

# Criteria for Mixed-Mode Fracture Prediction in Ductile Material

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## Abstract

In order to assess a crack in a ductile structure subjected to mixed mode I-II loading, the tensile-shear fracture mode competition must be considered. Then it is possible to determine the critical load and the crack kinking angle.

A mixed-mode fracture criterion is proposed for predicting the onset and the direction of a ductile crack growth under mixed mode loading. The transition between tensile type fracture and shear type fracture (TS-transition) will be evaluated using the J-M<sup>P</sup> criterion proposed in (Li, Zhang, Recho, 2003, [1]).

In order to validate the criterion, the values of the fracture locus  $J_c$  as a function of M<sup>P</sup> and of the kink angle will be estimated by finite element simulation of the corresponding mixed-mode I/II fracture experiments performed in (Pironi, Dalle Donne 2001, [2]), (Dalle Donne, 1999, [3]). The connection of J-M<sup>P</sup> criterion with the criteria presented in [2] and [3] will be finally addressed.

## Introduction

The failure behaviour in ductile material cannot be predicted by elastic fracture models. For a decade, more and more experimental studies and numerical simulations have been devoted to the investigation of mixed mode ductile fracture. It is well known that both tensile and shear fracture mechanisms can occur at the crack tip in ductile material. The competition between tensile fracture (T-type fracture) and shear fracture (S-type fracture) determines the fracture toughness of a cracked metallic structure.

The value of the mixed-mode fracture toughness depends on the actual crack propagation mode and on the fracture parameter used. Ductile steel often displayed a decrease of fracture toughness with increasing mode II components [2]. In order to assess fracture toughness in a ductile cracked structure under mixed mode loading, the Tensile-Shear fracture competition must be considered at first. Then it is necessary to determine

the critical fracture loads and the crack bifurcation angles. So a series of criteria are needed. The first is the criterion of TS transition, which is used to determine the type of fracture, Tensile-type or Shear-type; secondly, the criterion of critical loading is needed for both types of fracture to estimate whether the crack begins to propagate under a mixed mode loading. The third one is the criterion of bifurcation angle. It can be used to predict the crack growth angle whatever the crack type is.

For elastic material, different criteria can be used to calculate the bifurcation angle. For example, the maximum circumferential stress  $\sigma_{\theta\theta max}$  criterion (Erdogan and Sih, 1963, [6]), the maximum energy release rate criterion (Palaniswamy and Knauss, 1978, [7]), the stationary strain energy density criterion (Sih, 1974, [8]), the crack tip opening displacement (or angle) criterion (Sutton et al. 2000, [5]), and so on. Recently we have developed the  $J$ - $M^p$  based criteria [1] to assess the propagation of a crack in elastic-plastic material under mixed mode loading.

The aim of this paper is to propose an experimental procedure to establish the mixed mode fracture criteria for predicting the onset and the direction of a ductile crack growth in structure components. The J-integral and the plastic mixity parameter  $M^p$  are used as basic fracture parameters in this investigation. The  $J$ - $M^p$  based criteria is adopted to evaluate the transition between Tensile type fracture and Shear type fracture (TS-transition) and the bifurcation angle of a crack. The critical loading angle of crack propagation is predicted by our new proposed method on the basis of the experimental results.

The fracture toughness is studied on the CTS specimen made of steel StE550. Finally, by comparing the numerical calculation with the experimental data, the validation of the proposed criteria is discussed.

## Proposed criteria

Combining experiment and numerical calculation, we propose a procedure to predict the crack propagation under mixed mode loading. Three criteria are introduced in the procedure. They are the TS transition criterion, the bifurcation angle criteria for tensile and shear crack, and the critical load criterion.

### *TS transition criterion*

When a crack exists in an elastic-plastic material, the angle of crack growth depends on the competition between cleavage tensile fractures (T-type fracture), related to the void growth and coalescence near the crack tip, and ductile shear fracture (S-type fracture), dependent on the plastic strain concentration ahead of the crack tip. Recently, we have developed the  $J$ - $M^p$  based criteria [1] in order to determine this crack growth angle.

