MEAN VALUE INFLUENCE IN FATIGUE – ON THE RATIONAL CHOICE OF MODEL COMPLEXITY

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Abstract

The Wöhler curve approach to fatigue life assessment is here extended to include variable amplitude loads with mean value influence. This is done through the definition of an equivalent load amplitude, which depends both on the usual damage accumulation hypothesis and on some mean value correction model. The method is exemplified by using the Gerber model, the mean stress sensibility model and different parametrisations of the crack closure model. All models are applied to three different data sets, including aluminium, welded steel, and cast steel specimens subjected to different spectrum loads. The results are interpreted in relation to the complexity of the different models and the trade off between complexity, number of tests and random scatter is discussed in the context of prediction uncertainty.

Introduction

In industrial practice the Wöhler curve is the main design tool for fatigue life. However, the concept of Wöhler curve design varies over different applications. In the simplest case, the strength of a component is determined by laboratory tests at constant amplitude by estimating one or two parameters, and service loads are predicted by some variant of equivalent stress ranges, usually based on the Palmgren-Miner cumulative fatigue hypothesis.

On the other hand, fatigue is known to be a very complex phenomenon, and a model close to reality should include not only the Wöhler parameters, but also for instance the local mean stresses, the yield stress, crack closure and threshold characteristics, the defect contents, or the grain size. However, these features may not be known to the engineer, at least not in the design stage, and therefore most of these must be excluded in engineering practice. In the case of empirical or semi-empirical modeling, where parameters are fitted from experimental results, there is a clear trade off between model complexity and knowledge about influentials that can be studied in mathematical terms. In this paper we will demonstrate this by studying the problem of including the mean stress influence in the simple Wöhler concept.

We start at a method of predicting fatigue life at variable amplitude using the concept of an equivalent stress range presented in Johannesson et al. [1]. The Palmgren-Miner hypothesis in combination with the Basquin equation leads to the following predicted life

$$N_{pred} = \hat{\alpha} \cdot S_{eq}^{-\hat{\beta}} , \qquad (1)$$

where the parameters $\hat{\alpha}$ and $\hat{\beta}$ are estimated from laboratory tests and S_{eq} is determined from the service stress spectrum

$$S_{eq} = \left(\sum_{i=1}^{n} \nu_i \cdot S_{a,i}{}^{\hat{\beta}} \right)^{1/\hat{\beta}} . \quad (2)$$

Here $\psi = \{\nu_i, S_{a,i}; i = 1,2,\ldots,n\}$ is the service stress spectrum represented by its load amplitudes $S_{a,i}$ and their corresponding relative frequencies ν_i, counted by for instance the Rain Flow Count method. This approach can also be described as the statistical model

$$\ln N = f(\psi;\alpha,\beta) + \varepsilon = f(\psi;\theta) + \varepsilon, \qquad f(\psi,\theta) = \ln\alpha - \beta \ln S_{eq} \quad (3)$$

where $\theta = [\alpha, \beta]$ and ε is an error term modelled as a random variable with variance σ^2. The parameters in the model are estimated by the least squares method, i.e. by performing a number of fatigue tests with different spectra and finding the parameter values that minimizes the squared error. The estimated parameter vector $\hat{\theta}$ is

$$\hat{\theta} = \arg\min_{\theta} \sum_{j=1}^{n} \left[\ln N_j - f(\psi_j;\theta) \right]^2 , \quad (4)$$

where N_j is the number of cycles to failure when using the spectrum ψ_j, and *n* is the number of reference tests. From the test results one can also find an estimate s^2 of the variance σ^2 of the error term ε:

$$s^2 = \frac{1}{n-p} \sum_{j=1}^{n} \left[\ln N_j - f(\psi_j;\hat{\theta}) \right]^2 , \quad (5)$$

where *p* is the number of parameters in the model, which in this simplest case equals two.

Mean value influence

The general formulation of Wöhler curve modelling by Eq. (3) makes it easy to include new variables and parameters in the model by extending the dimension of θ.

