APPLICATIONS OF THE THEORY OF CRITICAL DISTANCES TO THE PREDICTION OF BRITTLE FRACTURE IN METALS AND NON-METALS

D. Taylor
Department of Mechanical and Manufacturing Engineering, Trinity College Dublin, Dublin 2, Ireland
dtaylor@tcd.ie

Abstract

The theory of critical distances (TCD) proposes that the failure of a body containing a stress concentration (e.g. a crack or notch) can be predicted using elastic stress information in a critical region close to the notch tip. This paper investigates the use of TCD for predicting brittle fracture. The critical region is defined in terms of a characteristic material length constant, L, which is a function of the fracture toughness K_c and a failure stress, σ_o. For very brittle materials (ceramics), σ_o is equal to the plain-specimen strength but for polymers and metals σ_o has a larger value. Two complications arise: (i) there exist non-damaging notches whose strength is equal to the plain-specimen strength, and; (ii) strength varies with the degree of constraint. These effects can be incorporated into TCD allowing predictions of experimental data for many types of materials and stress concentration features.

Introduction

This paper is concerned with the prediction of brittle fracture, defined as any failure which occurs by crack initiation and/or propagation in a rapid, unstable manner. Stress concentrations such as notches and cracks frequently cause brittle fracture even in relatively ductile materials. Traditional methods for the prediction of failure in materials use one of two approaches: (i) failure occurs when the maximum stress (or strain) in the body reaches some critical value, say the ultimate tensile strength σ_u; (ii) failure occurs when the stress intensity associated with a crack reaches some critical value: the fracture toughness K_c. Unfortunately these approaches only work in a limited number of cases; approach (i) works only for plain (i.e. unnotched) specimens in simple tension, or for notches which are so large that the local gradient of stress near the notch is negligible. Even mild stress gradients (e.g. bending loads applied to plain specimens) cause difficulties for this approach. Approach (ii), on the other hand, only works for long, sharp cracks; it is known to break down if the crack length is physically short (sub-millimetre) or if applied to notches having a significant root radius. In practice, then, many industrial components have stress-concentration features which neither of these approaches can handle, and several different theories have been suggested to solve this problem.

It has been recognised for some time that the theory of linear elastic fracture mechanics (LEFM), in order to be strictly valid, requires another length constant. Irwin [1] recognised this in his use of the plastic zone size, r_y, as an addition to the physical crack length. Broberg [2] made the following argument: if we assume that failure of material near the crack tip occurs at some critical stress, σ_c, then, given the evidence that the nominal fracture stress, σ_f, is a function of the crack length, a, it follows on dimensional grounds that another length constant is required, which will be a material parameter, L, so that we can write $\sigma_f = \sigma_c f(a/L)$. Over the years, various theories have arisen which use this length constant, explicitly or implicitly, calculated in various different ways. In particular, a number of workers have proposed that failure can be predicted by modifying the critical stress idea (approach (i) above) so that the stress to be used is not the maximum stress (at the notch root) but the stress at a point located at a certain distance from the notch. Other workers have used the average stress on a line drawn from the notch. These two methods, illustrated in fig.1, which we call the point method (PM) and line method (LM), were first proposed by Peterson [3] and Neuber [4], respectively, for use in fatigue. They seem to have been independently discovered for use in the prediction of brittle fracture in fibre composites [5], where they are frequently used today.

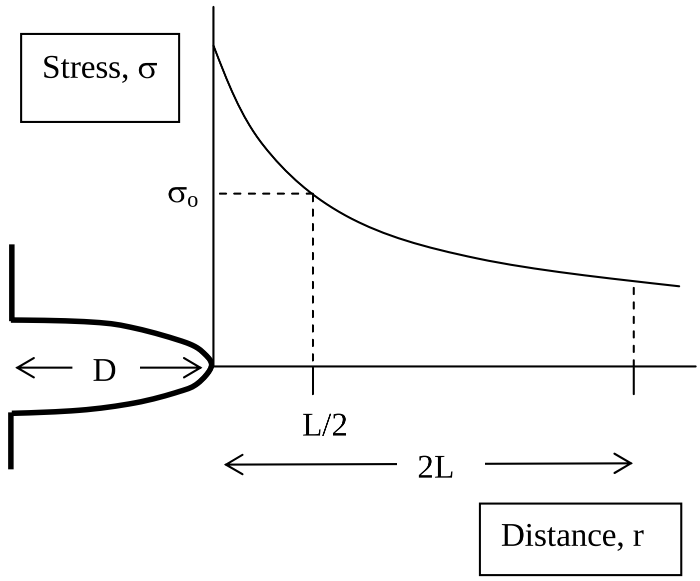

Fig.1: Critical distance methods: the PM uses the stress at a point r=L/2 from the notch root; the LM uses the average stress over a distance r=0 to 2L

