IMPACT SHEAR LOADING OF BI-MATERIAL STRIPS, A COMPARISON OF NUMERICAL AND GLOBAL ENERGY ANALYSIS

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Introduction

Fig.1. shows a model of a test configuration which has been studied extensively [e.g. 1] to observe dynamic crack growth along a weak interface between two dissimilar materials. Here we will restrict the analysis to strips of equal height, h, but having different elastic modulii E and densities ρ. The loading is effected by impacting one arm with a projectile at a velocity V which propagates the crack of length a at a velocity $\dot{a}$. The stresses are predominantly uniaxial compression but the crack propagates in shear, or mode II. It has been observed that the propagation speeds are greater than a shear wave velocity of the system [1] and are sometimes referred to as intersonic. It has also been observed that very high accelerations occur during the initial phase of propagation.

An earlier paper [2], applied a global dynamic energy balance analysis to this problem and compared solutions with experimental data from [1]. This analysis is summarised here and then compared with a numerical finite volume analysis [3], which describes the crack growth via a Cohesive Zone Model (CZM), using only the shear traction-separation relation, i.e. pure mode II.

Steady State Analysis

The static energy release rate when the upper arm is loaded with a stress σ_1 is given by [2],

$$G_S = \left(\frac{\sigma_1^2}{2E_1}\right)h \cdot \chi(\beta), \qquad \beta = {E_2}\big/{E_1} \tag{1}$$

and

$$\chi(\beta) = \frac{\beta(1+\beta)}{\left(1+14\beta+\beta^2\right)}.$$

Note that for $\beta = 1$, a homogenous specimen, $\chi = \frac{1}{8}$ while for the case of $\beta \gg 1$, a stiff lower arm, $\chi \to 1$. For polymer-metal systems $\beta \approx 20-60$ and χ is in the range $0.6-0.8$.

If the crack is moving at a constant velocity $\dot{a}_0$ then for a loading velocity V the strain in the upper, loaded, section becomes,

$$e_1 = \left(\frac{Vt}{\dot{a}_0 t}\right) \quad \text{and} \quad \sigma_1 = E_1\left(\frac{V}{\dot{a}_0}\right) \tag{2}$$

giving

$$G_S = \frac{E_1 h}{2}\left(\frac{V}{\dot{a}_0}\right)^2 \chi.$$

It should be noted that the steady state assumption is that the end deflection is Vt and the crack length is $\dot{a}_0 t$ which will be true only at long times.

In the dynamic case there is change in kinetic energy given by [2],

$$\frac{\mathrm{d}U_k}{b\,\mathrm{d}a} = \left(\frac{\sigma_1^2}{2E_1}\right) h\chi\left(\frac{\dot{a}_0}{\dot{a}_C}\right)^2$$

where

$$\dot{a}_C^{-2} = \phi\, c_1^{-2} + (1-\phi)c_2^{-2}$$

$$c_1^2 = E_1\Big/\rho_1, \quad c_2^2 = E_2\Big/\rho_2$$

and

$$\phi = \left(\frac{\beta}{1+\beta}\right)\left(\frac{13+2\beta+\beta^2}{1+14\beta+\beta^2}\right). \tag{3}$$

The dynamic G is,

$$G = G_S - \frac{\mathrm{d}U_k}{b\,\mathrm{d}a}$$

and for a constant value of $G = G_0$ we have,

$$\left.\begin{array}{c} G_0 = \dfrac{E_1 h\chi}{2}\left(\dfrac{V}{\dot{a}_C}\right)^2\left[1-\left(\dfrac{\dot{a}_0}{\dot{a}_C}\right)^2\right] \\[4mm] \text{or} \quad \left(\dot{a}_0\Big/\dot{a}_C\right) = \left[1+\dfrac{2G_0}{E_1 h\chi}\left(\dfrac{\dot{a}_C}{V}\right)^2\right]^{-\frac{1}{2}} \end{array}\right\} \tag{4}$$

$\dot{a}_C$ is the characteristic wave speed for the interface and is a function of the wave speed in each arm, c_1 and c_2. For polymer metal systems ϕ is in the same range, $0.6-0.8$, as χ.

Note that for very low G_0 or very high V values $\dot{a}_0 \to \dot{a}_C$. For polymer-metal combinations $\dot{a}_C \sim 1600$ m/s.