In the case of a crack in an elastic-plastic material under mixed mode loading, Shih (1981) [9] showed that the stresses, strains and displacements fields near the crack tip are dominated by the HRR singularity, and can be characterized by two parameters, the  $J$ -integral and the mixity parameter  $M^p$ . The mixity parameter  $M^p$  is defined as follows:

$$M^p = \lim_{r \rightarrow 0} \frac{2}{\pi} \tan^{-1} \left| \frac{\sigma_{\theta\theta}(\theta = 0)}{\sigma_{r\theta}(\theta = 0)} \right| \quad (1)$$

The main idea of the  $J$ - $M^p$  based criterion is as follows: the TS-transition occurs at a value of  $M^p$ ,  $M^p_c$ , that is a material constant.  $M^p$  varies from 0 to 1. When  $M^p = 0$ , it is the

case of pure mode II and when  $M^p = 1$ , it is the case of pure mode I. A numerical method has been developed (Li, 1998, [4]) to determine these two parameters for a crack subjected to mixed mode loading.

Experimental studies showed that, for an elastic-plastic material, there is a transition from T-type fracture to S-Type fracture. This transition is controlled by the critical value of the mixity parameter  $M_c^p$  which can be determined by means of experiments according to the critical fracture toughness  $J_{IC}$  and  $J_{IIC}$  ( $J_{IC}$  is obtained from a pure mode I tensile test and  $J_{IIC}$  from a pure mode II shear test). If the mixity parameter  $M^p$  for a given loading case is greater than  $M_c^p$ , the crack will propagate by T-type fracture. On the other hand, if  $M^p$  is smaller than  $M_c^p$ , the crack will propagate by S-type fracture

### ***Bifurcation angle criterion for a tensile crack***

When the crack grows in tensile manner, the maximum hoop stress (MHS) criterion is widely used in crack bifurcation angle assessment. It means the crack will propagate in the direction of the maximum circumferential stress  $\sigma_{\theta\theta_{max}}$ . For elastic-plastic cracks, the angles of the near-tip can be represented as function of  $M^p$  and hardening coefficient  $n$ . These relationships have been given in Fig. 1 [1].

