and B are material parameters related to the microstructure.

In the following it is assumed that the kinematic relations of the theory of simple materials are still valid. The boundary $\mathcal{S}$ of the body is divided into $\mathcal{S}_U$ and $\mathcal{S}_T$ where displacement rates $\dot{\bar{U}}_i$ are prescribed on $\mathcal{S}_U$ and surface tractions $\bar{T}_i$ are prescribed on $\mathcal{S}_T$. Application of the Principle of Virtual Displacements then leads to the following result

$$\frac{\partial}{\partial X_j} < \Sigma_{ij} > (\underline{X}) \quad = \quad 0 \qquad\qquad \forall \underline{X}, \underline{X} \in \mathcal{B} \tag{4}$$

$$\dot{E}_{ij}(\underline{X}) = \frac{1}{2}\left(\frac{\partial \dot{U}_i(\underline{X})}{\partial X_j} + \frac{\partial \dot{U}_j(\underline{X})}{\partial X_i}\right) \qquad \forall \underline{X}, \underline{X} \in \mathcal{B} \tag{5}$$

$$< \Sigma_{ij} > (\underline{X})\, n_j(\underline{X}) = \bar{T}_i(\underline{X}) \qquad \forall \underline{X}, \underline{X} \in \mathcal{S}_T \tag{6}$$

$$\dot{U}_i(\underline{X}) = \dot{\bar{U}}_i(\underline{X}) \qquad \forall \underline{X}, \underline{X} \in \mathcal{S}_U \tag{7}$$

where the abbreviation

$$< \alpha > (\underline{X}) := \frac{1}{\int\limits_{\mathcal{B}} \varphi(\underline{X}, \underline{X}')dV'} \int\limits_{\mathcal{B}} \varphi(\underline{X}, \underline{X}')\alpha(\underline{X}')dV' \tag{8}$$

is used and the n_j are the components of the unit normalvector on $\mathcal{S}$. Because of the assumptions mentioned above, the kinematic relations (5) and the kinematic boundary conditions (7) are identical with these of the theory of simple materials whereas the equilibrium conditions (4) as well as the dynamic boundary conditions (6) are nonlocal. Due to the use of (8) it is no longer necessary to point out explicitly if a variable refers to a point $\underline{X}$ or $\underline{X}'$. Therefore, this indication will be dropped in the following.

Proper constitutive equations are needed to complete the system of equations (4) - (7) as discussed next. The nonlocal work theorem

$$\int\limits_{\mathcal{B}} < \Sigma_{ij} > \dot{E}_{ij} dV = \int\limits_{\mathcal{S}} < T_i > \dot{U}_i dA \tag{9}$$

with $< T_i > = < \Sigma_{ij} > n_j$ can be obtained which shows that at every macroscopic point the nonlocal stresses do work on the strain rates formulated as usual within the theory of simple materials.

As discussed in [7] the nonlocal correction can be related to the microscopic surface tractions and the displacement rates which acts on the outer surface of a considered volume element. However, this volume element is no longer a representative one. Unfortunately, this would lead to very complicate constitutive equations and therefore another approach is proposed here. A volume element is considered, where neither linear displacement rates nor constant tractions can be assumed on the outer surface of the volume element which means that the Hill-Mandel Lemma is no longer valid. A model based on the theory of simple materials assumes eighter linear displacement rates or constant tractions. This leads to relatively simple constitutive equations by means of local macroscopic stresses $\Sigma_{ij}(\underline{X})$, strain rates $\dot{E}_{ij}(\underline{X})$ and internal variables. However, such a model would give incorrect results. On the other hand, another volume element with eigther linear displacement rates or constant surface tractions is considered but according to (9) the nonlocal macroscopic stresses $< \Sigma_{ij} > (\underline{X})$ and the local strain rates $\dot{E}_{ij}(\underline{X})$ are related to it. However, if instead of $\Sigma_{ij}(\underline{X})$ the nonlocal stresses $< \Sigma_{ij} > (\underline{X})$ act together with the same local strain rates $\dot{E}_{ij}(\underline{X})$ assuming validness of the Hill-Mandel Lemma, the internal variables of the model must change. The argumentation above implies the following scheme for the calculation: 1. Calculation according to the theory of simple materials. 2. Calculation of the resulting nonlocal stress state. 3. Replacement of the stresses Σ_{ij} in the constitutive equations by the nonlocal stresses $< \Sigma_{ij} >$ and solving the constitutive equations for known nonlocal stresses in order to obtain the correction with respect to the internal variables.

