# NUMERICAL ANALYSIS OF CRACKED COMPONENTS UNDER ROLLING CONTACT LOADING

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## Abstract

The problem of a cylinder moving on a semi-plane, carrying an oblique edge crack is studied by the Weight Function (WF) method. A general formulation of the WF is proposed, from which the Green's Function (GF) giving the crack opening displacement (COD), is deduced. An iterative procedure is adopted for studying the conditions of partial crack closure. The method is applied to evaluate the influence exerted on the crack loading by the non uniform contact compliance when the cylinder crosses the crack mouth. In particular, the error induced by assuming the theoretical Hertzian nominal stress distribution is discussed by comparison with correct WF solutions and with accurate Finite Element analyses. A parametric study accounting for different friction conditions between the crack surfaces and different contact conditions between the moving cylinder and the cracked semi-plane is performed and typical $K_I$ and $K_{II}$ histories produced by the cylinder movement are evaluated.

## Introduction

Many mechanical components (e.g. gears, bearings, rail wheels,…) suffer surface damage due to contact fatigue. In many cases, the contact between the moving bodies produces globally elastic strain that locally varies as a consequence of the relative movement of the bodies in contact. The corresponding loading cycle can promote initiation and growth of oblique edge cracks, initially loaded in mixed fracture mode (I+II) [1-3].

The lack of symmetry of the fracture problem makes the analysis of the fatigue crack growth not so simple, as many evaluations of fracture parameters ($K_I$ and $K_{II}$) have to be performed for complex stress distributions. Indeed, the local high stress gradients are produced not only by the travelling contact but also by possible surface treatments, that usually induce compressive residual stresses near the surface[4]. Moreover, the possibility for the lubricant to be entrapped or pumped into the crack increases the complexity of the problem [5,6].

During a loading cycle, conditions of partial closure are usually experienced by the crack. In this case, the problem is no more linear [6,7] and the mutual forces between the crack faces strongly influence the crack configuration and the Stress Intensity Factor (SIF) values.

The Weight Function (WF) method was verified to be a very useful tool for solving this kind of problem [8-10]. The authors have recently proposed a general matrix formulation of the WF for an inclined edge crack in a semi-infinite body [10]. On this basis, an analytical formulation of the Green Function (GF) has been obtained [11], thus allowing the COD components to be determined by direct integration under a completely general loading condition. Based on the WF and GF, an iterative procedure was developed to obtain the conditions of partial crack closure generated during the loading cycle [8,12]. Starting from a first attempt of the crack configuration, this procedure modifies location and extension of the closed regions step-by-step, until compatibility and equilibrium conditions are fulfilled on the entire crack within a fixed tolerance. It is worth noting that, when WF and GF are known, the problem can be completely solved on the basis of the nominal stress distributions (normal and shear components). In the WF approach, the nominal stress is considered the stress acting along the crack line in an equivalent uncracked body. In many cases the nominal stress can be easily determined, as it is not influenced by the presence of the crack. This is the case of a point-like force moving on the semiplane. The nominal stress produced by the point like loading can be obtained analytically by assuming the Bousinnesque solution [13].

In the present paper, the WF approach is applied for studying the problem of a cylinder rolling on the surface of a semi-plane carrying an oblique edge crack. This is a more complex problem because the presence of the crack influences the distribution of pressure on the surface, and consequently the nominal stress distribution. Indeed, the crack produces a not negligible effect on the local compliance of the semiplane. When the cylinder is in contact nearby the crack mouth, the distribution of the contact pressure is no more equal to that predicted by the Hertz theory. However, an accurate estimate of the effective pressure distribution can be obtained by a Finite Element analysis in which the crack is modelled along with possible contact between the crack faces. When the correct pressure distribution is evaluated, the correct nominal stress can be obtained by superposing Boussinesque solutions in the uncracked semiplane. Eventually, FM parameters can be efficiently calculated by the usual methods based on the WF and GF. It is worth noting that the FM analysis based on the WF is theoretically correct if conducted in this way.