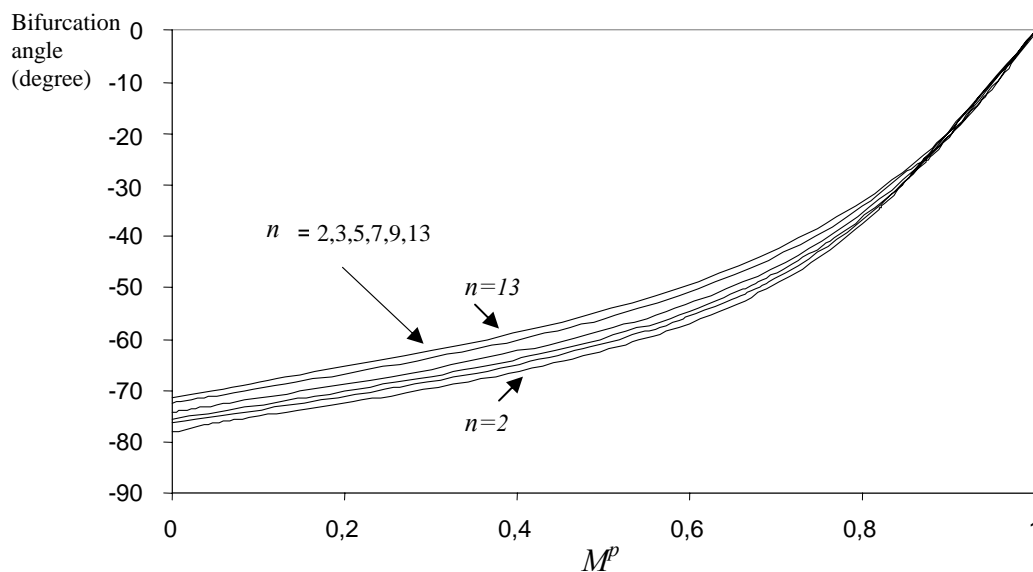


Figure 1 Bifurcation angles of a tensile crack according to the MHS criterion

### ***Bifurcation angle criterion for a shear crack***

When the crack grows in shear manner, the situation is more complicated. The crack will follow one of the slips lines near the crack tip. According to Li et al.[1], although several slip bands can be observed to develop from the crack tip, only two slip bands are possible directions of the shear crack growth because the shear stress along these two slip bands are larger comparing to those along others. These two bands are approximately at  $45^\circ$  with

respect to the initial crack plane, and, nearly along the crack plane. The bifurcation angles versus  $M^p$  and  $n$  are plotted in Figure 2.

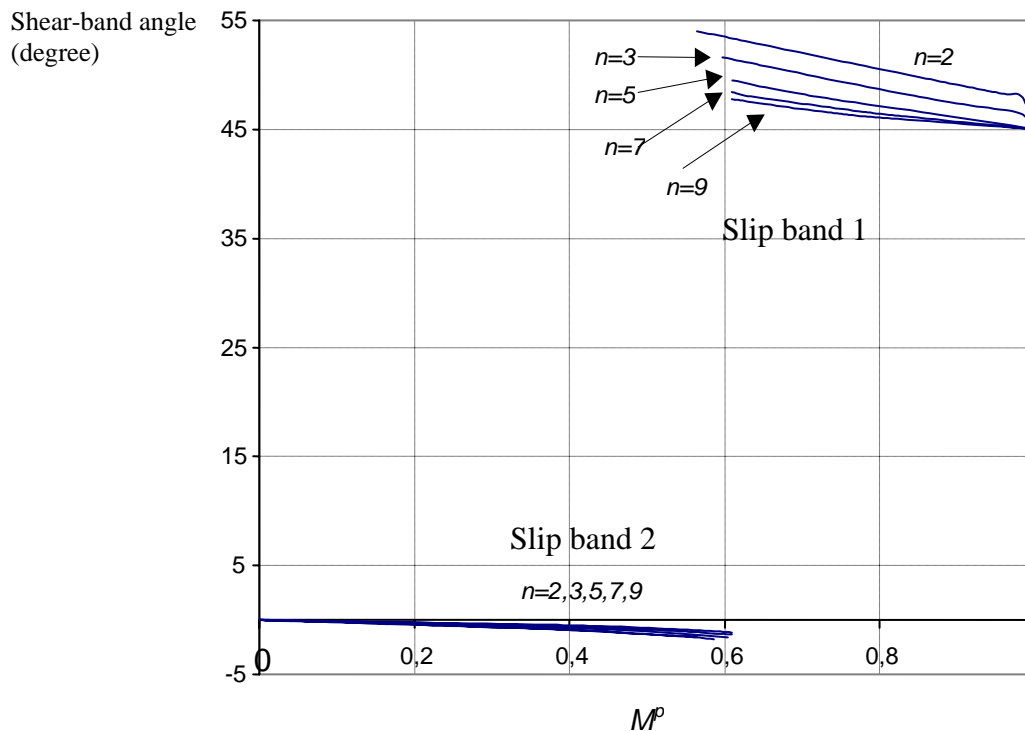


Figure 2 Bifurcation angles of a shear crack according to the slip band criterion

### ***Critical load criterion***

It is well known that the values of the mixed mode fracture toughness depend on the actual crack propagation mode and on the fracture parameter used. Ductile steels often displayed a decrease of fracture toughness and tearing resistance curves with increasing mode II components, whereas materials with a lower ductility generally showed higher mode II crack resistance values [2]. On the basis of the experiment performed by Dalle Donne and Pironi [2], a criterion is developed to assess the initial propagation of a mixed mode crack.