Transient Crack Growth

The steady state solution is not valid at initiation since $\dot{a} = 0$ at that time. A solution to describe this behaviour may be found [2] by considering a perturbation from the steady state, p, such that,

$$\dot{a} = \dot{a}_0 t + p$$

and imposing the fixed dynamic G ; G_0, as before. The equation of motion for p becomes,

$$\ddot{p}t^2 + 2\left(\frac{\dot{a}_C}{\dot{a}_0}\right)^2 p = 0 \tag{5}$$

which has a solution of the form,

$$p = \text{const} \cdot t^{\frac{1}{2} \pm i\varepsilon} \quad , \quad \varepsilon = \frac{1}{2}\sqrt{8\left(\frac{\dot{a}_C}{\dot{a}_0}\right)^2 - 1} \; .$$

This has a real part of;

$$p = \tau^{\frac{1}{2}}\left[B_1 \sin(\varepsilon \ln \tau) + B_2 \cos(\varepsilon \ln \tau)\right] \tag{6}$$

where $\tau = \frac{t}{t_0}$ and t_0 is the initiation time. The constants B_1 and B_2 are determined from the boundary conditions $t = t_0$, $p = -\dot{a}_0$ and $\ddot{p} = 0$ for which $t_0 = \frac{a_0}{\dot{a}_0}$ and a_0 is the initial crack length. The crack growth functions become;

$$\left.\begin{array}{l} a = a_0\left[\tau - \dfrac{\tau^{\frac{1}{2}}}{\varepsilon}\sin(\varepsilon \ln \tau)\right] \\[2ex] \dot{a} = \dot{a}_0\left[1 - \tau^{-\frac{1}{2}}\left(\cos(\varepsilon \ln \tau) + \dfrac{1}{2\varepsilon}\sin(\varepsilon \ln \tau)\right)\right] \\[2ex] \ddot{a} = \dfrac{2}{\varepsilon}\dfrac{\dot{a}_C^2}{a_0}\tau^{-\frac{3}{2}}\sin(\varepsilon \ln \tau) \end{array}\right\} \tag{7}$$

For short times $\dot{a} \propto (a - a_0)^{\frac{2}{3}}$ and the maximum acceleration is approximately when $\ln \tau = \frac{2}{3}$, i.e. $\tau \approx 2$; i.e. $\ddot{a}_{max} \approx 0.24\dfrac{\dot{a}_C^2}{a_0}$. For polymer systems with $a_0 = 50$ mm, as in [1],

$\ddot{a}_{max} \approx 10^7 \, \text{m}\big/\text{s}^2$. Figures 2 and 3 show $\dot{a}\big/\dot{a}_0$ and $\dddot{a}a_0\big/C^2$ as a function of $\ln \tau$ for various $\dot{a}\big/\dot{a}_C$ values, respectively.

The Numerical Model

A Finite Volume method is developed for the simulation of the transient shear crack problem. Global material behavior is approximated as linear elastic while a Cohesive Zone Model (CZM) is employed to define the local non-linear separation process. The main properties of the method are its simplicity, conservativeness and efficiency. The simplicity stems from the fact that the method uses the original integral linear-momentum conservation law as the basis for discretisation. The conservative character of the procedure is preserved both globally and locally, which leads to physically meaningful and accurate results even when very small number of computational cells are employed. The segregated, iterative solution algorithm results in low memory requirements and is directly applicable to non-linear problems. However, due to the iterative treatment of the inter-equation coupling, the solution procedure may result in increased CPU time, particularly for problems involving bending.

The method has been successfully applied to various fracture problems, and more recently for predicting branching in PMMA specimens [3]. This is the only work where good quantitative agreement is obtained between experiments and numerical predictions of dynamic, multiple crack branching by using initially rigid traction-separation laws. The use of the same in the Finite Element procedures does not result in branching. The explanation to this may be found in the facts that FV uses implicit time differencing, while the tractions equilibrium is considered on the cell faces and not in an average volume sense as would be the case with standard FE procedures. This is particularly important when using CZM, since cohesive tractions act on the cell (element) faces.

Comparisons

The numerical simulation of the shear crack propagation was conducted for the case with $\chi = 1$, i.e. the lower part of the bi-material strip was assumed rigid (Fig. 1). The top part of the specimen is considered to be linear elastic with the following properties: $E = 1$ GPa, $\nu = 0.3$ and $\rho = 1000$ kg/m^3. The solution domain and the boundary conditions used in the analysis are shown in Figure 4. The domain is discretised with 100 x 10 computational cells or control volumes (CV) in longitudinal and transverse directions, respectively. Plane stress conditions are used in the analysis. The time step is chosen such that the Courant number is equal to 1.

Consider traction **t** acting at a point on the crack boundary. For given geometry and boundary conditions, it is reasonable to assume that

$$\mathbf{t} \cdot \mathbf{i}_1 \text{ and } \mathbf{t} \cdot \mathbf{i}_3 \ll \mathbf{t} \cdot \mathbf{i}_2 \tag{8}$$

where $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$ are unit vectors in the normal and two tangential directions, respectively (Fig. 4). For simplicity, the small traction components in normal direction 1 and tangential direction 3 are therefore neglected, and only the shear (mode II) component is considered. The shear cohesive law is assumed to be of Dugdale form with the cohesive strength of 100 MPa and fracture resistance of 1 kJ/m^2.