This correct WF approach is more efficient from a computational viewpoint as compared to the complete FE analysis including the FM parameters evaluation. Indeed, the evaluation of the effective pressure distribution does not require very refined FE models, which, on the contrary, are necessary for accurate evaluations of FM parameters (particularly in the presence of crack closure). However, if the preliminary FE analysis could be avoided, the efficiency of the calculus should be greatly increased. For this reason, an approximate approach is also considered by assuming for the loading Hertzian pressure distribution (not modified by the presence of the crack). In this case, the WF solution is approximate even though the WF is exact as the nominal stress distribution is approximate. However, it is shown that for some FM parameters, crack lengths and loading conditions the approximate solution can be used in practice as the errors are not very large.

A parametric analysis was performed by considering several ratios between contact extension and crack length, and two friction conditions between crack edges. For these cases the complete FE solutions are compared to the correct and approximate solutions based on the WF.

## Problem definition

A scheme of the problem is reported in fig.1: a semi-infinite body carrying an oblique edge crack having length $a$ inclined by an angle $\theta$ with respect to the normal at the surface. A cylinder of radius $R$ is moving on the surface and the quantity $L$ indicates the position of the cylinder axis with reference to the origin located on the crack mouth. $W$ indicates the contact force per unit width applied in the normal direction. The Hertzian pressure distribution (fig. 1a) is characterised by the maximum $p_{max}$, and the contact half-width $b$, quantities related to the load $W$, the cylinder radius $R$ and the elastic constants. The quantities $p_{max}$, and $b$, completely defining the contact when the cylinder is far from the crack mouth, were used as scaling factors by which stresses and lengths were normalised respectively. When the cylinder approaches the crack mouth, the pressure distribution is modified as schematically represented in fig. 1b. This distribution depends on the presence of the crack and on the contact between the crack surfaces and it cannot be evaluated by simple analytical methods.

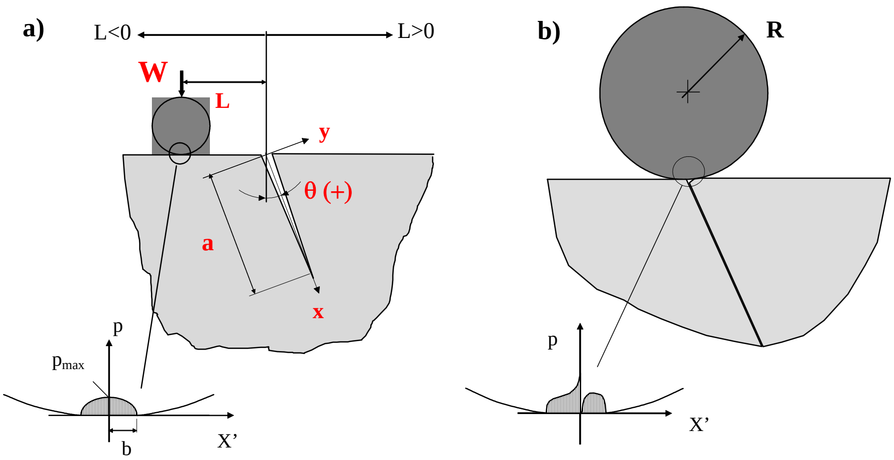


FIGURE 1. Scheme of the problem

## Analysis of the crack under the effective contact conditions

A FE analysis was carried out, by using the Ansys 5.7® code, to evaluate the effective contact pressure distributions for different positions of the cylinder. A two dimensional model was built up using about $2 \cdot 10^5$ four-nodes quadrilateral solid elements (stiff42). The contact between the cylinder and the semi-plane and between the crack faces was modeled by point to surface gap elements (contact48). The mesh was refined particularly at the crack mouth, in order to obtain an accurate reproduction of the local contact pressure distribution. A global picture of the model and a detail at the crack mouth is shown in fig. 2.

