STRAIN ENERGY DENSITY PARAMETER APPROACH TO THE NON-LOCAL THEORY

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Abstract

The paper presents a definition of the strain energy density parameter for description of fatigue of materials, machine elements and structures subjected to random loading. In the case of multiaxial random loading, a generalised energy criterion based on the energy density parameter of normal and shear strains acting in the critical plane was applied. This criterion was used for determination of the equivalent strain energy density parameter, reducing the multiaxial stress state to the uniaxial one. The paper also contains a review of models for determination of non-local stresses and strains under stress gradients in the material. Next, an equation for calculation of the non-local equivalent strain energy density parameter in the critical plane was proposed.

Introduction

The main aims of this paper are an approach to stress gradients in circular sections of bars, using the energy parameter and its introduction to description of multiaxial fatigue as well as elaboration of an efficient algorithm for fatigue strength evaluation under multiaxial service loading. The stress gradient occurs in two cases: under bending or torsion of smooth bars or any loading of elements with stress concentrators. In all such cases it is difficult to determine real stresses and strains on the surface or their distributions in the section. Using simple equations, it is possible to determine only nominal stresses on the surface and in the round bar section for bending and torsion on the assumption that the stress distributions are linear along the radius. The problem becomes more complicated in the case of stress concentrations caused by notches presence. If the notches are present, determination of stress and strain normal distributions in the bar sections requires numerical methods. In the case of stress concentrators, it seems to be right to model non-linear stress and strain distributions with simple analytical expressions. In this paper, the authors modelled the pseudoelastic strains and stresses for bars with ring notch by pure bending and pure torsion. Next, a method of determination of non-local parameter of fatigue damage in the energy approach is proposed.

Energy parameter of damage

A change of strain energy density has been applied in theory of plasticity for many years. It has been proposed as a parameter for fatigue description [1]. The proposed model does not include separation of strain energy density into elastic and plastic parts, as in the case of the SWT parameter [2]. In order to distinguish tension and compression in a fatigue cycle, the following equation for determination of the strain energy density parameter history was proposed

$$W(t) = 0.5\sigma(t)\varepsilon(t)\,\mathrm{sgn}[\sigma(t),\varepsilon(t)] = 0.5\sigma(t)\varepsilon(t)[\mathrm{sgn}(\sigma(t)) + \mathrm{sgn}(\varepsilon(t))] \qquad (1)$$

Eq. (1) expresses positive and negative parameters of strain energy density in a fatigue cycle and it is possible to distinguish energy under tension and energy under compression.

Assuming that $W(t)$ – Eq. (1) – is the fatigue damage parameter, it is possible to rescale the standard characteristics of cyclic fatigue $(\sigma_a - N_f)$ for high-cycle fatigue and to obtain a new characteristic $(W_a - N_f)$:

$$W_a = 0.5\sigma_a^2 E^{-1} .$$
(2)

This characteristic can be brought to

$$\log N_f = A' - m' \log W_a .$$
(3)

The proposed generalized energy criterion [3] has the following form:
(1) Fatigue is caused by the part of strain energy density corresponding to work of the normal stress $\sigma_\eta(t)$ on the normal strain $\varepsilon_\eta(t)$, i.e. $W_\eta(t)$ and work of the shear stress $\tau_{\eta s}(t)$ on the shear strain $\varepsilon_{\eta s}(t)$ in direction $\bar{s}$, on the plane with normal $\bar{\eta}$, i.e. $W_{\eta s}(t)$;

(2) Direction $\bar{s}$ on the critical plane agrees with the mean direction where the strain energy density is maximum, $W_{\eta s max}(t)$;

(3) In the limit state, the material effort is determined by the maximum value of linear combination of strain energy density parameters, $W_\eta(t)$ and $W_{\eta s}(t)$, i.e.

$$\max_t \{\beta W_{\eta s}(t) + \kappa W_\eta(t)\} = Q$$
(4)

where β, κ and Q – material constants obtained from simple fatigue tests, applied for selection of a particular form of the criterion (4).

