MODELLING OF COHESIVE STRESSES IN A PLASTIC ZONE AROUND THE CRACK TIP IN A STRAIN-HARDENING MATERIAL

Dragan Pustaić[1] and Boris Štok[2]
[1] University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture
Ivana Lučića 5, Zagreb, Croatia, e-mail: dragan.pustaic@fsb.hr
[2] University of Ljubljana, Faculty of Mechanical Engineering ,
Aškerčeva 6, Ljubljana, Slovenia, e-mail: boris.stok@fs.uni-lj.si

Abstract

The present article is an extension of the authors' earlier publications, e.g. Pustaić and Štok [1-3], in which Dugdale strip yield zone model was used for modeling some elastic-plastic fracture mechanics parameters. In those investigations the authors had kept the assumption about constant cohesive stresses within the yield zone and the assumption about elastic-perfectly plastic model of a material as well. In this paper we wish to define a model, which will be closer to the real elastic-plastic state of a material within a yield zone. An assumption of variable cohesive stresses within a yield zone is introduced. In this way we would like to get a model for describing plastic yielding around a crack tip in a strain-hardening material. On the base of the Dugdale strip yield model for the strain-hardening materials the formulae for calculating the magnitude of a plastic zone and the crack tip opening displacement (CTOD) were derived. The calculation was performed for several different values of strain hardening exponent n and two different Ramberg-Osgood coefficients α .

Introduction

A thin infinite center cracked plate (CCT) with an embedded straight crack of the length $2a$ is considered. The plate is loaded by a monotonically increasing tensile stress $\sigma_{yy}^{\infty} = \sigma_{\infty}$ in a direction perpendicular to the crack plane, Fig. 1. The plate is in the state of plane stress determined by the components $\sigma_{xx}(x,y)$, $\sigma_{yy}(x,y)$ and $\sigma_{xy}(x,y)$ of the stress tensor. It is assumed that the plate is made of a ductile material, therefore the plastic zones around crack tips are occured. Our aim is to investigate the magnitude r_{p} of plastic zones, as well as the magnitude of the crack tip opening displacement δ_{t} (CTOD) in a plate made of strain hardening material. One of the first models by which it was possible to determine these parameters was the Dugdale strip yield zone model, Dugdale [4]. Although it simplified the real physical picture of occurrences around a crack tip, it was very successfully applied for solving many engineering problems of elastic-plastic fracture mechanics. This model describes a yield zone as a narrow strip band, extending ahead from the crack tip and lying in the direction of a crack plane. Accordingly, he postulated the existence of an imaginary elastic crack composed of a physical blunt crack of length $2a$ and a supplementary cracked zone extended ahead at both tips of the virgin sharp crack for a distance r_{p}, the length of the supplementary crack being equal to the length of the plastic zone around the crack tip, Fig. 1. The elastic response due to the external loading is superposed by the elastic response due to the application of the cohesive stresses. Because of the assumed elastic approach both responses are characterized by the stress singularity, their intensities being given by the stress intensity factors (SIF) K_{ext} and K_{coh}, respectively. But, since in reality the stress singularity, introduced by the elastic approach, does not occur due to plastic yielding it has to be cancelled by imposing

$$K\left(a + r_{\text{p}}\right) = K_{\text{ext}}\left(a + r_{\text{p}}\right) + K_{\text{coh}}\left(a + r_{\text{p}}\right) = 0 \ . \tag{1}$$

The original Dugdale strip yield zone model was implied the application of the Tresca yield criterion and an *elastic-perfectly plastic* material model of a plate. It assumed *constant cohesive stresses* in the whole yielding zone whose magnitude is equal to the tensile yield stress of a material, i. e. $\sigma_{\text{Y}} = \sigma_0$.

