FINITE ELEMENT MODELLING OF MULTIPLE COHESIVE DISCRETE CRACK PROPAGATION IN REINFORCED CONCRETE BEAMS (I): THE MODEL AND ITS COMPUTER IMPLEMENTATION

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Abstract

A finite element model for fully automatic simulation of multiple discrete crack propagation in reinforced concrete (RC) beams is presented in this paper. The discrete cracks are modelled based on the cohesive/fictitious crack concept using nonlinear interface elements with a bilinear tensile softening constitutive law. The model comprises an energy-based crack propagation criterion, a simple remeshing procedure to accommodate crack propagation, two state variable mapping method to transfer structural responses from one FE mesh to another, and a local arc-length algorithm to solve the system equations characterised by material softening. The bond-slip behaviour between reinforcing bars and their surrounding concrete is modelled by a tension-softening element. This paper discusses some key aspects of the model and its computer implementation. A detailed case study is presented in the companion paper.

Keywords: Finite element analysis, local arc-length method, cohesive crack model, multiple crack propagation, reinforced concrete beams

Introduction

Numerical modelling of reinforced structures (RC) structures such as beams, plates and shells presents significant challenges to the engineering community because of their complex nonlinear structural behaviour. The nonlinearities arise mainly from two major material factors: plasticity of the reinforcement and compressive concrete, and cracking of the concrete. Other nonlinearities are due to bond-slip interaction between the reinforcements and concrete, aggregate interlock of the cracked concrete, dowel action of reinforcements, concrete creep and shrinkage etc. Although great effort has been made in developing numerical models in recent years, a recent review [1] indicates that only trivial structural problems with simple geometries have so far been solved. One of the most challenging issues has been the accurate prediction of concrete cracking.

It is now well known that in order to accurately model cracking behaviour of normal-sized structures such as RC beams, the fracture process zone (FPZ) must be properly modelled to consider the gradual energy dissipation during cracking. This means models based on nonlinear fracture mechanics rather than linear elastic fracture mechanics should be used. Two types of crack models, i.e., smeared crack model and discrete crack model, have been

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widely used for modelling FPZ. In modelling single tensile crack propagation in plain concrete beams, both discrete and smeared models are able to satisfactorily predict crack trajectories and load-displacement responses [2]. However, both models have only achieved limited success in modelling multiple distributed crack propagation in structures such as RC beams, with the smeared crack model being far more popular because of its computational convenience [1]. Due to the necessity of a remeshing procedure to accommodate crack propagation, the discrete crack model has been rarely used to model multiple concrete cracking (e.g. [3-6]). For these few studies, most were concerned with bond-slip behaviour and used predetermined crack paths or pre-specified initial crack locations. To the best knowledge of the authors, an automatic finite element modelling of multiple discrete crack propagation in RC structures exhibiting complex fracture behaviour is still not available.

This paper presents a nonlinear finite element (FE) model for automatic modelling of multiple discrete crack propagation in RC beams. The key aspects of the model and its computer implementation are discussed. A detailed case study of modelling an RC beam with well-documented test data is presented in the companion paper [18].

FEM for multiple cohesive discrete crack propagation

The discrete crack FE model for RC beams consists of six key components: a cohesive crack model (CCM) considering the tensile softening behaviour of concrete, a bond-slip model considering interaction between steel reinforcement and concrete, an energy based crack propagation criterion, a remeshing procedure, mesh-mapping techniques transferring structural responses from one FE mesh to another, and a numerical solution technique to solve nonlinear equation systems involving material softening. These important aspects are briefly described in the following sections.

Cohesive crack model (CCM)

The cohesive crack model (CCM) assumes that a fracture process zone (FPZ) exists ahead of a real crack tip. The FPZ has the capability of transferring stresses through mechanisms such as aggregate interlock and material bonding until the crack opening displacement (COD) reaches a critical value. The CCM has become the basis of nonlinear discrete crack modelling and has been incorporated into some finite element codes in the form of 2-dimensional four-node, six-node and 3-dimensional eight-node interface elements to model mode-I and mixed-mode crack propagation (e.g., [2, 3, 7-9]). The four-node interface element developed by Gerstle and Xie [7] is adopted to represent cohesive cracks in this study for its simplicity.