If the maximum value of $W(t)$ is greater than Q, then damage leading to material fatigue is accumulated. The random process $W(t)$ can be understood as a stochastic process of the material fatigue effort. Positions of the vector directions $\bar{\eta}$ and $\bar{s}$ are determined with one of three procedures proposed in [3]. Here, it is assumed that $\kappa = 1$ and $Q = W_{af}$, and the critical plane with normal, where $\hat{l}_\eta, \hat{m}_\eta, \hat{n}_\eta$ - mean direction cosines, $\bar{i}, \bar{j}, \bar{k}$ - versors of axes in the 0xyz c $\bar{\eta} = \hat{l}_\eta\bar{i} + \hat{m}_\eta\bar{j} + \hat{n}_\eta\bar{k}$ ccoordinate system, is determined by normal stresses, and position of the vector $\bar{s} = \hat{l}_s\bar{i} + \hat{m}_s\bar{j} + \hat{n}_s\bar{k}$, where $\hat{l}_s, \hat{m}_s, \hat{n}_s$ are the mean direction cosines $\bar{s}$ in relation to the 0xyz axis in this plane, is determined by the direction of the dominating shear stress:

$$\max_t \{\beta W_{\eta s}(t) + W_\eta(t)\} = W_{af} .$$
(5)

The following formula for the equivalent parameter of normal strain energy density results

$$W_{eq}(t) = 0.5\beta \cdot \tau_{\eta s}(t)\varepsilon_{\eta s}(t)sgn[\tau_{\eta s}(t),\varepsilon_{\eta s}(t)] + 0.5\sigma_\eta(t)\varepsilon_\eta(t)sgn[\sigma_\eta(t),\varepsilon_\eta(t)],$$
(6)

where:

$$\sigma_\eta(t) = \hat{l}_\eta^2\sigma_{xx}(t) + \hat{m}_\eta^2\sigma_{yy}(t) + \hat{n}_\eta^2\sigma_{zz}(t) + 2\hat{l}_\eta\hat{m}_\eta\sigma_{xy}(t) + +2\hat{l}_\eta\hat{n}_\eta\sigma_{xz}(t) + 2\hat{m}_\eta\hat{n}_\eta\sigma_{yz}(t),$$
(7)

$$\varepsilon_\eta(t) = \hat{l}_\eta^2\varepsilon_{xx}(t) + \hat{m}_\eta^2\varepsilon_{yy}(t) + \hat{n}_\eta^2\varepsilon_{zz}(t) + 2\hat{l}_\eta\hat{m}_\eta\varepsilon_{xy}(t) + 2\hat{l}_\eta\hat{n}_\eta\varepsilon_{xz}(t) + 2\hat{m}_\eta\hat{n}_\eta\varepsilon_{yz}(t),$$
(8)

$$\tau_{\eta s}(t) = \hat{l}_\eta\hat{l}_s\sigma_{xx}(t) + \hat{m}_\eta\hat{m}_s\sigma_{yy}(t) + \hat{n}_\eta\hat{n}_s\sigma_{zz}(t) + 2\hat{l}_\eta\hat{m}_s\sigma_{xy}(t) + 2\hat{l}_\eta\hat{n}_s\sigma_{xz}(t) + 2\hat{m}_\eta\hat{n}_s\sigma_{yz}(t),$$
(9)

$$\varepsilon_{\eta s}(t) = \hat{l}_\eta \hat{l}_s \varepsilon_{xx}(t) + \hat{m}_\eta \hat{m}_s \varepsilon_{yy}(t) + \hat{n}_\eta \hat{n}_s \varepsilon_{zz}(t) + 2\hat{l}_\eta \hat{m}_s \varepsilon_{xy}(t) + 2\hat{l}_\eta \hat{n}_s \varepsilon_{xz}(t) + 2\hat{m}_\eta \hat{n}_s \varepsilon_{yz}(t). \quad (10)$$