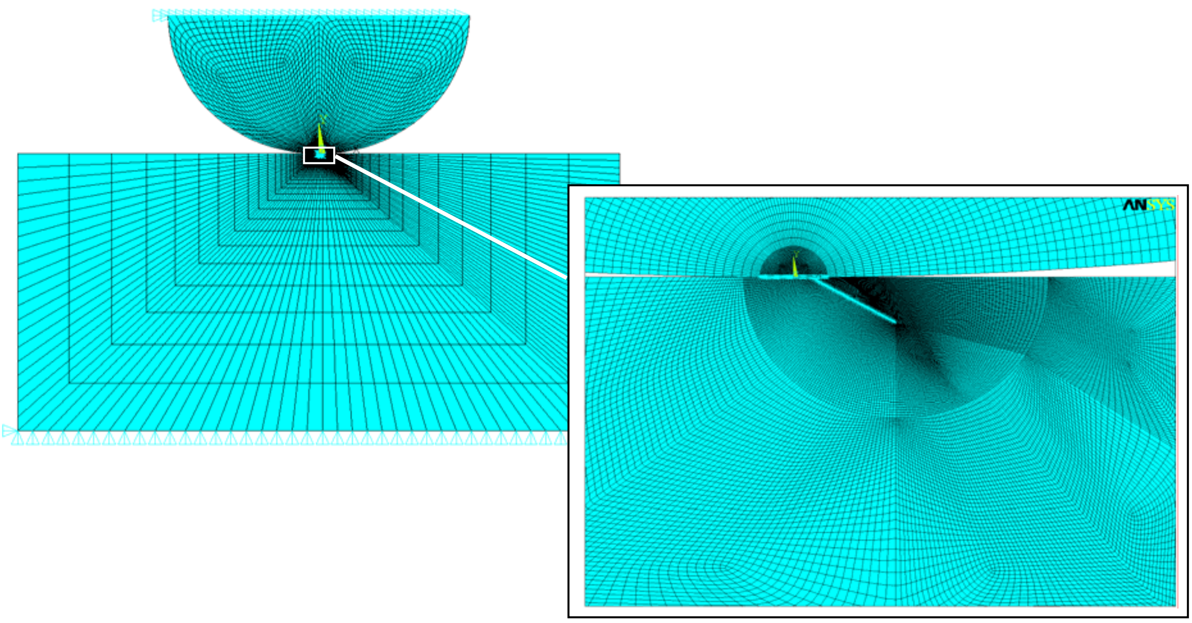


FIGURE 2. FE model built up for the analysis

The analysis was performed for a crack inclined by an angle $\theta = +60°$, and several $a/b$ ratios were considered. Since the friction forces between the crack faces counteract the sliding, a friction coefficients equal to zero produces the maximum effect on the contact pressure distortion. For this reason the frictionless condition between the crack faces was adopted for the analysis. The results showed that when the distance $|L|$ of the cylinder from the crack mouth is greater than 2*b*, the pressure distribution between cylinder and semi-plane is not influenced by the presence of the crack and the Hertzian solution holds. The model was used also for a complete FM solution including the evaluation of COD components, of forces arising between crack faces in the closed regions and of the SIFs. These values were used for comparison.In fig. 3, pressure distributions evaluated for $a/b$ =12 are shown for several cylinder positions. It can be observed that, for $|L|$=2*b*, the pressure distribution is already regular symmetric and nearly equal to the Hertzian solution. In fact, the perturbation of the pressure distribution is significant when the contact spreads across the crack mouth. In this case, the stiffer region ($X$<0 in the examined case) tends to transfer a higher fraction of the load $W$ and a pressure peak arises in this more stiff side of the crack mouth.

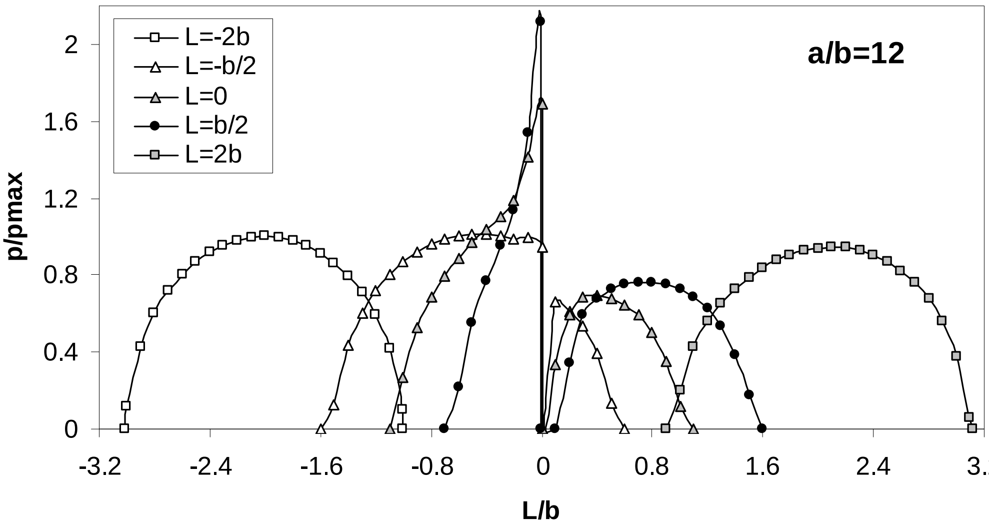


FIGURE 3. Contact pressure distribution originated by different cylinder position with respect to the crack mouth