Figure 1 shows schematically the FPZ in concrete where two interface elements are used to model the FPZ. The softening behaviour of concrete in these cohesive interface elements is modelled with Petersson's bi-linear curve [10]. Figure 2 shows the bi-linear COD-traction curve with a typical unloading path indicated. The model assumes an "irreversible unloading path" which is linear elastic from its current point at $w = w^*$ to the origin with the corresponding secant stiffness. If it is loaded again, it follows the unloading path up to $w = w^*$ and then follows the original COD-traction curve.

Bond-slip model

The reinforcing bars are modelled using elastic-perfectly plastic truss elements. A simple yet rational bond-slip model for the bond behaviour between reinforcement bars and concrete proposed by Ingraffea *et al.* [3] is adopted. Assuming that the bond-slip behaviour between a reinforcing bar and its surrounding concrete is mainly governed by localised secondary cracking of concrete near a primary crack, the model lumps all the nonlinearities caused by bond-slip due to secondary cracking onto a special 2-node "tension-softening" element. This tension-softening element is characterised by a softening stress-crack opening constitutive relationship, which is dependent on the concrete strength, the diameter and form of the reinforcing bar and loading conditions [8].

Figure 3 illustrates a tension-softening element connected with two 4-node cohesive interface elements modelling the primary crack and two truss elements modelling the reinforcing bar.

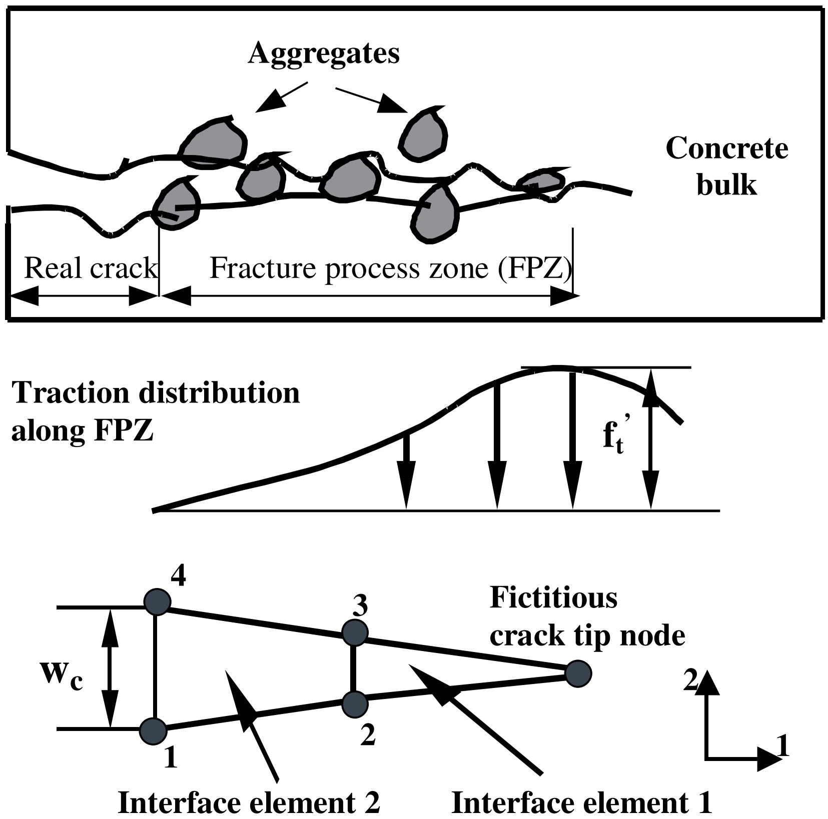

FIGURE. 1 Modelling of FPZ with 4-node interface elements (after Xie [8])