We here make such extensions by including the mean values of the load cycles into the model. This can be done in several ways, resulting in different functions $f_k(\psi;\theta)$, where *k* indicates different concepts like the Gerber correction, a linear model in the Haigh diagram, or a crack closure approach. The different concepts include different number of additional parameters, and the resulting estimated models will give decreased variances of their error terms. We will formulate each mean stress correction method by the following method: Each cycle with amplitude s_a and mean s_m is transformed into a damage equivalent cycle with amplitude $\tilde{s}_a^{(k)} = h_k(s_a, s_m)$ at mean zero. An equivalent load can then be constructed, corrected for the mean stress effect, by using the amplitudes $S_{a,i}^{(k)} = \tilde{s}_{a,i}^{(k)} = h_k(s_{a,i}, s_{m,i})$ in Eq. (2), and this modified equivalent load can be used both for estimation and prediction.

The first correction is according to Gerber

$$\tilde{s}_a^{(1)} = h_1(s_a, s_m) = \frac{s_a}{1-(s_m/s_u)^2},$$

where s_u is the ultimate tensile strength for the material. If the tensile strength is known beforehand, no additional parameter need to be estimated.

The second correction is a linear curve in the Haigh diagram,

$$\tilde{s}_a^{(2)} = h_2(s_a, s_m) = s_a + M s_m,$$

where M is the so-called mean-stress-sensibility of the material [2]. The new parameter M may be known for some materials, but will here be included in the parameter vector θ and estimated from fatigue tests.

The third and fourth corrections are based on the crack closure concept. Here we use the simplified assumption that there exist a constant crack closure level, and then the following effective stress range is defined for each stress cycle:

$$\Delta S_{eff,i} = \begin{cases} S_{\max,i} - S_{cl} & \text{if} \quad \mathrm{S}_{\mathrm{min,i}} < S_{cl} \\ S_{\max,i} - S_{\min,i} & \text{if} \quad \mathrm{S}_{\mathrm{min,i}} \geq S_{cl} \\ 0 & \text{if} \quad \mathrm{S}_{\mathrm{max,i}} \leq S_{cl} \end{cases}.$$

The constant closure level S_{cl} depends on the spectrum, and two different models of this dependence will be used here. First we will use a simple crack closure formula, here called proportional closure,

$$S_{cl}^{(p)} = S_{\min G} + c(S_{\max G} - S_{\min G}),$$

where c is the closure parameter and $S_{\min G}$ and $S_{\max G}$ are the global minimum and maximum in the spectrum, respectively. The corresponding equivalent amplitude is

$$\tilde{s}_a^{(3)} = h_3(s_{\max}, s_{\min}, \psi) = \frac{\Delta S_{eff}^{(p)}}{2}.$$

The fourth correction is based on another empirical model for crack closure, namely the DuQuesnay/Topper formula [3]:

$$S_{cl}^{(Q)} = a S_{\max G}\left[1 - \left(\frac{S_{\max G}}{S_y}\right)^2\right] + b S_{\min G},$$

where a and b are the closure parameters and S_y is the cyclic yield strength of the material, and the formula is based on the global maximum and minimum as in the previous model. The corresponding equivalent amplitude is

$$\tilde{s}_a^{(4)} = h_4(s_{\max}, s_{\min}, \psi) = \frac{\Delta S_{eff}^{(Q)}}{2}.$$

Parameter estimates from this model appear to be strongly correlated which sometimes leads to numerical difficulties. Therefore a variant of this model is used, where the ratio between the parameters are regarded as constant, $b/a = 1.8$, and thereby reducing the complexity of the closure model to one parameter.

Optimal complexity

By comparing each decrease in error terms with the influence of the mean value variation one can draw conclusions about what model is the most appropriate at different applications and how many laboratory tests that are needed to make use of the extra information in mean values. A general result for linear regression models [4] shows that the model should be chosen that minimize

$$U_{n,p}^2 = s_{n,p}^2 \cdot \left(1 + \frac{p}{n-1-p}\right), \tag{6}$$

where p is the number of parameters in the model, n is the number of tests and $s_{n,p}^2$ is the estimated error variance Eq. (5), which depends on both the number of tests and the model complexity. In our case the models are not linear in the parameters, but the formula Eq. (6) is still used as an approximate criterion for model choice.