In the present paper we wish to define a new micro mechanical model, which will better describe the real elastic-plastic state of a material within a yield zone. If we wish to describe a strain-hardening effect of a material we must introduce an assumption about *variable cohesive stresses* within a yield zone. There are several papers discussing the application of the Dugdale model by strain-hardening materials, for example Hoffman and Seeger [5], Chen *et al*. [6], Neimitz [7] and so on. Hoffman and Seeger defined the cohesive stress distribution on the plastic zone as a function of the plastic zone length r_{p}, taking into account the strain hardening exponent *n* of a material. The cohesive stress distribution should be clearly and reliably defined if we wish to obtain an accurate CTOD value. Hoffman and Seeger pointed out in their article that: "even if the exact stress distribution is taken as $p(x)$, the calculated CTOD still has large discrepancies from the expected CTOD". Chen *at al*. [6] suggest that we could use the relation between *J*-integral and CTOD (J / CTOD = const.) as a judgment about reliability of assumed cohesive stress distribution.

Stresses and displacements in the direction of the crack plane

Considering the fact that due to symmetry the shear stress vanishes when $\operatorname{Im} z = 0$, the governing equations in the theory of a complex variable $(z = x + \text{i}y)$ of the mathematical theory of elasticity, Muskhelishvili [8], can be expressed in terms of one single Westergaard function $Z(z)$, as derived by Sih [9]

$$\begin{aligned} &\sigma_{xx} + \sigma_{yy} = Z(z) + \overline{Z(z)} \\ &\sigma_{yy} - \sigma_{xx} + 2\text{i}\sigma_{xy} = 2A - (z - \bar{z})Z'(z) \\ &2\mu(u + \text{i}v) = \frac{1}{2}\left[\frac{3-\nu}{1+\nu}\int Z(z)\text{d}z - \int \overline{Z(z)}\text{d}\bar{z} - (z - \bar{z})\overline{Z(z)}\right] - A\bar{z} \,, \end{aligned} \tag{2}$$

where *A* is a real constant, μ is the shear modulus and ν is Poisson's ratio. On the *x*-axis, the condition $z - \bar{z} = 0$ is fulfilled, so from second equation in the system (2) we get

$$\sigma_{yy}(x,0) - \sigma_{xx}(x,0) = 2A \,. \tag{3}$$

Normal stresses $\sigma_{xx}(x,0)$ and $\sigma_{yy}(x,0)$ will be at the same time *principal stresses*. It follows from eq. (3) that difference between principal stresses of the axis *x* will be *constant* and equal to the double value of the real constant *A*. Its magnitude can be determined from boundary conditions in infinity and in present example according to Fig. 1 it amounts $A = \sigma_{yy}^{\infty}/2 = \sigma_{\infty}/2$, where $\sigma_{\infty} > 0$.

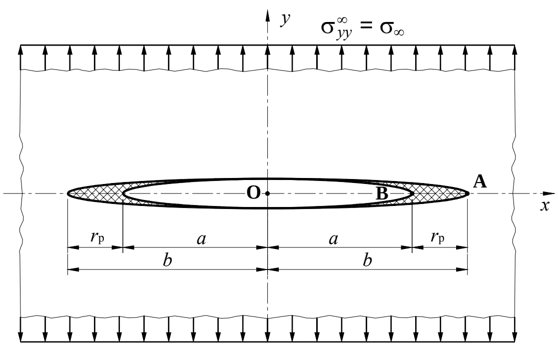

FIGURE 1. Center cracked specimen subjected to remote loading.

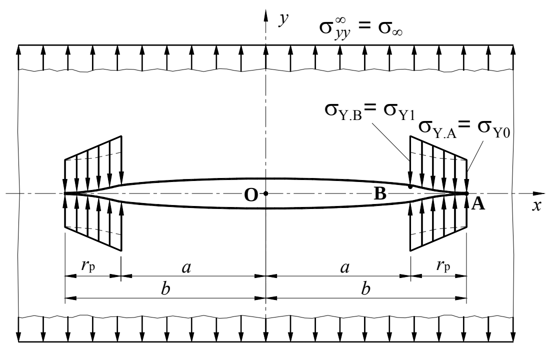

FIGURE 2. Linear distribution of cohesive stress in the yielded zone.