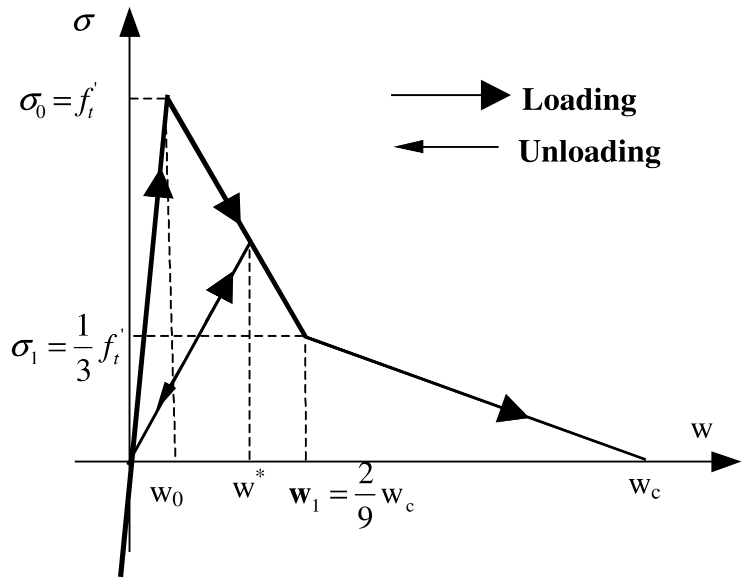

FIGURE. 2 COD- traction curve of nonlinear interface elements

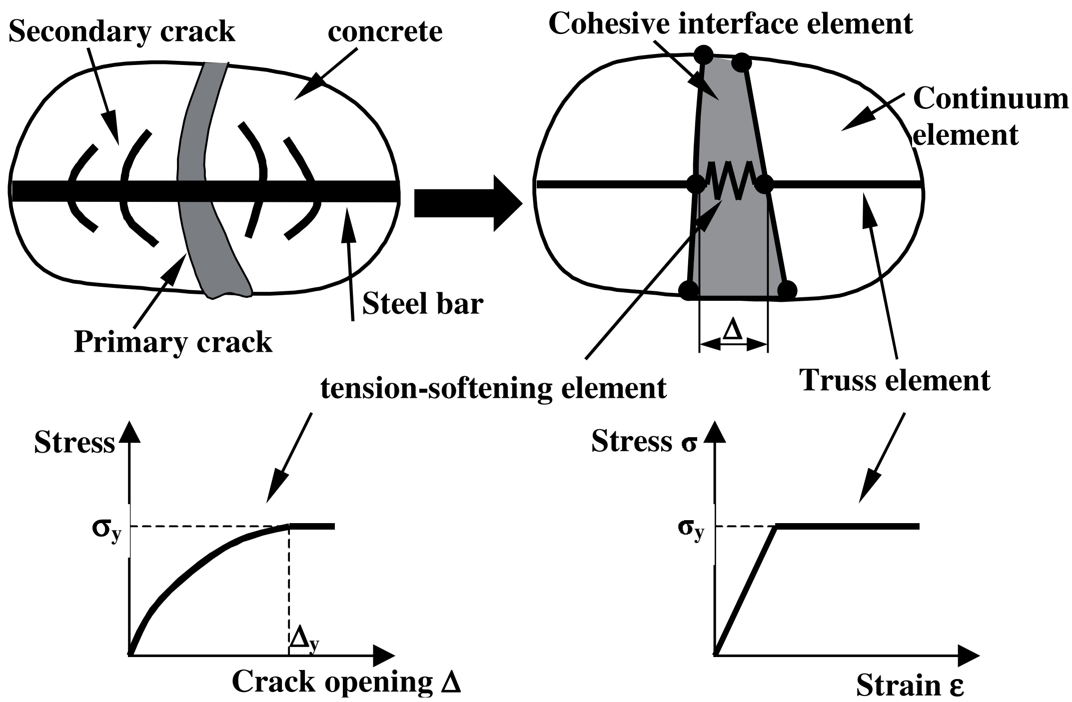

FIGURE. 3 Bond-slip modelling with a tension softening element (after Xie [8])

Crack initiation and propagation criteria

When the principal tensile stress at a node exceeds the tensile strength of the concrete, a crack perpendicular to the principal tensile stress is assumed to initiate.