## WF analyses

***Approximate solution with the Hertzian contact pressure distribution***

By neglecting the effect of the crack on the contact pressure, the Hertzian distribution (fig. 1a) with half-width $b$ and maximum $p_{max}$ was assumed in the approximate evaluation for any position $L$ of the cylinder. The nominal stress distribution in the uncracked body was calculated by the analytical solution of Boussinesque [12] for a point like force. To this purpose, the pressure distribution was reduced to an equivalent system of 100 point like forces in bell-like distribution over the interval $[L-b, L+b]$. A parametric study was carried out to evaluate the influence produced by the parameters on the crack behaviour. In particular, the following parameters were considered: crack inclination ($\theta$), ratio between crack length and contact half width ($a/b$), position of the cylinder ($L/a$), friction between crack faces ($\mu$) and friction coefficient in the contact between the cylinder and semi-plane ($\phi$). An extensive discussion can be found in [13,14], in the following, some results are reported for comparison. In fig. 4, the SIF components $K_{\mathrm{I}}$ and $K_{\mathrm{II}}$ are plotted versus the position of the cylinder *L/a*, for θ=40°, μ=0.1, ϕ=0 and different values of the *a/b* ratios. Dimensionless SIFs values are normalized by the factor: $K_o = W \cdot \sqrt{\dfrac{\pi}{a}}$.

In this example can be verified that, as in any other similar conditions, the crack experiences mixed mode loading only when the cylinder is near the crack mouth (0<*L/a*<0.4). When the cylinder is external to that interval, the crack is subjected to pure mode II. Moreover, mode II seems to quantitatively prevail in the whole *L/a* domain.

By increasing the friction coefficient between the crack faces, lower SIF values were predicted. However, the most important consequence of increasing μ, is that different SIFs histories are produced for opposite versus of the relative movement. This effect can be observed in fig. 5, where $K_I$ vs. $K_{II}$ loci, due to rightward (+) and leftward (-) cylinder movement are reported. In general, a crack with positive θ, as in fig. 1, experiences more intense SIF cycles when the cylinder moves rightward. This behaviour was observed for every crack inclination (θ≠0°), and the difference between rightward and leftward *K*-cycles increases when either the crack inclination $|\theta|$ or the friction coefficient are increased.

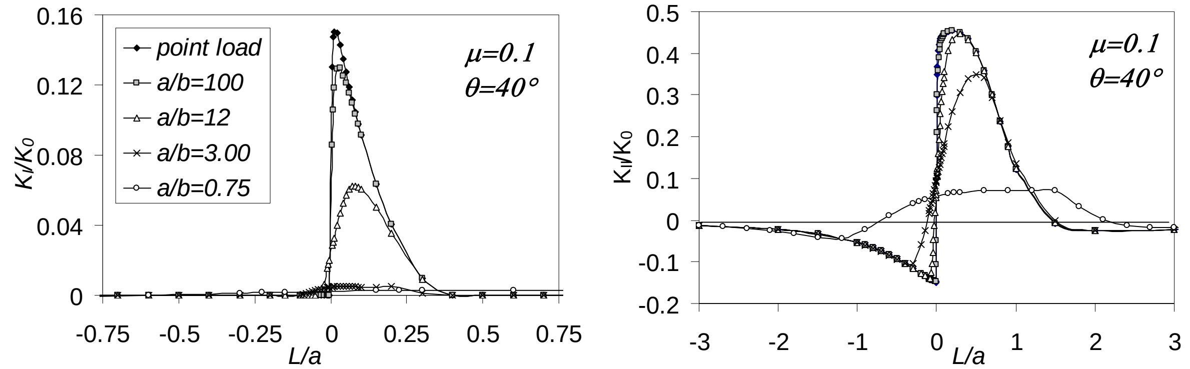


FIGURE 4. $K_{\mathrm{I}}$ and $K_{\mathrm{II}}$ vs. *L/a* for different *a/b* ratios