Modelling of local distributions of strains and stresses

When a notch occurs, it is important to know stress distribution in the bar section. This distribution is linear only in smooth bars under elastic strains and the stress gradient is constant. In a notched bar subjected to bending, normal stress distribution is nonlinear. Weiss [4] analysed tension of the notched element and he proposed a simple equation for distribution of pseudoelastic stresses, assuming the elastic body model under tension. From analyses [5] it appears that the following equations can be applied for modelling of distributions of pseudoelastic stresses in the notched bar sections under bending:

$$\sigma_{azz}(x,y) = \sigma_{an} k_{tb} \frac{x}{R} \sqrt{\frac{\rho_o a_x}{\rho_o a_x + 6R(a_x - x)}} , \qquad \sigma_{ayy}(x,y) = \sigma_{an} \nu k_{tb} \frac{x}{R} \sqrt{\frac{\rho_o a_x}{\rho_o a_x + 6R(a_x - x)}} , \qquad (11)$$

$$\sigma_{axx}(x,y) = \sigma_{an} \frac{x}{R} \left[\frac{k_{tb\sigma}}{\nu} \sqrt{\frac{\rho_o a_x}{\rho_o a_x + 6R(a_x - x)}} - \frac{k_{tb\varepsilon}}{\nu} \frac{\rho_o a_x}{\rho_o a_x + 2R(a_x - x)} \right.$$

$$\left. - \nu k_{tb\sigma} \frac{x}{R} \sqrt{\frac{\rho_o a_x}{\rho_o a_x + 2R(a_x - x)}} \right] \qquad (12)$$

According to Agnithorii [6], distributions of the stresses coming from torsion can be written as

$$\tau_{ayz}(x,y) = \tau_{azy}(r) = \tau_{an} k_{tt} \frac{r}{R} \sqrt{\frac{\rho_o}{2R + \rho_o - 2r}} \qquad (13)$$

where σ_{an} - nominal stress amplitude, R – maximum bar radius, $r = \sqrt{x^2 + y^2}$ - distance from the specimen axis, ρ_o - radius in the notch bottom, $a_x = \sqrt{R^2 - y^2}$ is the maximum range of changes x under the given y, $k_{tb\sigma}$ - stress coefficient of the notch action under bending, $k_{tb\varepsilon}$ - strain coefficient of the notch action under bending, k_{tt} - theoretical coefficient of the notch action under torsion.

Having distributions of pseudoelastic normal stresses $\sigma_{azz}(x,y)$ and shear stresses $\tau_{az\varphi}(x,y)$, we can determine distributions of pseudoelastic strain energy densities

- under bending $\qquad W_a^e(x,y) = \dfrac{\sigma_{azz}^2(x,y)}{2E}$, $\qquad (14)$

- under torsion, $\qquad W_a^e(x,y) = \dfrac{\tau_{az\varphi}^2(x,y)}{4G} + d\dfrac{\tau_{a\varphi z}^2(x,y)}{4G}$, $\qquad (15)$

when the fatigue fracture plane is perpendicular to the bar axis $\left(\hat{\alpha}_\eta = 0 \right)$, d=0 and when the fatigue fracture plane is inclined at $\hat{\alpha}_\eta = \pi/4$, d=1 and there is a relation between circumferential stresses in the polar and rectangular coordinates $\tau_{az\varphi}^2 = \tau_{azy}^2 + \tau_{azx}^2$.

Determination of non-local strain energy densities in the critical plane

The first non-local theories were applied for description of linearly elastic heterogeneous materials and for solving problems of crack mechanics where high stress gradients were

observed near sharp cracks. Theories of non-local elasticity allows for removal of high stresses and to obtain regular finite stress in the all area. Borino *et al.* [7] presented the non-local theory of symmetric damages, describing a change of non-local damage $\overline{d}$. They expressed the weight function W(x,y) using the delta function δ(x,y).

The stress gradient occurs under bending, torsion and any loading of notched elements. From comparison of fatigue lives under tension-compression and alternating bending [8] it appears that the material can be loaded by a greater amplitude under bending in comparison with tension-compression. Under tension, damages are accumulated in all the material volume, and under bending they are accumulated mainly in external layers of the material volume. Thus, a special attention should be paid to non-local approach to fatigue life determination, especially when the stress gradients occur. Such approach has been used for a long time by determination of non-local stresses $\overline{\sigma}$ and strains $\overline{\varepsilon}$.

Saouridis and Mazars [9] were engaged in determination of the non-local strains. They paid attention to necessity of non-local approach to crack mechanics in the volume where stress and strain gradients occurred. The authors proposed to determine the matrix of non-local strains $\overline{\varepsilon}_{SM}$ using matrices of local strains $\widetilde{\varepsilon}$ in a certain sphere volume V with a diameter 2L. Stabler and Baker [10] proposed a mathematical model applied for damage mechanics description, being an extended version of the Mazars non-local model where the damage development was analysed with use of definition of the damage surface.