A proper crack propagation criterion is needed to determine when and in which direction a crack will propagate. The CCM assumes that the crack propagates when the maximum principal stress of the crack tip node reaches the concrete tensile strength. This stress-based assumption has been used in most previous studies (e.g. [11]), but it requires very fine crack-tip meshes to predict accurate nodal stresses. This makes the remeshing procedure very complex. Xie [8] developed an energy-based cohesive crack propagation criterion

$$G - \mathbf{u}^T \frac{\partial \mathbf{f}}{\partial A} = 0 \tag{1}$$

in which **u** is the displacement vector; **f** is the cohesive forces; *A* is the crack surface area; and *G* is the total strain energy release rate (SERR) calculated by

$$G = -\frac{1}{2}\mathbf{u}^T \frac{\partial \mathbf{K}}{\partial A}\mathbf{u} + \mathbf{u}^T \frac{\partial \mathbf{P}}{\partial A} \tag{2}$$

where **K** is the total stiffness matrix of the elastic bulk and **P** the total equivalent nodal force due to external and body forces. The value of G in Eq. 2 can be accurately calculated by a simple virtual crack extension (VCE) technique [8]. The second term in Eq. 1 can be explicitly derived using the four-node interface elements [8]. Because the convergence rate for energy entities is faster than stresses in FE analysis, crack propagation can be modelled accurately even using coarse meshes [2, 8, 9].

The crack is assumed to propagate in the direction perpendicular to the maximum principal tensile stress at the crack tip node.

Remeshing procedure

An efficient remeshing procedure is paramount in discrete crack modelling to accommodate crack propagation. Existing procedures can generally be classified into two categories. One may be termed "remove-rebuild" algorithms, in which a new crack-tip node is determined by extending a specified crack growth increment in the calculated propagation direction. The original mesh within a certain range around the new crack-tip node is then completely removed. A complex procedure is followed to form the new crack and regenerate the mesh within this range where a regular rosette is added. Examples of this category include those developed by Wawrzynek and Ingraffea [12] and Bocca et al [11].

The other category of algorithms may be termed "insert-separate" algorithms (e.g. [8-9]). In this procedure, a new edge from the old crack-tip node is first inserted into the local mesh in the propagation direction. The intersection point of this edge with the original mesh is used as the new crack-tip node. The new crack is then formed by separating those nodes along the line through the new and old crack-tip nodes. A rosette can finally be added to refine the mesh around the crack tip node. Because this procedure does not need to completely remove and rebuild the new crack-tip mesh as the "remove-rebuild" procedure does, fewer elements are affected and the procedure is much simpler. The "insert-separate" [8] is used in this study.

Mesh mapping techniques

After remeshing, the structural state variables from the old mesh need to be accurately mapped to the new mesh as their initial values to be used in the next loading step to ensure numerical convergence and accuracy. The most widely used mapping methods are inverse isoparametric mapping (e.g. [13]) and direct interpolation (e.g. [14]). Both methods are implemented in the model.

Local arc-length methods

The material softening represented by nonlinear traction-COD relations (Fig. 2), together with constant boundary changes due to discrete crack propagation, poses significant challenges in finding a robust and efficient solution procedure, especially when post-peak responses are desired. A recent comparative study [2] has shown that the local arc-length algorithms are much more superior than the global ones in terms of numerical robustness and efficiency. The term "global" used herein means that the arc-length constraint equations include all the degrees of freedom of the whole structure whereas only selected degrees of freedom of dominant nodes are included in a "local" arc-length method. Based on a comparative study [2], May and Duan's [15] local updated normal plane constraint equation is used here.