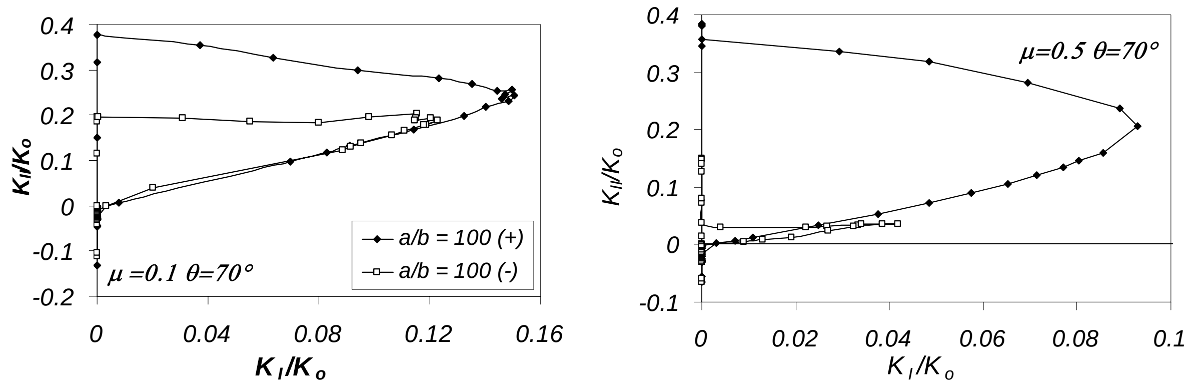


FIGURE 5. $K_{\mathrm{II}}$ vs. $K_{\mathrm{I}}$ loci for opposite versus of the cylinder movement: rightward (+) leftward (-). (a) moderate friction μ=0.1, (b) high friction μ=0.5.

As a crack inclined by an angle $+\theta$ with the cylinder moving leftward is completely equivalent to a crack inclined by an angle $-\theta$ with the cylinder moving rightward, the previous results can be used to study the effect of positive and negative crack inclination with respect to the cylinder movement direction. The dependence of the ranges $\Delta K_{\mathrm{I}}$ and $\Delta K_{\mathrm{II}}$ on the inclination angle, are reported in fig. 6 for rightward movement over the whole interval -∞<*L/a*<+∞. Figure 6 demonstrates that the most critical conditions are experienced by the cracks with positive inclination (fig. 1).

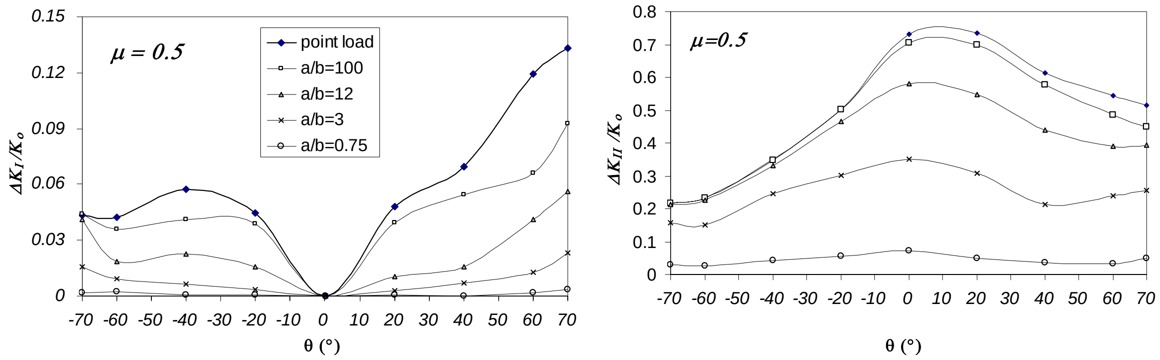


FIGURE 6. $\Delta K_I$ and $\Delta K_{II}$ vs. $\theta$ for $\mu$=0.5

***WF solution with the correct pressure distribution and comparison***

The WF was used to determine the SIFs and the COD values considering the effective (FE evaluated) pressure distributions. The nominal stress within the uncracked body was determined with proper discretization of the FE pressure distribution using 100 point-forces , the Boussinesque solution and the superposition principle. As the partial crack closure and the corresponding pressure distribution and COD components along the crack faces were evaluated also by the FE, a comparison with the correct WF results was carried out.