The numerical results and comparison with the analytical solution are to follow.

References

1. Rosakis, A. J. (2002), *Advances in Physics*, Vol 51, N°4, pp 1189-1257

2. Williams J. G. in press, *J.Eng.Fract.Mech.* 2004

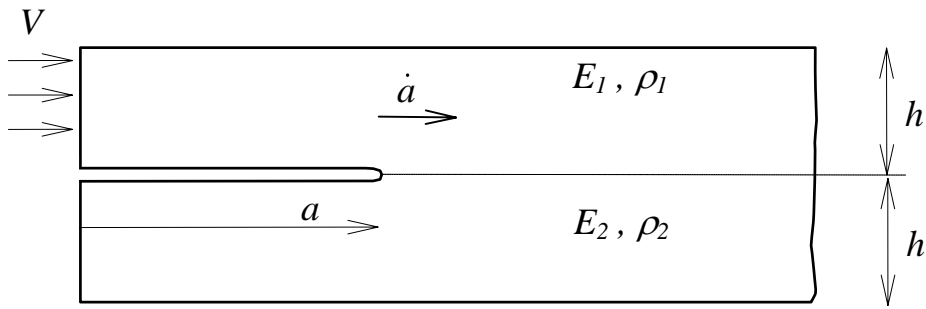3. Murphy, N. and Ivankovic, A., in press, *J.Eng.Fract.Mech.* 2004

4.

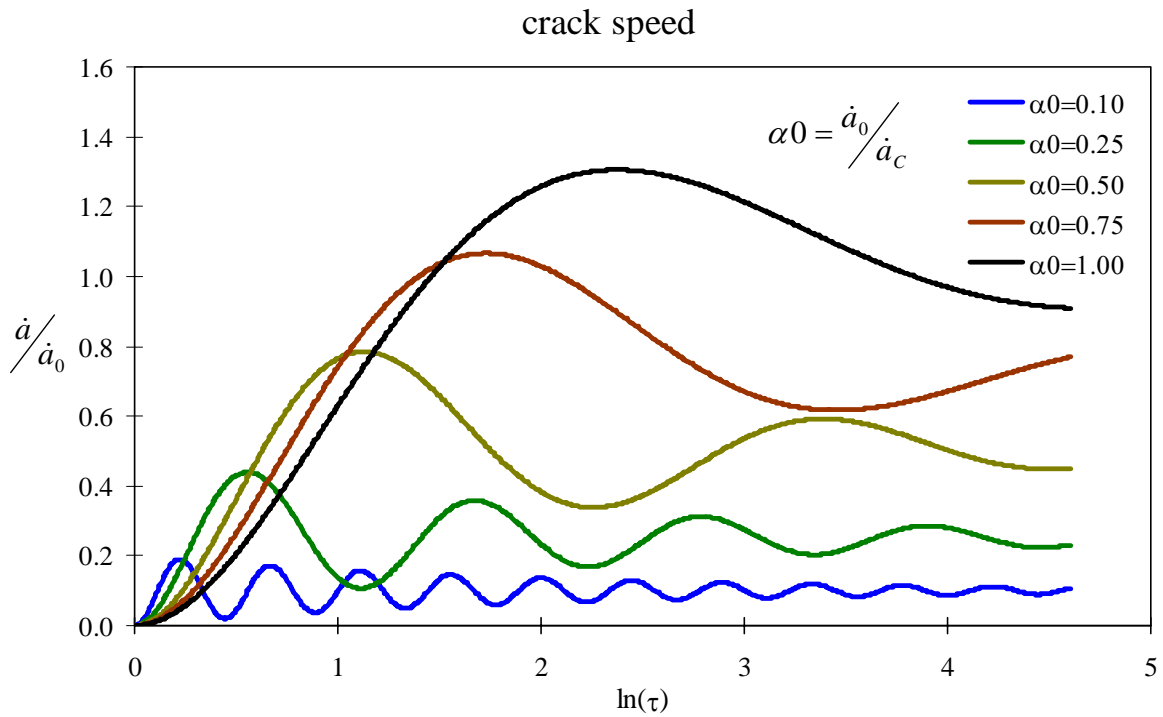Fig 1: Bi-material Strip in Shear Loading.