Tresca's and Mises' yield criteria

The cohesive zones around the crack tips are considered as narrow bands of yielded material extending from the tips of the physical blunt crack in the direction of the crack plane, Fig. 1. First plastic strains occur in the tips of the physical crack when the *equivalent stress* $\bar{\sigma}$ becomes equal to the initial yield stress σ_{Y0} of the material under uniaxial tension. The occurrence of yielding in the points around the crack tips, in those points lying on the straight line of the crack plane, depends upon principal stresses $\sigma_{xx}(x,0)$ and $\sigma_{yy}(x,0)$. The equivalent stress $\bar{\sigma}$ can be determined from *Tresca's yield criterion* or from *Mises' criterion*. Tresca's criterion can be written in the analytical form as

$$\left[(\sigma_{xx}-\sigma_{yy})^2-\sigma_{\mathrm{Y0}}^2\right]\left[\sigma_{xx}^2-\sigma_{\mathrm{Y0}}^2\right]\left[\sigma_{yy}^2-\sigma_{\mathrm{Y0}}^2\right]=0\,, \tag{4}$$

and Mises' as

$$\sigma_{xx}^2-\sigma_{xx}\,\sigma_{yy}+\sigma_{yy}^2=\sigma_{\mathrm{Y0}}^2\,. \tag{5}$$

If the stress $\sigma_{xx}(x,0)$ is eliminated from the systems of equations (3) and (5), we get a quadratic equation from which we can determine the cohesive stress $\sigma_{yy}(x,0)=\sigma_{\mathrm{Y}}(x,0)$. This equation is expressed as

$$\sigma_{yy}^2(x,0)-\sigma_{\infty}\sigma_{yy}(x,0)-\left(\sigma_{\mathrm{Y0}}^2-\sigma_{\infty}^2\right)=0\,. \tag{6}$$

The solution of these equation gives the magnitude of cohesive stress in that point of cohesive zone in which Mises' yield criterion is satisfied. This will be the point on the boundary of the cohesive zone, i.e. on the border between plastic and elastic range.

Modeling of cohesive stresses in the plastic zone in the case of isotropic hardening of a material

It is assumed that the plate material has the property of *isotropic hardening*. According to the one hypotheses of hardening, the *parameter of hardening* κ is equal to the *equivalent plastic strain* $\bar{\varepsilon}_{\mathrm{p}}$, i.e. $\kappa=\bar{\varepsilon}_{\mathrm{p}}$. This hypothesis is known under the name of *strain hardening hypothesis*. In the yielded zone around the crack tip, $a\le \operatorname{Re} z=x\le b=a+r_{\mathrm{p}}$, the equivalent stress $\bar{\sigma}$ will change from a point to a point, in dependence on the level of equivalent plastic strain $\bar{\varepsilon}_{\mathrm{p}}$, and at a particular level of external loading. The equivalent stress $\bar{\sigma}$, in the range of strain hardening, will always be equal to the *current yield stress* of the material σ_{Yc}, where $\sigma_{\mathrm{Yc}} > \sigma_{Y0}$. It means that the magnitude of the *cohesive stresses* in the yielded zone will change in dependence on the level of equivalent plastic strain $\bar{\varepsilon}_{\mathrm{p}}$, i.e. $\sigma_{yy}(x,0) = \sigma_{\mathrm{Y}}(x,0) = F(\bar{\varepsilon}_{\mathrm{p}}) = f(\sigma_{\mathrm{Yc}})$. The equivalent plastic strain $\bar{\varepsilon}_{\mathrm{p}}$ must be a function of the elastic-plastic fracture mechanics parameters such as, the crack tip opening displacement δ_{t} (CTOD) and the length of the plastic zone r_{p}, i.e. $\bar{\varepsilon}_{\mathrm{p}} = f(\delta_{\mathrm{t}}, r_{\mathrm{p}})$. The similar assumptions were introduced in the papers of Chen *at al.* [6], Neimitz [7], Wnuk and Legat [10].