Three data sets

Three different applications have been investigated with respect to the mean value influence, one including welded mild steel (SP data), one with plain specimens made of two different aluminium alloys for aircraft applications from Saab (Saab data) and one with an automotive suspension arm of cast steel from Volvo Trucks (Volvo data). All data sets contain variable amplitude fatigue lives where the mean value differs between the applied spectra. Some spectrum tests were replicated and based on these results the standard deviations of the inherent scatter have been estimated for comparison. The mean value concepts given above have been used in different ways. In fact, sometimes the parameters in the models are available from literature or previous tests and then they do not need to be estimated. This would result in more precise predictions, since the loss of precision according to Eq. (6) depends on the number of estimated parameters p. However, literature values may not always be trusted, which will be seen in our results. The results are here only illustrated in figures illustrating their prediction abilities in view of the different variance measures. The complete results are available in the report [5].

Specimens and spectra

The SP test object was a load carrying butt welded specimen of mild steel with the ultimate tensile strength 454 MPa. The spectra were originally constructed in purpose of testing the validity of the Palmgren-Miner rule and study the irregularity factor on fatigue life. Two types of load sequences were constructed, characterized by their level crossing spectra, one appearing as concave and one as convex in the level crossing plot. Each type was constructed in two versions, with irregularity factors 0.99 and 0.50, respectively. By using different scale factors and offsets these spectra were used here to study the mean value influence.

The Volvo test objects were suspension arms made from nodular cast steel with the ultimate tensile strength 510 MPa with quite rough surfaces at the locations of maximum stress. The load sequences were constructed from a raw signal measured at a proving ground for cars. The signal was filtered in different ways giving stress spectra with different contents of small cycles. Since the original sequence was quite irregular one could expect a certain mean value influence of the deletion of small superimposed cycles.

The Saab objects were aluminium specimens of two different alloys, with ultimate tensile strengths of 496 MPa and 538 MPa, respectively. Two types of specimens were used, with or without an initial artificial defect. In this paper we only discuss results from one of the four sets, namely the 538 MPa material without initial defect, while the other results can be found in [5]. The load sequences used in this investigation were three different fighter aircraft lower surface wing root manoeuvre spectra and one Gaussian spectra. All spectra were scaled and given different offsets with the purpose of investigating mean value effects.

Results

The Saab data results in Figure 1 show considerable improved prediction ability when mean value influences are included. The standard deviation of the residuals decreases from 0.49 with no correction to 0.28 when using the DuQuesnay closure model with one or two free parameters. The same model with fixed parameters from [6] gives worse results than the reference case and also the proportional closure model is discriminated. The number of tests is here large enough to make the prediction variance very close to the residual variance and one can afford to have the largest complexity, including four parameters, in the modelling. Comparing the resulting error measures with the replicate standard deviation shows that there is still a significant model error which is not captured by the mean value correction.

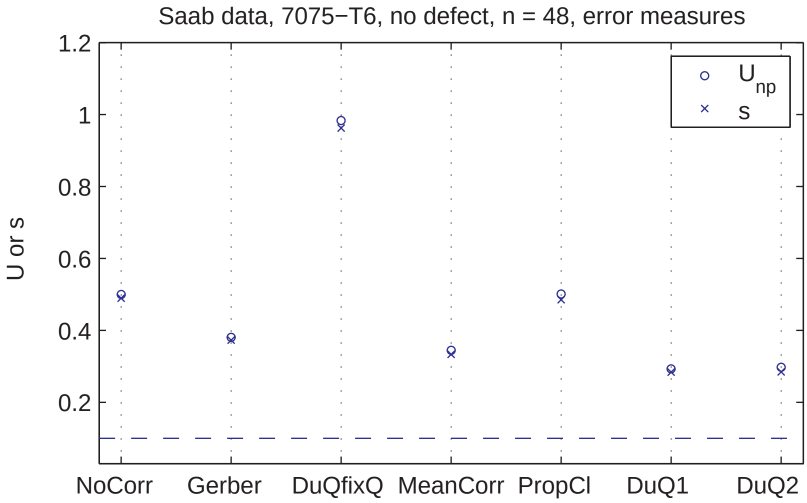

Figure 1. Error measures for one of the Saab specimen types. The broken line shows the estimated standard deviation due to inherent scatter.

The Volvo results are shown in Figure 2. Here, the results are based on only nine tests which results in a large difference between prediction and residual variation. This is in particular emphasized in case of more complex models. The smallest residual scatter around the models is not significantly larger than the inherent scatter. The conclusion from the results is that the performed tests are not sufficient to extract any mean value influence, but the simplest model should be used, without taking mean values into account.