crack speed

Fig 2: Variations of velocity with time, $\tau = \dfrac{t}{t_0}$. $\dot{a}_0$ is steady state speed, t_0 is initiation time.
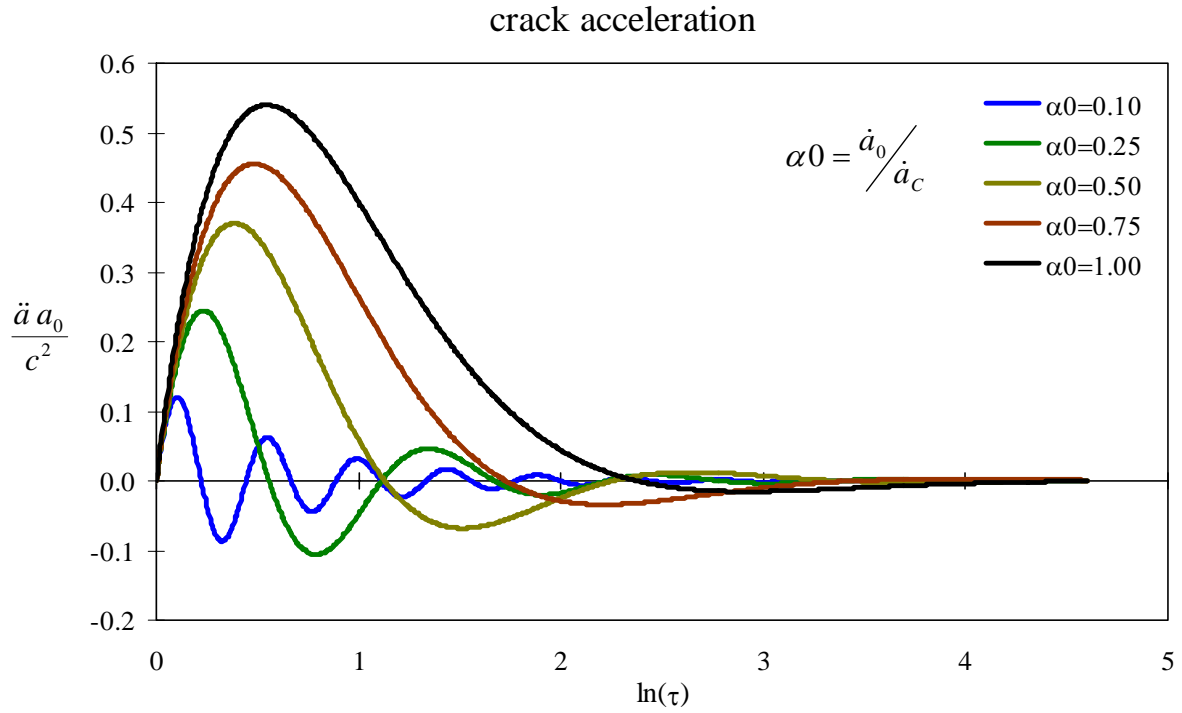
crack acceleration

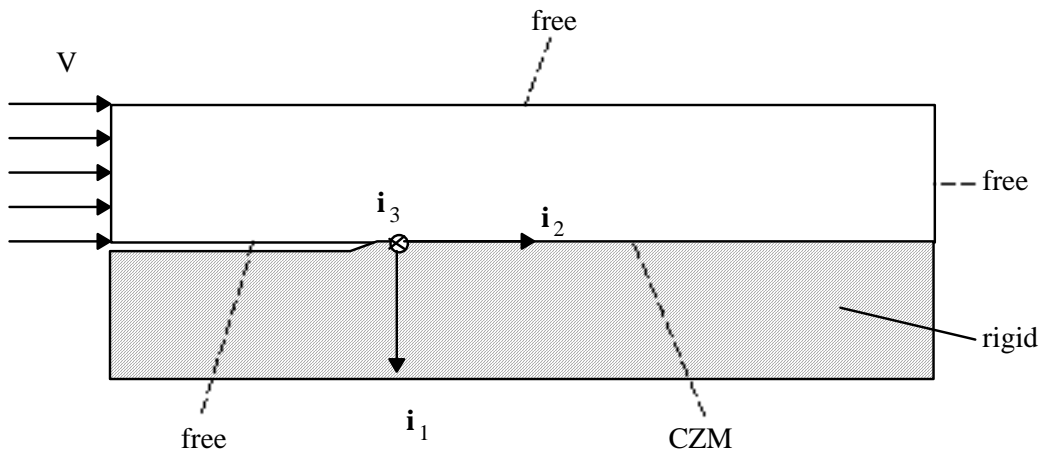Fig 3: Variations of acceleration with time. Stable as $\ddot{a} \to 0$ as $t \to \infty$.

Fig 4: Solution domain and boundary conditions