In the paper [6], the authors took same cohesive stress distribution on the plastic zone $p(x)$ as it was defined in the paper [5], namely

$$p(x) = \sigma_0 \left(\frac{r_{\mathrm{p}}}{x-a} \right)^{1/(n+1)} . \tag{7}$$

They adopted this expression in their paper in order to calculate the elastic-plastic fracture mechanics parameters r_{p} and δ_{t} in an infinite center cracked plate subjected to a remote tension (CCT). But, they didn't answer the question, how are the obtained results accurate? Is the expression (7) founded on the experimental observations? In his paper [11], the author shown that within the strip yield zone the cohesive stress $\sigma_{yy}(x,0)$ rises considerably above the initial yield stress σ_0 due to the geometrical constraints only. The author modified the Dugdale strip yield model in order to include the work hardening properties of a material. He adopted the constitutive equation and proposed the distribution of the elastic-plastic strains within the cohesive zone in the form

$$\varepsilon_{yy}(x,0) = \varphi \frac{\delta(x)}{\delta_{\mathrm{t}}}, \tag{8}$$

where $\delta(x)$ defines the opening of the crack faces within the strip yield zone and φ is a scaling factor which was defined in the paper [7]. The magnitude of the parameter $\delta(x)$ could be taken as suggested in the same article, namely as

$$\delta(x) = \delta_{\mathrm{t}} \left(1 - \frac{x-a}{r_{\mathrm{p}}} \right)^{4\sigma_\infty/\sigma_0} . \tag{9}$$

In the paper [7] author suggests the general formula to compute the cohesive stress within the strip yield zone σ_{SYZ}. Wnuk and Legat [10] submitted a two-parameter nonlinear function for representing the cohesive stress distribution within the cohesive zone. This function is assumed in the form (the designations are adopted those in the present paper)

$$\sigma_{yy}(x,n,\alpha) = \sigma_0 \left(\frac{x-a}{r_{\mathrm{p}}} \right)^n \cdot \exp\left[\alpha \left(1 - \frac{x-a}{r_{\mathrm{p}}} \right) \right], \qquad \text{or} \tag{10}$$

$$\sigma_{yy}(\lambda,n,\alpha) = \sigma_0 \lambda^n \cdot \exp\left[\alpha (1-\lambda) \right], \qquad \lambda = (x-a)/r_{\mathrm{p}} . \tag{11}$$

The parameters α and n are the cohesive zone parameters or the state variables. Similar result for cohesive stress distribution was obtained in the Guo's paper [12].

Magnitude of cohesive zone around the crack tip

Non-linear variation of cohesive stress in the yielded zone

In this section we shall repeat and critically discuss some results concerning on the Dugdale strip yield model for strain hardening materials, as they have been given in the article [6]. The cohesive stress distribution within the plastic zone is given by expression (7). The stress intensity factor, the length of the plastic zone r_p and the crack tip opening displacement δ_t, for a center cracked plate (CCT) according to Fig. 1, could be obtained by means of a Green function (weight function) method. The Green function for an elastic center crack in an infinite plate subjected to a remote tension is given by the expression

$$m(x,b)=2\sqrt{\frac{b}{\pi}}\left(b^2-x^2\right)^{-1/2}. \tag{12}$$

The stress intensity factor (SIF) can be expressed by means of a weight function (12) as

$$K_{\mathrm{coh}}(b)=\int_a^b p(x)\cdot m(x,b)\,\mathrm{d}x. \tag{13}$$

If we introduce a new variable ξ, so it is $x=a+r_p(1-\xi)=b-r_p\xi$ ($\xi=1$ at the point B, and $\xi=0$ at the point A), we can transform the expression (7) and write him in the following form

$$p(\xi)=\sigma_0(1-\xi)^{-1/(n+1)}. \tag{14}$$

After inserting the expressions (12) and (14) in the formula (13) we get

$$K_{\mathrm{coh}}(b)=\sqrt{\frac{2}{\pi}r_p}\cdot\sigma_0\int_0^1\frac{1}{(1-\xi)^{1/(n+1)}}\cdot\frac{1}{\left[\xi\left(1-\frac{r_p}{2b}\xi\right)\right]^{1/2}}\mathrm{d}\xi. \tag{15}$$