The first step is the mixed mode experiment. The fracture tests are conducted on CTS specimens, which are mounted on the loading device shown in Fig. 3 so that they can be subjected to different loads with different loading angles. Several loading angle such as  $0^\circ$ ,  $45^\circ$ ,  $75^\circ$ , and  $90^\circ$  are selected in the experiments. The crack begins to propagate at a (experimentally evaluated) value of crack tip opening of 0.19mm. For each loading angle, the crack tip opening, crack length and applied load are recorded at the same time (for details see [2]). Therefore, the critical load is the load corresponding to a crack initiation.

The second step consists in a numerical calculation. For a given loading angle and the corresponding critical load, the  $J_c$  and  $M^p$  can be calculated numerically by means of FEM [4]. Consequently, the  $J_c$  versus  $M^p$  fracture locus of the material can be obtained. For any loading angle, one can assess the corresponding fracture toughness  $J_c$  from this curve.

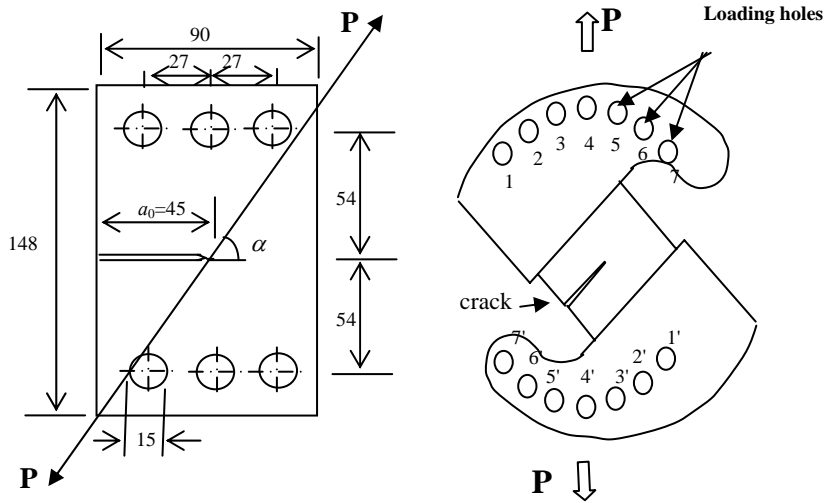


Figure 3 mixed-mode loading device (dimensions in mm)

To sum up, the proposed procedure for predicting the crack propagation are described as follows:

- (1) Determination of the fracture locus  $J_c-M^P$
- (2) Calculation of  $J$  and  $M^P$  for a given loading
- (3) Comparison between  $M^P$  and  $M_c^P$ . If  $M^P > M_c^P$ , the crack will propagate by T-type fracture, if  $M^P < M_c^P$ , the crack will propagate by S-type fracture
- (4) Comparison between  $J$  and  $J_c$ . If  $J > J_c$ , crack will begin to grow
- (5) Determination of the bifurcation angle (and crack growth path)

## Validation of the procedure

In this study, the fracture tests were conducted on 4mm thickness and 90 mm width CTS specimens of the fine grained structural steel StE550, the yield strength is about 580 MPa and ultimate strength is about 650 MPa, the critical mixity parameter  $M_c^P$  is about 0.77, which is issued from elastic mixity parameter  $M_c^e = 0.68$ . A fatigue pre-crack was introduced up to  $a/w \approx 0.6$ . The specimens were tested under  $0^\circ$ ,  $15^\circ$ ,  $45^\circ$ , and  $75^\circ$  loading with respect to the crack axis. When loading angle  $\alpha = 0^\circ$ , this is the case of pure mode II loading, on the other hand, when  $\alpha = 90^\circ$ , it is pure mode I loading. The fracture toughness data are presented in terms of  $J_c$  and  $M^P$  in this investigation. Table 1 lists the experimental results.