McDowell [11] used the non-local strain $\overline{\varepsilon}_{MD}(y)$ in the point, averaged in volume V_o on the basis of local strain distribution at the distance Z from the chosen point at the axis y. Also Borst *et al.* [12] proposed a damage model using the non-local equivalent strain. Belytschko and Lasry [13] used non-local strain for analysis of weakening materials around the chosen point. Also Simone *et al.* [14] applied the non-local equivalent strain in crack mechanics. They determined the non-local equivalent strain in the crack tip and at a certain distance from it. As Peerlings *et al.* [15], they showed that non-local equivalent strain $\widetilde{\varepsilon}$ had the finite maximum value $\widetilde{\varepsilon}_0$ in the crack tip. According to the non-local elasticity model proposed by Eringen *et al.* [16], the stress field value in the crack tip is finite and similar as in the model of non-local damages considered by Simone. In this case, the maximum occurs at the same distance from the crack tip along the cracking line. Droz and Bazant [17] applied the non-local vector of derivatives after plastic strains for evaluation of damage propagation.

In order to determine stresses and electric fields near the crack tip for piezoelectric materials, Zhou *et al.* [18] developed theory of electroelastic crack mechanics and non-local theory used for modelling and analysis of cracks in such materials. This theory enabled to eliminate difficulties at determination of stresses and displacements near crack tips.

Under complex stress state, Neuber [19] proposed to determine non-local stress at the point y_o using the equivalent stress σ_v by its averaging at a certain length ρ^*, which depends on the material and the yield point. Dyskin [20] determined non-local microscopic plastic stresses in the cracking zone from a certain length d, along which plasticity occurred. Dorgan and Voyiadjis [21] tested application of non-local theory for description of non-local damage behaviour and plastic hardening. They proposed the equation expressing non-local plasticity as the function of potential plasticity. The occurring gradients allow for finding non-local behaviour of the materials and understand collective behaviour of defects, e.g. displacements. The applied non-local theory assumes that in the point x, non-local value $\overline{A}$ being a variable of the internal local state A, is expressed as the weight function in volume V at a small distance from x. Taylor [22] proposed to determine the non-local stress range at the crack length „a" from the local stress ranges. Seweryn and Mróz [23] determined the non-local normal and shear stress in the plane of a characteristic dimension d_o connected with the

critical value of the stress intensity factor according to I K_{IC}. Qilafku [24] used the non-local equivalent stress and joined it with the stress gradient. Finally, the authors proposed to determine the non-local normal stress at a certain length x_{eff} The obtained non-local stress can be applied for determination of the fatigue notch coefficient K_f. Sonsino *et al.* [25] introduced the normalized coefficient of the stress gradient, connected with the radius in the notch bottom ρ_o at transition from the less diameter (d) the greater one (D). The strain reaching 90% of the maximum value on the surface is the calculation strain. Its value is determined at a distance from the surface. Another approach to stress gradient usability was presented by Papadopoulos and Panoskaltsis [26]. They introduced a coefficient dependent on the stress gradients in three directions to their multiaxial fatigue criterion based on the maximum amplitude of shear stress τ_{max} and the maximum hydrostatic pressure p_{max}. Bomas *et al.* [27] used the integral from stress at a certain volume V. According to the authors, the notch effect is a particular case of the dimension scale effect. Morel ad Palin-Luc [28] proposed to average the local normal and shear stresses in a certain volume in the case of determination of fatigue life under the complex stress state for a high cycle regime. Filippini [29] determined the stress gradient coefficients χ for different loading cases and element shapes. Qylafku *et al.* [30] averaged the stress in a certain volume in a chosen plane or at a certain length. Next, the authors use this non-local stress for determination of the notch effect. Sonsino *et al.* [31] averaged the stress at a certain length in order to determine a fictious radius in the notch bottom, enabling determination of the notch coefficient based on the theoretical notch coefficient.