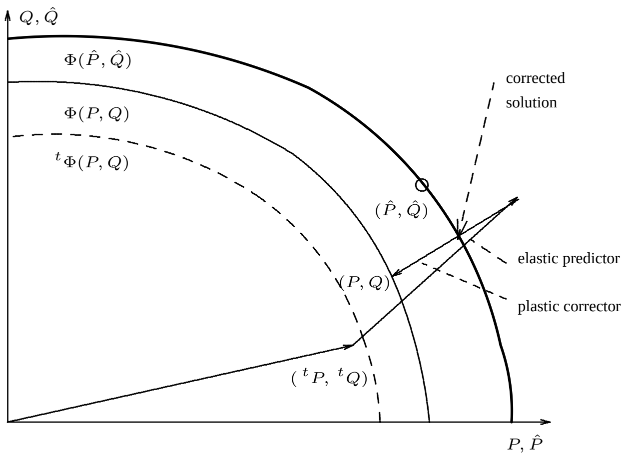

FIGURE 1. Schematic representation of the nonlocal correction

Application to the Gurson-model

The Continuum Damage Model considered here consists of the yield condition

$$\Phi = \left(\frac{Q^2}{\sigma_Y(\bar{\varepsilon})}\right)^2 + 2q_1 f^* Cosh\left(\frac{1}{2}q_2\frac{P}{\sigma_Y(\bar{\varepsilon})}\right) - 1 - q_3(f^*)^2 \tag{10}$$

with

$$Q = \sqrt{\frac{3}{2}\Sigma'_{ij}\Sigma'_{ij}}\ ,\ P = -\Sigma_{kk} \tag{11}$$

and evolution equations for the internal variables of the model. The Σ'_{ij} in (11) are the components of the deviatoric part of the Cauchy stress tensor and $\sigma_Y(\bar{\varepsilon})$ stands for an averaged stress versus plastic strain curve of the matrix material. The fit parameters q_1, q_2, q_3 in (10) have been introduced by Tvergaard [10] into the model, originally proposed by Gurson [3] to get a better agreement between the predictions of the Gurson model with the results obtained by cell model calculations. To take into account the loss of stress carrying capacity associated with void coalescence, the modified damage parameter f^* as a piecewise linear function of the void volume fraction f

$$f^* = \begin{cases} f & f \le f_c \\ f_c + \kappa(f - f_c) & f > f_c \end{cases} \quad \text{with} \quad \kappa = \frac{f_U^* - f_c}{f_F - f_c} \tag{12}$$

was proposed in Tvergaard and Needleman [11]. The parameter f_U^* is related to q_1 by $f_U^* = 1/q_1$ if $q_3 = q_1^2$ is used. The void volume fraction where void coalescence starts is indicated by f_c and the void volume fraction at final fracture is denoted by f_F.

Following Aravas Aravas [12], the normality rule can be used in order to obtain

$$\frac{\partial \Phi}{\partial Q}\dot{E}_P + \frac{\partial \Phi}{\partial P}\dot{E}_Q = 0 \tag{13}$$

with the two scalar plastic strain variables E_P and E_Q. Here, nucleation of voids is not considered. Therefore, from incompressible matrix behaviour follows

$$\dot{f} = (1 - f)\dot{E}_P\,. \tag{14}$$

Finally, the evolution equation for the equivalent plastic strain $\bar{\varepsilon}$ is given by

$$\dot{\bar{\varepsilon}} = \frac{-P\dot{E}_P + Q\dot{E}_Q}{(1-f)\sigma_Y(\bar{\varepsilon})} \,. \tag{15}$$

The Gurson-model described above has been implemented into the finite element program ABAQUS Hibbit et al. [13] assuming small elastic strains and employing a predictor-corrector method which will not be discussed here. More detailed informations can be found in Aravas [12].

From the prediction obtained by the Gurson-model described above, the nonlocal stress variables

$$\hat{P} = -\frac{1}{3} < \Sigma_{ij} > \tag{16}$$

$$\hat{Q} = \sqrt{\frac{3}{2} < \Sigma_{ij} >' < \Sigma_{ij} >'} \tag{17}$$

can be determined at every integration point. In order to calculate the nonlocal corrections with respect to f, $\bar{\varepsilon}$, E_P and E_Q the following system of equations has to be solved

$$0 = \left(\frac{\hat{Q}^2}{\sigma_Y(\bar{\varepsilon})}\right)^2 + 2q_1 f^* Cosh\left(\frac{1}{2}q_2\frac{\hat{P}}{\sigma_Y(\bar{\varepsilon})}\right) - 1 - q_3(f^*)^2 \tag{18}$$

$$0 = \frac{\partial\Phi}{\partial\hat{Q}}\dot{E}_P + \frac{\partial\Phi}{\partial\hat{P}}\dot{E}_Q \tag{19}$$

$$\dot{f} = (1-f)\dot{E}_P \tag{20}$$

$$\dot{\bar{\varepsilon}} = \frac{-P\dot{E}_P + Q\dot{E}_Q}{(1-f)\sigma_Y(\bar{\varepsilon})} \tag{21}$$