Table 1 experimental results

Loading angle $\alpha$	$75^\circ$	$45^\circ$	$15^\circ$	$0^\circ$
Critical load (KN)	27	30	37	35
Fracture toughness $J_c$ (KN.m)	71	56	39	26
Mixity parameter $M^P$ (FEM results of Eq.1)	0.98	0.89	0.52	0

The fracture locus  $J_c-M^P$  is shown in Fig.4. For a given loading condition, we can determine easily whether the crack begins to propagate or not from Fig.4. For example,

when the crack is subjected to 35 kN, 30° loading, we get  $M^p = 0.72$  and  $J = 55$ . From Fig4, when  $M^p = 0.72$ ,  $J_c$  is about 47, therefore the crack will grow.

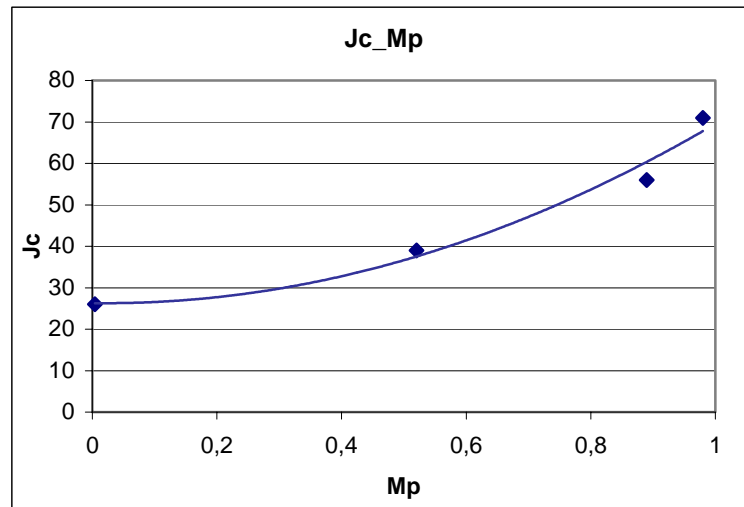


Figure 4 fracture curve as a function of  $J_c$  and  $M^p$

In order to verify the  $J$ - $M^p$  criteria, we calculate the bifurcation angle for each loading condition and compare them with the experimental data. Fig. 5 shows the bifurcation angle of 45° loading. Table 2 lists the results of kink angles.

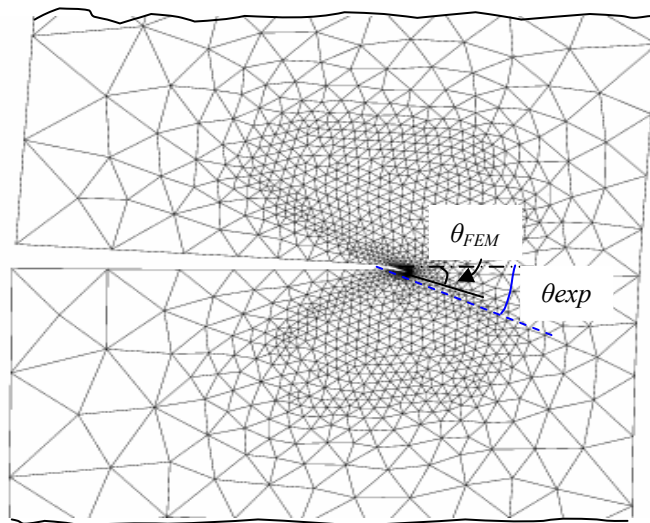


figure 5 bifurcation angle of 45° loading

Table 2 results of bifurcation angles  $\theta$

Loading angle $\alpha$	75°	45°	15°	0°
$M^p$ (FEM result of Eq.1)	0.98	0.89	0.52	0
Fracture type	T-type	T-type	S-type	S-type
Numerical results	-12°	-23°	-1.7°	0°
Experimental results	-13.5°	-27.6°	5.7°	-11.1°