In figs. 7 and 8, contact pressure and COD (normalised by the factors: $p_0 = \dfrac{2W}{\pi a}$, $v_0 = \dfrac{a \cdot p_0}{E}$ respectively) are compared. A quite good agreement between the results obtained by the two methods can be verified. This was considered a further confirmation of the validity of the WF and of the iterative algorithm proposed in [8] for determining the conditions of partial crack closure.

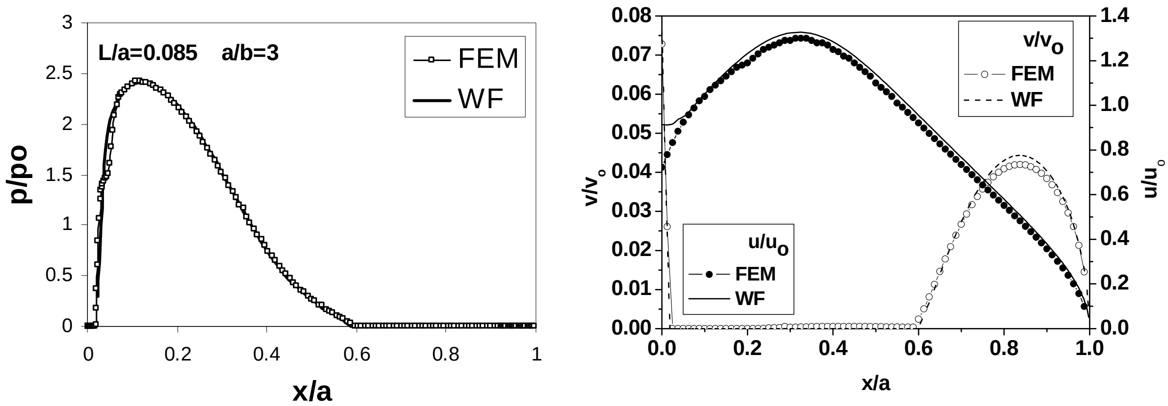


FIGURE 7. Comparison between pressure distributions (a) and COD components (b) obtained by the FEM and the correct WF methods

The analysis was then addressed to compare the SIFs values, obtained by means of the two WF based approaches. An example of the comparison is reported in fig. 8. As expected, the differences between the two solutions are significant, particularly for $K_I$ , only within a limited interval of cylinder positions near the crack mouth. The approximate solution based on the Hertzian pressure distribution tends to overestimate both $K_I$ and $K_{II}$. This result is consistent with the observation that the stiffer side of the plane body tends to withstand a portion of the load $W$ larger than that predicted by the Hertzian solution. Indeed, the solution of a point-like load travelling on the surface [14], demonstrated that when the force is in the stiffer portion lower SIFs values are produced. As a consequence, the pressure peak located in the stiffer portion is less effective on producing the SIF values and, consequently, the approximate approach is conservative. Figure 8 refers to a specific value of the *a/b* ratio, however similar trends were obtained for any other analysed *a/b* ratio.

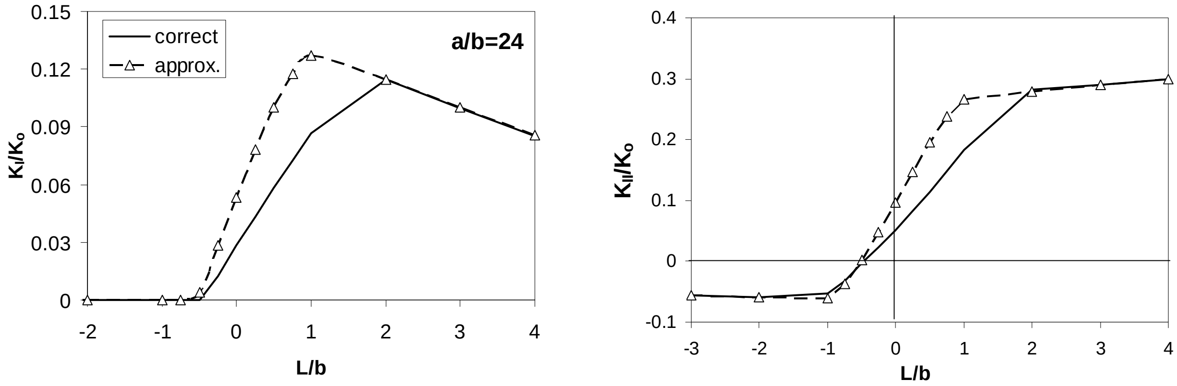


FIGURE 8. $K_{I\,(correct)}$ and $K_{I\,(approx)}$ (a), $K_{II\,(correct)}$ and $K_{II\,(approx)}$ (b) as a function of the cylinder distance from the crack mouth