Lemaitre and Chaboche [32] assumed that density of the released energy in the material was the sum of the elastic and plastic parts, $\psi^e(\varepsilon^e, \overline{d})$ and $\psi^p(\overline{r}, \overline{d})$, where ε^e was the elastic strain, $\overline{d}$ - non-local damage, and $\overline{r}$ - non-local equivalent plastic strain. Vree *et al.* [33] applied two different non-local models for analysis of defect behaviour in structures made of brittle materials. The authors considered the above models in order to use them for crack simulation and their location.

Palin-Luc and Lasserre [34] proposed a criterion based on the strain specific energy averaged at a cycle T and along the material volume V^* in the case of the stress gradient occurrence. The main source of non-locality in crack mechanics is interaction between the adjacent microcracks. Using the theory based on the system of microcrack interaction, Pijaudier-Cabot and Bazant [35] considered the model of non-local damages. The calculation argument for idea of non-local damages is necessity of reduction of damage location to areas with non-zero volume. The authors applied (like Saanouni *et al.* [36]), a non-local damage using crack energy for an area unit G_f and energy dissipated in the elementary volume W_s.

Different authors proposed different weight functions: linear, in form of the Gaussian function, modified Gaussian functions, Dirac etc. Averaging is performed at the interval or in the plane or in a certain volume.

From the above review it results that in literature we can meet non-local stresses $\overline{\sigma}$, strains $\overline{\varepsilon}$, damage functions $\overline{D}$, and sometimes their derivatives. We can expect that the non-local methods can be efficiently used also in the case of the strain energy density parameter in the critical plane. Since the fatigue cracking occurs on a certain area, introduction of the non-local strain energy strain energy parameter $\overline{W}$ in the critical plane seems to be possible.

$$\overline{W}_{eq}(t) = \frac{1}{S_f(t)} \int\limits_{S_f(t)} W_{eq}(x, y, t) dS \ , \tag{16}$$

The integration area $S_f(x,y,t)$ is this part of the area $S(x,y)$ of the critical plane where the following inequality is valid $\left|W_{eq}(x,y,t)\right| \geq W_{min}$.

The area $S_f(x,y,t)$ includes all the points of the area $S(x,y)$, in which moduli of history of the strain energy density parameter $W_{eq}(t)$ are equal to the level W_{min} or greater.

The minimum strain energy density parameter is defined as $W_{min} = \dfrac{(\sigma *_{af})^2}{2E}$, where $\sigma *_{af} = c\sigma_{af}$ is the limit stress below which - as it is assumed − fatigue damages do not accumulate and which can be related to the fatigue limit.

The coefficient c is dependent on the material and it can be determined from the fatigue test results obtained under constant-amplitude tension − compression.

The non-local strain energy density parameter in a given moment can be expressed as

$$\overline{W}_{eq}(t) = \begin{cases} \dfrac{1}{S_f(t)} \int_{S_f(t)} W_{eq}(x,y,t)dS & \text{when} \quad W_{eqmax}(t)>0 \quad \text{and} \quad W_{eqmax}(t)>W_{min} \\ W_{eqmax}(t) & \text{when} \quad -W_{min}<W_{eqmax}(t)<(t)W_{min} \\ \dfrac{1}{S_f(t)} \int_{S_f(t)} W_{eq}(x,y,t)dS & \text{when} \quad W_{eqmax}(t)<0 \quad \text{and} \quad W_{eqmax}(t)<-W_{min} \end{cases} \tag{17}$$

where $W_{eqmax}(t)$ is the maximum value of the strain energy density parameter in the considered section.

Conclusions

1. A new approach to the stress gradients in circular sections of smooth and notched bars was presented. The considered bars were subjected to combined tension with torsion and the new approach uses the non-local equivalent strain energy density parameter in the critical plane. It was efficiently introduced to the algorithm for calculations of fatigue life under random loading.
2. The equivalent strain energy density parameter in the critical plane superposes the normal strain energy from bending and the shear strain energy from torsion and includes signs of strain and stresses, so under random loading with zero expected values we obtain the centred stochastic process, histories of which can be schematised by the widely known algorithms of cycle counting without any undesirable effects.
3. For simplification of determination of non-local strains and stresses, analytical equations were proposed. They can be applied for description of distributions of local strains and stresses in circular sections of smooth and notched bars from bending and torsion, which sufficiently approximate the calculation results obtained with the finite element method on the assumption of perfect elasticity of the material.

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