The local arc-length formulation at one loading increment can be written as

$$\sum_e \nabla(\Delta\mathbf{u}^{\mathrm{T}})\cdot\nabla(\Delta\mathbf{u}^{k}) = l^2,\ \ (k = 2,3,4.....) \tag{3a}$$

$$\delta\lambda^1 = \frac{l}{\sqrt{\sum_e \nabla(\delta\bar{\mathbf{u}}^{\mathrm{T}})\cdot\nabla(\delta\bar{\mathbf{u}})}} \tag{3b}$$

$$\delta\lambda^k = \delta\lambda^1 - \frac{\sum_e \nabla(\delta\bar{\mathbf{u}}^{\mathrm{T}})\cdot(\nabla(\Delta\mathbf{u}^{k-1}) + \nabla(\delta\mathbf{u}^{*}))}{\sum_e \nabla(\delta\bar{\mathbf{u}}^{\mathrm{T}})\cdot\nabla(\delta\bar{\mathbf{u}})},\ \ (k = 2,3,4.....) \tag{3c}$$

where l is the specified arc-length; $\mathbf{u}$ is the displacement vector; λ is the loading factor; the symbols Δ and δ represent incremental and iterative changes respectively; k is the iterative number; and $\delta\bar{\mathbf{u}}$ and $\delta\mathbf{u}^*$ are iterative displacement vectors [16]. Eq. 3a defines the constraint equations. Eq. 3b determines the loading factor at the beginning of a loading increment $\delta\lambda^1$ whilst Eq. 3c determines the iterative load factors.

The summation in Eq. 3 is calculated in an element-by-element way. Only the elements contributing to structural nonlinearity are included in the constraints and they are termed dominant elements. In the present study, these are the nonlinear interface elements. The symbol ∇ denotes the relative displacement vector (RDV) of dominant elements,

$$\nabla(\mathbf{a}) = \begin{bmatrix} a_1 - a_n & a_2 - a_1 & a_3 - a_2 & \cdots\cdots & a_n - a_{n-1} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

in which $\mathbf{a}$ is any displacement vector in Eq. 3.

At the beginning of each loading step, the arc-length l (Eq. 3) must be determined to ensure the efficiency of the algorithms. The arc-length of the i^{th} loading increment l_i is often determined from [16]

$$l_i = \left(\frac{N_d}{N_{i-1}}\right)^m l_{i-1},\ \ i=2,\ 3,\ 4\ldots\ldots \tag{5}$$

where N_{i-1} and l_{i-1} are the iteration number and arc-length used at the $(i-1)^{\text{th}}$ loading step respectively. N_d is a desired optimum iteration number which is problem-dependent. The coefficient m ranges from 0.25-0.5 [17]. A default value of 0.5 is used in this study. Eq. 5 tends to adjust l to keep the iteration number around N_d.

The arc-length at the first loading increment l_1 can be determined from Eq. 3b by specifying an initial loading factor based on a proper reference loading condition, e.g., $\delta\lambda^1 = 0.1$. Special considerations are needed for the loading increment (e.g. j) when interface elements are first added because all the nodal displacements are used in Eq. 3b (i.e., global arc-length method) before whilst local arc-length is used hereafter. To ensure continuity and convergence, Eq. 5 is modified as

$$\begin{cases} \delta\lambda^1{}_i = \lambda_{ini} & i = 1,......j \\ l_i = \left(\dfrac{N_d}{N_{i-1}} \right)^m l_{i-1} & i = j+1,...... \end{cases} \tag{6}$$

in which λ_{ini} is the initial loading factor for the first j increments. A default value of λ_{ini} =0.1 is used in this study.