The SP results are shown in Figure 3. Here, apparently none of the models can catch the physical behaviour, but the Palmgren-Miner rule fails. This conclusion is based on the observation that the inherent standard deviation is almost half of the residual standard deviation for the best fitting model. Among the different mean value correction methods the two DuQuesnay models with fixed parameters give the lowest residual variances.

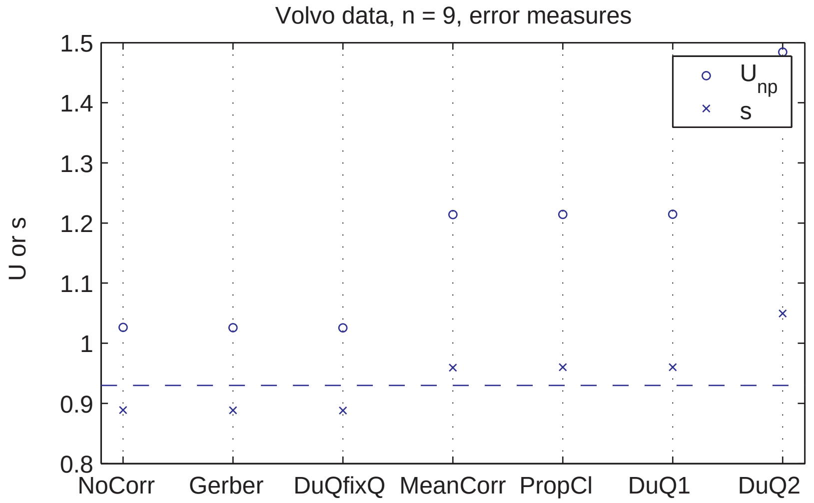

Figure 2. Error measures for the Volvo data set. The broken line shows the estimated standard deviation due to inherent scatter.

The first of these uses parameters from [6] and the other from a previous test on the same subject as at the present investigation. The DuQuesnay parameters from both sources seem to describe the present situation good enough and the alternatives of estimating them (DuQ1 and DuQ2) only increase the complexity without any gain in model precision.

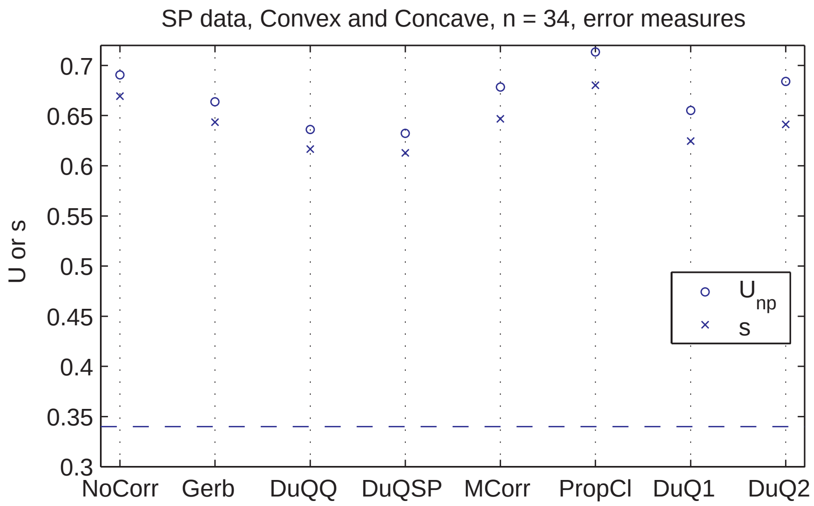

Figure 3. Error measures for the SP data set. The broken line shows the estimated standard deviation due to inherent scatter.

Discussion

For the Volvo results it is clear from Figure 2 that the optimal choice is to disregard any possible mean value correction. The lowest prediction uncertainty is obtained without any mean value correction and the two models with fixed parameters do not improve the prediction capacity. This illustrates the fact that if the number of tests is small and the inherent scatter is large it is useless to introduce complicated models. In fact, in this particular case, no model errors can be detected since no difference can be seen between the regression standard deviation and the replicate standard deviation.