if the nonlocal yield condition (18) indicates plastic flow. Otherwise, the considered point behaves linearly elastic. Application of the Euler backward integration scheme to system of equations (18) - (21) finally leads to one nonlinear equation

$$(f - {}^t f)(\hat{P}\frac{\partial\Phi}{\partial\hat{P}} + \hat{Q}\frac{\partial\Phi}{\partial\hat{Q}}) + \sigma_Y(\bar{\varepsilon})\Delta\bar{\varepsilon}(1-f)^2\frac{\partial\Phi}{\partial\hat{P}} = 0 \tag{22}$$

for $\bar{\varepsilon}$, where the left-hand superindex t indicates the value of the considered variable at the start of the increment. Because $\hat{P}$ and $\hat{Q}$ are known, f^* can be obtained from (18) as a function of $\bar{\varepsilon}$

$$f^* = \frac{q_1}{q_3}Cosh\left(\frac{1}{2}q_2\frac{\hat{P}}{\sigma_Y(\bar{\varepsilon})}\right) - \sqrt{\frac{q_1^2}{q_3^2}Cosh^2\left(\frac{1}{2}q_2\frac{\hat{P}}{\sigma_Y(\bar{\varepsilon})}\right) + \frac{\hat{Q}^2}{\sigma_Y^2(\bar{\varepsilon})q_3} - \frac{1}{q_3}} \tag{23}$$

and f and f^* are related by (12). The iterative loop consisting of the prediction by means of the Gurson-model formulated within the theory of simple materials and nonlocal correction can be applied until convergence with respect to the internal variables is achieved. However, instead of an iterative algorithm an explicit scheme was chosen for the nonlocal correction. Once the new stress state related to the theory of simple materials for the actual time increment is determined, the nonlocal averaging is carried out. The corrected values for f and $\bar{\varepsilon}$ determined by solving (22) are then used as start values for the next time increment. A schematical representation of the nonlocal correction is shown in Figure 1.

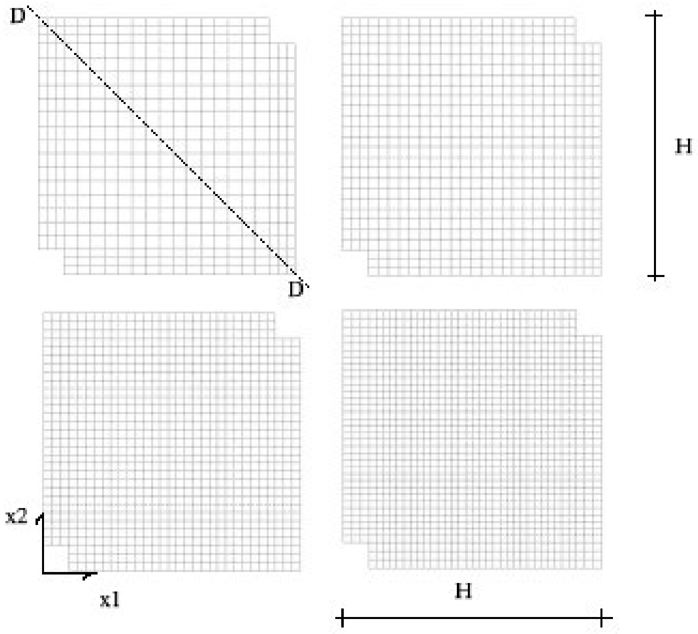

FIGURE 2. Finite element meshes and used denominations

Numerical example

In order to discuss the regularization behaviour of the model the problem shown in Figure 2 is considered. The calculations have been carried out with four different finite element meshes varying the size of the four-node finite elements within the localization zone. The finite element meshes as well the denominations used in the following can also be found in Figure 2. According to the number of elements within the central part the denominations 15x15, 20x20, 25x25 and 30x30 are used in the following in order to refer to the different meshes. The boundary conditions

$$\begin{aligned} u_1(x_1, x_2 = 0) &= u_2(x_1 = 0, x_2) = 0 \\ u_1(x_1 = H, x_2) &= u_2(x_1, x_2 = H) = \bar{u} \end{aligned} \tag{24}$$

have been prescribed and the displacement $\bar{u}$ was applied linearly with respect to time starting from 0.0 mm up to 7.5 mm. The Young modulos $E = 200$GPa, the Poisson number $\nu = 0.3$ and linear hardening with a plastic tangent modulus $E_t = 1$GPa were used. An initial void volume fraction $f_0 = 0.1$ and $f_c = 1$, $\kappa = 1$ have been assumed. All nonlocal calculations have been carried out with $A = 0.5$ and $B = 2$.