The specification of N_d is very important for numerical efficiency and stability in solving problems involving nonlinear interface elements. As cracks propagate, more interface elements are added and thus more RDVs are included in Eq. 3b. Using a constant N_d therefore tends to shorten arc-length l calculated by Eq. 7 and leads to smaller incremental loading factors and thus more increments. Based on extensive simulation experience, an empirical rule is proposed as follows

$$N_d = N_{\text{int}}{}^{\eta} + N_{d0} \tag{7a}$$

where N_{int} is the number of interface elements varying with loading increments and N_{d0} is the initial desired iteration number. $N_{d0} = 10$ is used in this study. The coefficient η is dependent on N_{int} and whether the tangential or the secant stiffness is used. It is proposed that, for secant-stiffness based algorithms,

η =1.5 when $N_{int} \leq 10$ and η = 1.2 when $N_{int} > 10$ (7b)

whilst for tangential stiffness based algorithms,

η=1.1 when $N_{int} \leq 10$ and η = 1.0 when $N_{int} > 10$ (7c)

The model adopts the modified Newton-Raphson iterative procedure with the convergence criterion based on the out-of-balance force factor, i.e.,

$$\frac{\|\mathbf{r}(\mathbf{u})\|}{\lambda_0 \|\mathbf{f}^e\|} \leq \beta \tag{8}$$

where $\mathbf{r}$ is the out-of balance force vector, $\mathbf{f}^e$ is the reference loading vector, β is the tolerance, λ_0 is the converged total loading factor of the last loading increment and the norms are Euclidean.

Using Eqs 3-8 the whole loading procedure is automatically simulated by specifying only four parameters: the reference loading vector $\mathbf{f}^e$, the initial loading factor λ_{ini}, the initial desired iteration number N_{d0} and the convergence tolerance β.

Computer implementation of the model

The cohesive crack model, the energy based crack propagation criterion and the remeshing procedure proposed by Xie [8] have been implemented in his program AUTOFRAP. This program, designed originally for workstations, has been transplanted to PCs and integrated with the other above-mentioned aspects into a FEA program with pre- and post-processing functions. Figure 4 shows a simplified flowchart of the program.