For the Saab results a completely different picture appears. Figure 1 shows that the most complicated model, the DuQuesnay model with two estimated parameters, gives the lowest prediction variance. Note, that the difference between the expected prediction variance and the regression variance is small when the number of test is large and that added variables have small influence in the factor $p/(n-p-1)$ in the prediction variance Eq. (6). The difference between estimating one or two parameters in the DuQuesnay model is small, which indicates that our initial estimate of the relationship between the two parameters happens to be good in this case. The proportional closure model is much worse than the other models with estimated parameters, which suggests that this simple way of modelling closure does not comply with physics. The fixed parameter DuQuesnay model also gives bad results. This shows that the model predictions are quite sensible to the parameters in this case and that it may be dangerous to trust literature data.

The SP results represent a case when the physical modelling is poor since the distance between the prediction error and the replicate error is considerable. This can be seen as a result of the different types of spectra, convex or concave, and treating each type separately would give better results. Nevertheless, if some of the models still must be used one can conclude that the DuQuesnay model with fixed parameters according to the earlier investigation is the best choice.

The DuQuesnay-Topper model for crack closure includes one material parameter and two empirical parameters. In our applications we have used this model in different ways, namely by estimating one or two parameters from the actual tests or use previous investigations for the parameter values. From our results we can conclude that these values are not generally applicable; for the Saab aluminium alloys they even gave worse model fit than for the simplest model. However, in the SP application we had access to a pair of parameters estimated from an earlier investigation on the same material and type of specimen and these parameters give the best predictions.

In general the choice between fixed or estimated parameters depends on the trade off between the precision in the fixed parameter values, the scatter, and the number of tests. For each specific engineering application judgements must be done with regard to such considerations. In many situations there may be small steps in material and component development and old parameter values may be safely used, maybe with some updating technique.

Our one parameter DuQuesnay model is a compromise between fixed and estimated parameters. It may be the fact that the differences between different materials on crack geometries can be caught with only one parameter and that the relationship between the two parameters is fairly constant. Of course, such conclusions cannot be drawn on this small data set, but it could be a possible subject for further investigations.

It is interesting to note that both the closure concept with the DuQuesnay-Topper model and the mean stress sensibility model give improved models with respect to their prediction abilities. The two concepts are quite different since the mean stress sensibility model adjusts the amplitude individually with the local cycle mean, but the closure concept used here assumes a global closure level which each cycle is adjusted against. A possible extension of the closure concept is to allow a dynamic closure level. Such a model could be seen as a compromise between these two concepts, but is much more complicated to apply.

The classical Gerber correction is a fixed parameter mean stress sensibility model. It is not very successful in our applications, but this may be due to our uncritical use of the ultimate tensile stress for the pure material in the Gerber model. Maybe some considerations must be done with regard to the special weld properties for the SP case, for the stress concentrations in the SAAB case or for the surface roughness in the Volvo case. But, in that case an experience based fixed parameter in the mean stress sensibility model may be a good alternative.

Conclusions

We have used a general approach of estimating model parameters directly from experimental lives at variable amplitude fatigue tests. The flexibility of the approach makes it easy to apply different models for mean stress corrections and thereby demands rules for decisions in a specific industrial application. The concept of minimizing the prediction uncertainty has been presented as a tool for such decisions, which will depend on the trade off between model complexity, random scatter and the number of reference tests.

Among the investigated mean stress influence concepts the crack closure model with the DuQuesnay-Topper formulation, and the mean-stress-sensibility seems to be the best ones. They perform almost equally well, with a slight advantage for DuQuesnay-Topper. This is the case when the parameters are estimated from our data. Using literature data for parameters seems to be dangerous, since the sensitivity for the parameter values is quite large. However, in the one case where parameter values were available for the same specimens these proved to be useful.

References

1. Johannesson, P., Svensson, T., and de Maré, J., submitted to *Int J Fatigue,* 2003.
2. Schütz, W., *Z.f. Flugwissenschaften,* Vol. 15, pp. 407-419, 1967.
3. DuQuesnay, D. L., Topper, T. H., Yu, M. T., and Pompetzki, M. A., *Int J Fatigue*, Vol. 14, No. 1, pp. 45-50, 1992.
4. Breiman, L., and Freedman, D., *J Am Stat Assoc*, Vol. 78, pp. 131-136, 1983.
5. Johannesson, P., Svensson, T., FCC report 328-040109-164, Fraunhofer Chalmers Centre, Göteborg, Sweden.
6. Lynn, A. K., DuQuesnay, D. L., *Int J Fatigue* , Vol. 24, pp. 977-986, 2002.

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