At this point we could introduce an assumption about small crack tip plastic zone. Under small scale yielding (SSY) condition it could be taken $r_p/2b\approx 0$. After carrying out integration, the final result for $K_{\mathrm{coh}}(b)$ will be

$$K_{\mathrm{coh}}(b)=\sqrt{\frac{2}{\pi}r_p}\cdot\sigma_0\frac{\Gamma\left(\frac{1}{2}\right)\cdot\Gamma\left(\frac{n}{n+1}\right)}{\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)}=\sqrt{\frac{2}{\pi}r_p}\cdot\sigma_0\,B\left(\frac{1}{2},\frac{n}{n+1}\right), \tag{16}$$

where $\Gamma(x)$ stands for the gamma function or the Euler's integral of second type and $B(x,y)$ is the beta function or the Euler's integral of first type. This result we have to put in eq. (1) with opposite sign because the stress intensity factor $K_{\mathrm{coh}}(b)$ takes the negative value if it calculates for a real direction of the cohesive tensile stresses.

Finally, we get the plastic zone length r_p in front of the crack tip, normalized to the initial crack length a as

$$\frac{r_p}{a}=\frac{\pi}{2}\left(\frac{\sigma_\infty}{\sigma_0}\right)^2\cdot\left[\frac{\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)}{\Gamma\left(\frac{n}{n+1}\right)}\right]^2\Bigg/\left\{1-\frac{\pi}{2}\left(\frac{\sigma_\infty}{\sigma_0}\right)^2\cdot\left[\frac{\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)}{\Gamma\left(\frac{n}{n+1}\right)}\right]^2\right\}. \tag{17}$$

The SIF, corresponding to a remote tension of a plate with an imaginary crack of length b, amounts to $K_{\text{ext}}(a+r_{\text{p}}) = \sigma_\infty \sqrt{\pi\left(a+r_{\text{p}}\right)}$.

Now we wish to obtain the plastic zone length r_{p} in front of the crack tip in a special case of *elastic – perfectly plastic material*, $n \to \infty$. Namely, the closed form solution for $K_{\text{coh}}(b)$ can be obtained in a case if $n = \infty$. It follows from eq. (15)

$$K_{\text{coh}}(b) = \sqrt{\frac{2}{\pi} r_{\text{p}}} \cdot \sigma_0 \int_0^1 \frac{\mathrm{d}\xi}{\sqrt{\left(1 - \frac{r_{\text{p}}}{2b}\xi\right)\xi}} \ . \tag{18}$$

After integration, the final result for $K_{\text{coh}}(b)$ yields

$$K_{\text{coh}}(b) = -4\sqrt{\frac{b}{\pi}} \cdot \sigma_0 \arctan\sqrt{\frac{r_{\text{p}}}{a+b}} \ . \tag{19}$$

By inserting this solution, together with the expression $K_{\text{ext}}(b) = \sigma_\infty \sqrt{\pi b}$, in the eq. (1) and after some rearrangements it is obtained

$$\arctan\sqrt{\frac{r_{\text{p}}}{a+b}} = \frac{\pi}{4}\frac{\sigma_\infty}{\sigma_0} \ , \tag{20}$$

or if we do some trigonometric transformations, we shall get the famous expression for the length of the plastic zone in front of the crack tip, according to the original Dugdale model for *elastic-perfectly plastic material* of a plate, i.e.

$$\frac{r_{\text{p}}}{a} = \sec\left(\frac{\pi}{2}\frac{\sigma_\infty}{\sigma_0}\right) - 1 \ . \tag{21}$$