Results and discussion

Together with the results of the calculations achieved with the nonlocal Gurson-type model, the predictions obtained by the use of the local Gurson model for the meshes 15x15 and 30x30 are shown in the Figure 3 as well in Figure 4. The results by means of the reaction

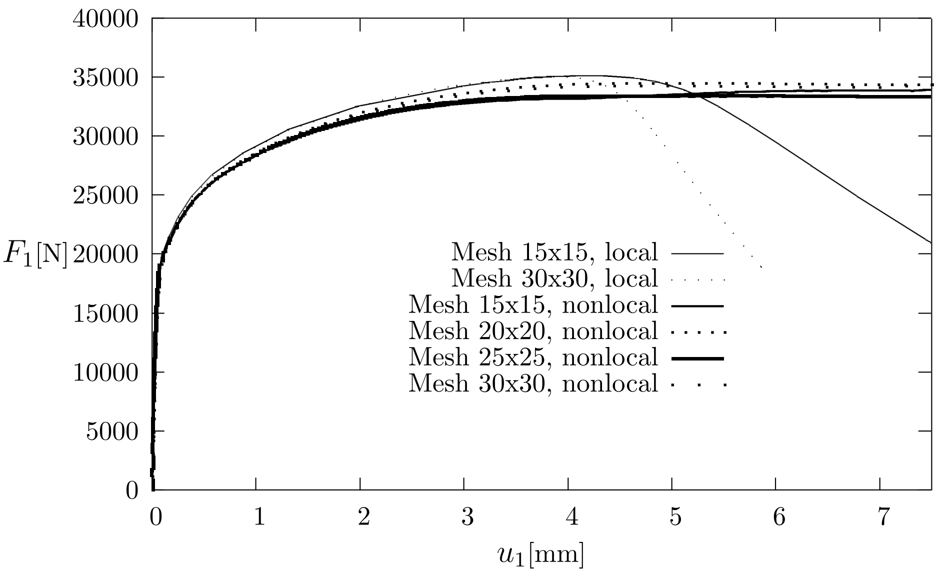

FIGURE 3. Reaction force in x_1 direction related to $\bar{u}$.

force which corresponds to the global displacement $\bar{u}$ in x_1 direction can be found in Figure 3. After the onset of localization, the local Gurson model predicts a rapid drop of the reaction force where the gradient depends strongly on the mesh density. In contrast to these results, the nonlocal model predicts independently of the mesh density almost identical load - displacement curves. The regularization effect of the nonlocal Gurson-type model can be demonstrated as well by means the equivalent plastic strain of the matrix material. In Figure 4, $\bar{\varepsilon}$ is shown along the line D-D indicated in Figure 2 for the deformed configuration at $\bar{u} = 6$mm. The local model predicts much higher values for $\bar{\varepsilon}$ and the specific values depend on the mesh density whereas the nonlocal model predicts almost identical values whithin the localization zone. Far from the localization zone the nonlocal model gives significantly higher values for $\bar{\varepsilon}$ then the local model. It is supposed here that this effect appears due to the choice of the nonlocal parameters A and B which represent a relatively large range of nonlocal influence.

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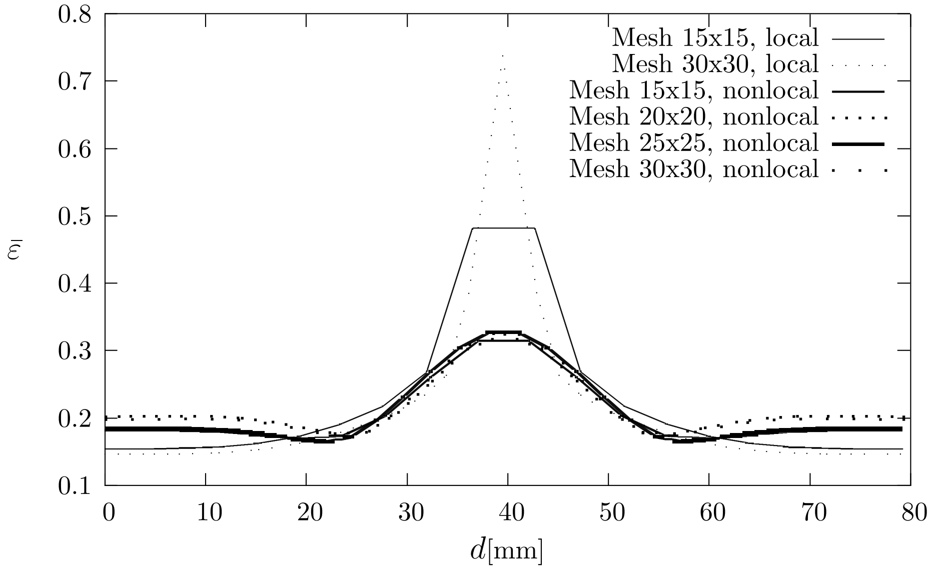

FIGURE 4. Spatial ditribution of $\bar{\varepsilon}$.

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