According to  $J-M^p$  criteria, when loading angles are  $75^\circ$  and  $45^\circ$ ,  $M^p$  are greater than  $M_c^p$  (0.77), they are T-type fracture and the crack grows in the direction of  $-12^\circ$  and  $-23^\circ$  with respect to the crack axis. When loading angle are  $15^\circ$  and  $0^\circ$ ,  $M^p$  are smaller than  $M_c^p$ , they are S-type fracture and the crack grows along one of slip bands, the crack growth angle are  $-1.5^\circ$  and  $0^\circ$ . The numerical results of T-type crack growth are close to the experimental data. On the other hand, the S-type crack growth angles calculated numerically follow a different trend with respect to the experimental values. In [2] it was postulated that the S-type crack growth is promoted by a slight positive triaxiality, which is not accounted for by the slip band criterion.

Finally, in order to verify the accuracy of our method for evaluating J-integral, we evaluate J values by another experimental based method [2], in which J ( $J_{\text{experiment}}$  in Fig. 6) is defined by

$$J = J_e + J_p \quad (2)$$

where,

$$J_e = \frac{K_I^2 + K_{II}^2}{E} \quad (3)$$

$$J_p = \eta_p \frac{U_p}{B(W-a)} \quad (4)$$

and  $U_p$  is calculated from the crack tip opening and shearing displacements  $\delta_I$ , and  $\delta_{II}$  respectively, monitored during experiments and  $\eta_p$  is the mixed-mode plastic  $\eta$  – factor [9]. Fig.6 shows the J results of  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  loading. It can be noted that our J values are in good agreement with the results of equation 2.

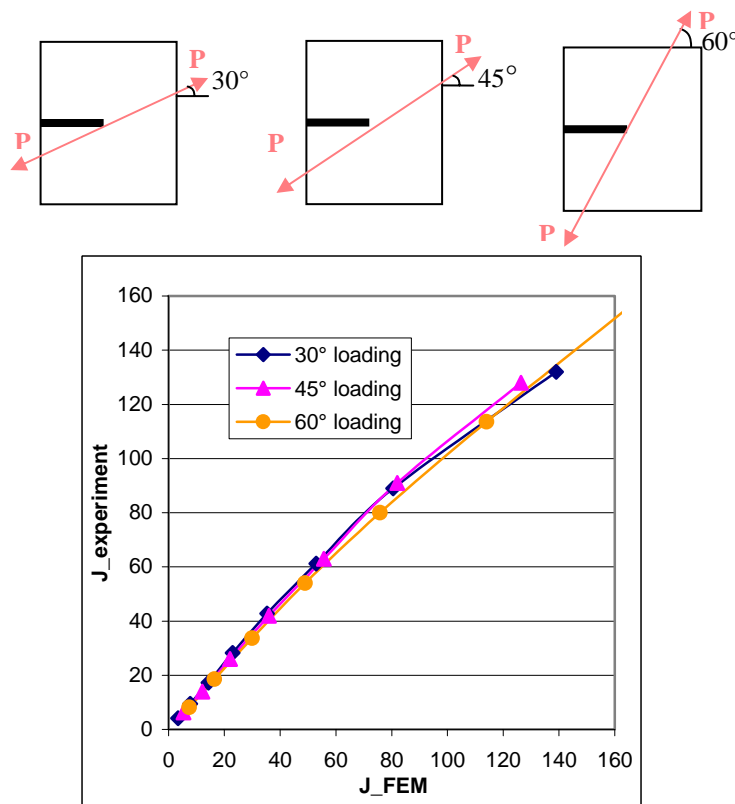


Figure 6 J results of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  loading

## 4. CONCLUSION

An experimental procedure was proposed to predict the propagation of a mixed mode crack in ductile material. This procedure includes three criteria to determine fracture type, to evaluate crack bifurcation angles, and to estimate the critical loads. This method is validated numerically and experimentally with CTS specimens of ferritic steel. The material constant  $M^p_c$  and critical loads of several loading angles are given by mixed mode experiments, and then the criteria can be determined by FEM calculation. So it is easy to realize.

Further experiments are needed to verify the fracture criteria so that they can be used in different material.

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