Linear distribution of cohesive stress in the yielded zone – special case

The magnitude of plastic zone r_{p} around the crack tip is determined under the assumption of *linear* distribution of *cohesive stress* $\sigma_{\text{Y}}(x)$ in the yielded zone, as shown in Fig. 2. The equation of the straight line on a part of equivalent elastic crack, through points B: $x = a$, $\sigma_{\text{Y.B}} = \sigma_{\text{Y1}}$ and A : $x = b = a + r_{\text{p}}$, $\sigma_{\text{Y.A}} = \sigma_{\text{Y0}}$, is expressed as

$$\sigma_{\text{Y}}(x) = \sigma_{\text{Y1}} - \frac{\sigma_{\text{Y1}} - \sigma_{\text{Y0}}}{b-a}(x-a) \ . \tag{22}$$

The stress intensity factor K_{ext} in a point A of a fictitious elastic crack, which corresponds to the external loading of the plate, according to Fig.1, is expressed as $K_{\text{ext}} = \sigma_\infty \cdot \sqrt{\pi\, b}$ .The stress intensity factor K_{coh} which corresponds to the linear distribution of cohesive stresses $\sigma_{\text{Y}}(x)$ within the yielded zone $a \le |\, x\,| \le b$, is determined by integration. As a matter of fact, we imagine that a concentrated tensile cohesive force $F = -\sigma_{\text{Y}}(x)\,\mathrm{d}x$ is acting on an infinitesimally small part of the cohesive zone. When we introduce expression (22) for $\sigma_{\text{Y}}(x)$, K_{coh} can be written as

$$K_{\text{coh}} = -\frac{1}{\sqrt{\pi\, b}} \int_a^b \left[\sigma_{\text{Y1}} - \frac{\sigma_{\text{Y1}} - \sigma_{\text{Y0}}}{b-a}(x-a)\right] \cdot \left(\sqrt{\frac{b+x}{b-x}} + \sqrt{\frac{b-x}{b+x}}\right) \mathrm{d}x \ . \tag{23}$$

After carrying out integration, we get

$$K_{\mathrm{coh}} = -2\sqrt{\frac{b}{\pi}} \cdot \left[\left(\sigma_{\mathrm{Y1}} + \frac{\sigma_{\mathrm{Y1}} - \sigma_{\mathrm{Y0}}}{b-a} \cdot a \right) \arccos\left(\frac{a}{b}\right) - \frac{\sigma_{\mathrm{Y1}} - \sigma_{\mathrm{Y0}}}{b-a} \sqrt{b^2 - a^2} \right]. \tag{24}$$

Since the cohesive stress $\sigma_{yy}(x) = \sigma_{\mathrm{Y}}(x)$ in the tip of equivalent elastic crack, point A, should not be singular, but must have definite magnitude, it means that the resulting stress intensity factor K in this point must be equal to zero, as it is defined with the expression (1). Finally, we get a transcendent equation from which we can determine, numerically, the length of the fictitious elastic crack b. This equation is written in the following form

$$\frac{\pi}{2}\sigma_\infty \left(1 - \frac{a}{b}\right) - \left(\sigma_{\mathrm{Y1}} - \sigma_{\mathrm{Y0}} \frac{a}{b}\right) \arccos\left(\frac{a}{b}\right) + \left(\sigma_{\mathrm{Y1}} - \sigma_{\mathrm{Y0}}\right)\sqrt{1 - \left(\frac{a}{b}\right)^2} = 0\,. \tag{25}$$

Using numerical methods, we can calculate the ratio a/b from equation (25), and from this ratio we can get the length of equivalent elastic crack b. Since $b = a + r_{\mathrm{p}}$, hence follows the length of the plastic zone r_{p} around the crack tip.

Determination of crack tip opening displacement (CTOD) in a strain-hardening material

The crack opening displacement $\delta_{\mathrm{D}}(x)$ (COD) in the Dugdale model is defined as

$$\delta_{\mathrm{D}}(x) = \delta_{\mathrm{ext}}(x) - \delta_{\mathrm{coh}}(x)\ . \tag{26}$$