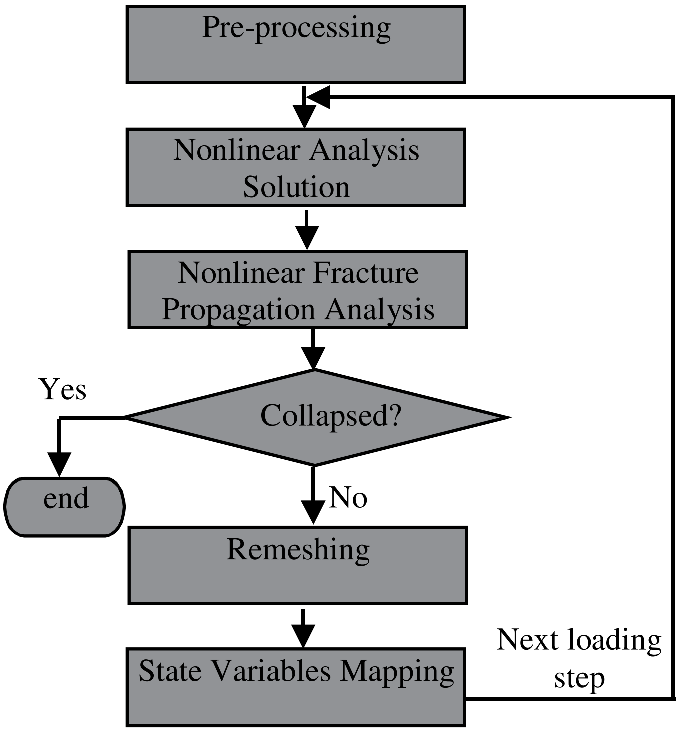

FIGURE.4 Key steps of discrete crack FEM

During remeshing, the finite element information changes constantly. This requires a robust method to execute a large number of manipulations on the mesh topology. Six arrays of type TYPE in FORTRAN 90, representing the six basic finite element entities, i.e., elements, nodes, edges, cracks, boundaries and materials, have been designed to carry out this task. Due to the flexibility of the dynamic memory allocation technique in FORTRAN 90, the sizes of these six arrays only need to be estimated by multiplying their sizes required for the initial FE mesh by an estimated size-expanding ratio to accommodate remeshing. The operations on the six arrays include enquiry, modification, deletion and addition of their sizes and properties. The most complex yet important manipulation is probably the deletion and addition of elements, which leads to the change of all relative topology information. In this model, all the topology and response information, together with their histories caused by continuous remeshing are stored in and dealt with by only manipulating the six arrays. This greatly facilitated the program development.

Conclusions

This paper has presented a finite element model for fully automatic simulation of multiple discrete crack propagation in reinforced concrete (RC) beams. The six key components of the model, i.e., the cohesive crack model, the bond-slip model, the energy-based crack propagation criterion, the remeshing procedure, the state variable mapping methods and the local arc-length algorithms, have been described. Its computer implementation is also briefly discussed.

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References

[1] ACI Report 446.3R-97, *Finite element analysis of fracture in concrete structures: state-of-the-art*, 1997.
[2] Yang, Z.J. and Proverbs, D. *Engng. Fract. Mech.*, vol. 71, 81-105, 2003.
[3] Ingraffea, A.R., Gerstle, W.H., Gergely, P. and Saouma, V., *ASCE J. Struct. Engng.*, vol. 110, 871-890, 1984.
[4] Yao, B.D. and Murray, D.W., *ASCE J. Struct. Engng*., vol. 119, 2813-2834, 1993.
[5] Prasad, M.V.K.V. and Krishnamoorthy, C.S., *Comp. Meth. App. Mech. Engng.*, vol. 191, 699-2725, 2002.
[6] Feenstra, P.H., De Borst, R. and Rots, J.G., *ASCE J. Engng. Mech.*, vol. 117, 754-769, 1991.
[7] Gerstle, W.H. and Xie, M., *ASCE J. Engng. Mech.*, vol. 118, 416-434, 1992.
[8] Xie, M., *Finite element modelling of discrete crack propagation*, PhD. Thesis, University of New Mexico, USA, 1995.
[9] Xie, M. and Gerstle, W.H., *ASCE J. Engng. Mech.*, vol. 121, 1349-1458, 1995.
[10] Petersson, P.E., *Crack growth and development of fracture zone in plain concrete and similar materials*, Report TVBM-1006, Lund Inst. of tech., Lund, Sweden, 1981
[11] Bocca, P., Carpinteri, A. and Valente, S., *Int. J. Solid Struct.*, vol. 27, 1139-1153, 1991.
[12] Wawrzynek, P.A. and Ingraffea, A.R., *Fini. Elem. Anal. Design*, vol. 5, 87-96, 1989.
[13] James, M.A., *A plane stress finite element model for elastic-plastic mode I/II crack growth*, PhD thesis, Kansas State University, 1998.
[14] Harbaken, A.M. and Cescotto, S., *Int. J. Num. Meth. Engng.*, vol. 30, 1503-1525, 1990.
[15] May, I.M. and Duan, Y., *Comp. Struct.*, vol. 64, 297-303, 1997.
[16] Crisfield, M.A., *Non-linear Finite Element Analysis of Solids and Structures, Volume II: Advanced Topics*, John Wiley and Sons, 1997.
[17] Bellini, P.X. and Chulya, A., *Comp. Struct.*, vol. 26, 99-110, 1987.
[18] Yang, Z.J. and Chen, J.F., *Proc., the 15th European Conference of Fracture*, Edited by Nilsson, F., KTH, Royal Institute of Technology, Sweden, August 11-13, 2004.