The relative errors introduced by the Hertzian approximation in the SIF amplitudes ($\Delta K_I$ e $\Delta K_{II}$ ) for a complete loading cycle $-\infty<L<+\infty$, are shown in figure 9. The errors reach their maximum values when the dimension of the contact area is comparable with the crack length, whereas they become negligible for either very small cracks or concentrated loads. Indeed, when $a/b>50$ the solution is nearly equal to that produced by the point-like force, i.e. not influenced by the compliance difference. On the other hand, the errors for very short cracks are small because the pressure distribution spreads over a large portion of the surface as compared to *a*.

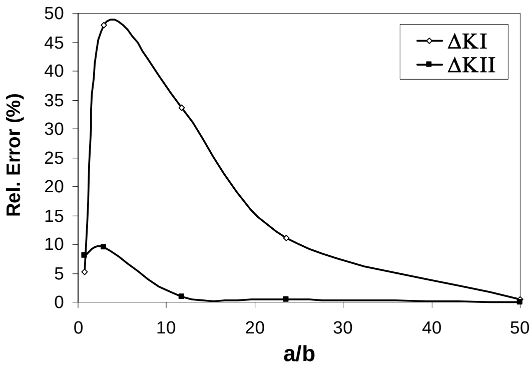


FIGURE 9. Relative errors in the $\Delta K_I$ and $\Delta K_{II}$ evaluation as a function of *a/b* ratio

An estimate of the effective SIF history at the crack tip can be obtained by defining an equivalent SIF, and evaluating the corresponding range $\Delta K_{eq} = K_{max} - K_{min}$. It is worth noting that both $K_I$ and $K_{II}$ are positive in the region of mixed mode of fracture, therefore $K_{max} = \max\sqrt{K_I^{\,2} + K_{II}^{\,2}}$ , whereas $K_{min}=K_{IImin}$ as the minimum (negative) value is obtained under pure mode II. In fig. 10 the $\Delta K_{eq}$ values are plotted vs. the *a/b* ratio for the two pressure distributions. It is interesting to observe that considering the range of equivalent SIF, the error introduced by the Hertzian approximation is quite small and on the conservative side.

Several analyses carried out to study the effect of the friction between the crack faces confirmed that the difference between the two solution become smaller when the friction is increased.

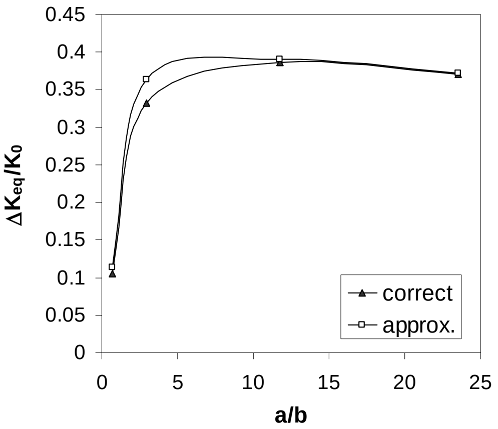


FIGURE 10. $\Delta K_{eq}$ vs. a/b ratio obtained by considering the two WF approaches

## Conclusions

The WF method was applied to analyse the problem of a cylinder travelling on the surface of a semi-plane carrying an inclined crack. The influence of the non-symmetric compliance of the cracked body was studied. The effective contact between cylinder and semi plane was evaluated by FEM and this distribution was employed as the input for a correct FM evaluation based on the WF. An approximate and very computationally efficient approximate approach was also applied by assuming for the contact pressure the Hertz solution.

The correct WF solution was compared with the result of an accurate FE model and a very good agreement was found in SIFs, COD and contact pressure between the crack faces. By the comparison between the correct and approximate WF approaches it was shown that the simple method produced a reasonable overestimate of the crack tip stress cycles. This allows the analyser to apply the approximate approach in the analyses involving several parameters or when the prediction of fatigue crack growth requires a lot of SIF evaluations.

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