Using the weight function method, as it was discussed in the paper [6], the crack tip opening displacement caused by uniformly distributed remote loading σ_∞ amounts to

$$\delta_{\mathrm{ext}}(a) = 4\frac{\sigma_\infty}{E}\sqrt{b^2 - a^2}\ , \tag{27}$$

while the crack opening displacement caused by cohesive stresses, distributed according to the expression (7), and under the small scale yielding condition will be

$$\delta_{\mathrm{coh}}(x) = \frac{2\sqrt{2}}{\pi}\frac{\sigma_0}{E}\frac{n+1}{n}\sqrt{b^2 - x^2} \cdot \sqrt{\frac{r_{\mathrm{p}}}{b}} \cdot \int_0^1 \frac{(1-\xi)^{n/(n+1)}}{(s-\xi)\cdot\sqrt{\xi}}\,\mathrm{d}\xi\ . \tag{28}$$

By inserting $x = a$ and $s = 1$ ($x = b - r_{\mathrm{p}}\, s$) we shall get the crack tip opening displacement $\delta_{\mathrm{coh}}(a)$ in the form

$$\delta_{\mathrm{coh}}(a) = \sqrt{\frac{8}{\pi}}\frac{\sigma_0}{E}\frac{n+1}{n}\sqrt{b^2 - a^2} \cdot \sqrt{\frac{r_{\mathrm{p}}}{b}} \cdot \frac{\Gamma\left(\frac{n}{n+1}\right)}{\Gamma\left(\frac{1}{2} + \frac{n}{n+1}\right)}\ . \tag{29}$$

If the plate material is *elastic-perfectly plastic* ($n = \infty$), the crack tip opening displacement, caused by cohesive stresses, will be

$$\delta_{\mathrm{coh}}(a) = \frac{8}{\pi}\frac{\sigma_0}{E}\sqrt{b^2 - a^2} \cdot \left[\arccos\left(\frac{a}{b}\right) - \frac{a}{\sqrt{b^2 - a^2}} \cdot \ln\left(\frac{b}{a}\right)\right]. \tag{30}$$

This expression, together with the expression (27), gives the famous formula for CTOD, based on the Dugdale model, for an elastic – perfectly plastic material

$$\delta_{\mathrm{D}}(a) = \frac{8}{\pi}\frac{\sigma_0}{E} a \cdot \ln\left[\sec\left(\frac{\pi}{2}\frac{\sigma_\infty}{\sigma_0}\right)\right] . \tag{31}$$

Conclusion

In this paper, we have tried to form a more reliable micro mechanism of plastic yielding in the yielded zone around the crack tip, which would better describe the real elastic-plastic state of the material in these zones. Our aim has been to model cohesive stresses in a plate made of a *strain-hardening material*. A hypothesis of variable cohesive stresses $\sigma_{\mathrm{Y}}(x)$ within the plastic zone has been introduced. Many authors tried to model a non-linear distribution of the cohesive stresses in a yielded zone in front of the crack tip, in the last decade, for example [5-7], [10-12] and so on. In the paper [6], the authors took same cohesive stress distribution on the plastic zone $p(x)$, as it was defined in the paper [5]. But, they didn't answer the question, how are the obtained results accurate? Is the expression (7) founded on the experimental observations? In the papers [7] and [11] the author modified the Dugdale strip yield model in order to include the work hardening properties of a material. He adopted the constitutive equation and proposed the distribution of the elastic-plastic strains within the cohesive zone in the form (8). Wnuk and Legat [10] submitted a two-parameter nonlinear function, in the form (10) or (11), for representing the cohesive stress distribution within the cohesive zone. Similar result for cohesive stress distribution was obtained in the paper [12].

In the present paper we have not succeeded to find an accurate law of distribution of cohesive stresses $\sigma_{\mathrm{Y}}(x)$ in the yielded zone, we have taken the law of distribution according to the expression (7). If this hypothesis were proved, then it would be possible to determine precisely, using analytical methods, the magnitude of the plastic zone around the crack tip r_{p} (17), as well as the crack tip opening displacement δ_{t} (CTOD), (29). Also, we have shown that if the cohesive stress $\sigma_{\mathrm{Y}}(x)$ in the yielded zone was *linearly* distributed, we could determine, in an analytical way, the length of the plastic zone r_{p} around the crack tip, by solving, numerically, the transcendent equation (25).

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