The application of LEFM theory shows that, in order to be applicable to sharp cracks, these two critical distances must have the values L/2 and 2L where:

$$L = (1/\pi)(K_c/\sigma_o)^2 \qquad (1)$$

The stress σ_o represents the strength of the material, but its definition is not straightforward (see below). The aim of the present paper is to investigate the use of the PM and LM for prediction of brittle fracture, using experimental data from the literature and our own labs.

Ceramic materials

Fig.2 shows data on the strength of ceramic materials SiC and Al_2O_3 [6,7], as a function of crack length (for sharp cracks) and notch root radius (for notches of constant length). It is clear that ceramic materials display significant short-crack effects – i.e. the fracture strength of small cracks deviates from that predicted by LEFM – a fact which must be taken into account when considering the effect of small defects on strength. The measured K_c value is approximately constant for long notches up to a critical root radius: this is very useful for the determination of fracture toughness in ceramic materials for which the introduction of long cracks into specimens is difficult. Predictions of these effects, made using the PM and calculating L using eqn.1, were very successful. Here the value of σ_o used was the tensile strength of plain, defect-free specimens, σ_u. Many other sets of data on ceramics were analysed with similar success.

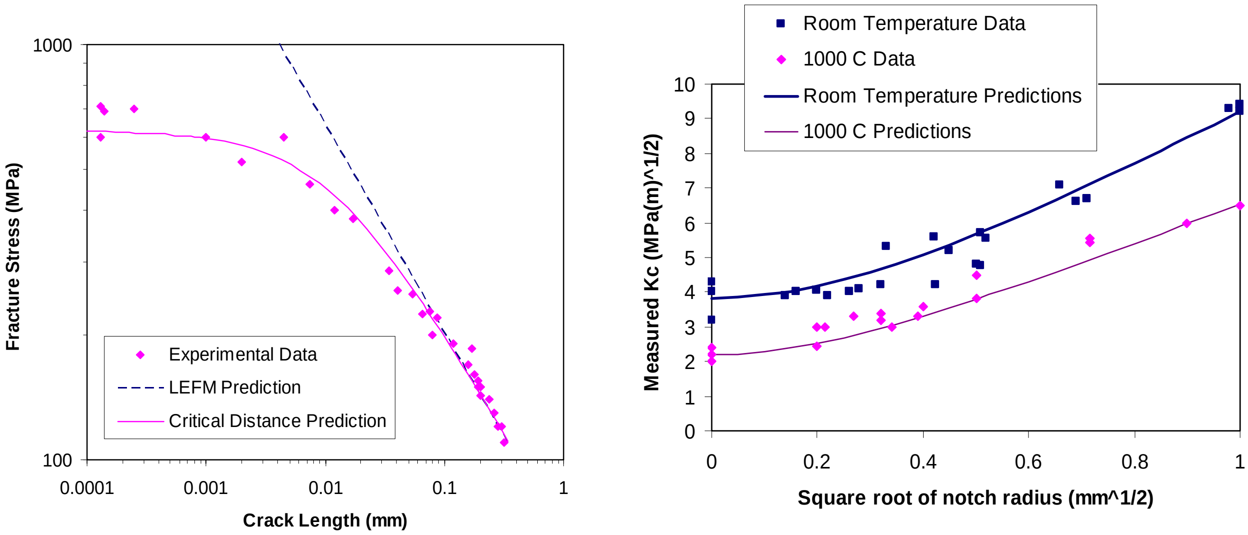

Fig.2: Experimental data on ceramic materials showing the effect of crack length (on the left) and notch root radius (on the right). Predictions using TCD (the point method PM).

Polymers

It was found that the TCD could also be used to predict data from brittle polymers such as PMMA (Perspex), PC (Polycarbonate) and PS (Polystyrene), but that the value of σ_o was no longer equal to σ_u. Optimum values of σ_o were larger, of the order of 2-3 times σ_u. This was also reported by Kinloch and Williams [8]: we call this the Modified TCD method. We conducted tests in our own laboratories on PMMA specimens containing a variety of notch shapes, including some 3D shapes such as surface hemispheres as well as 2D notches and holes. Fig.3 summarises the results.

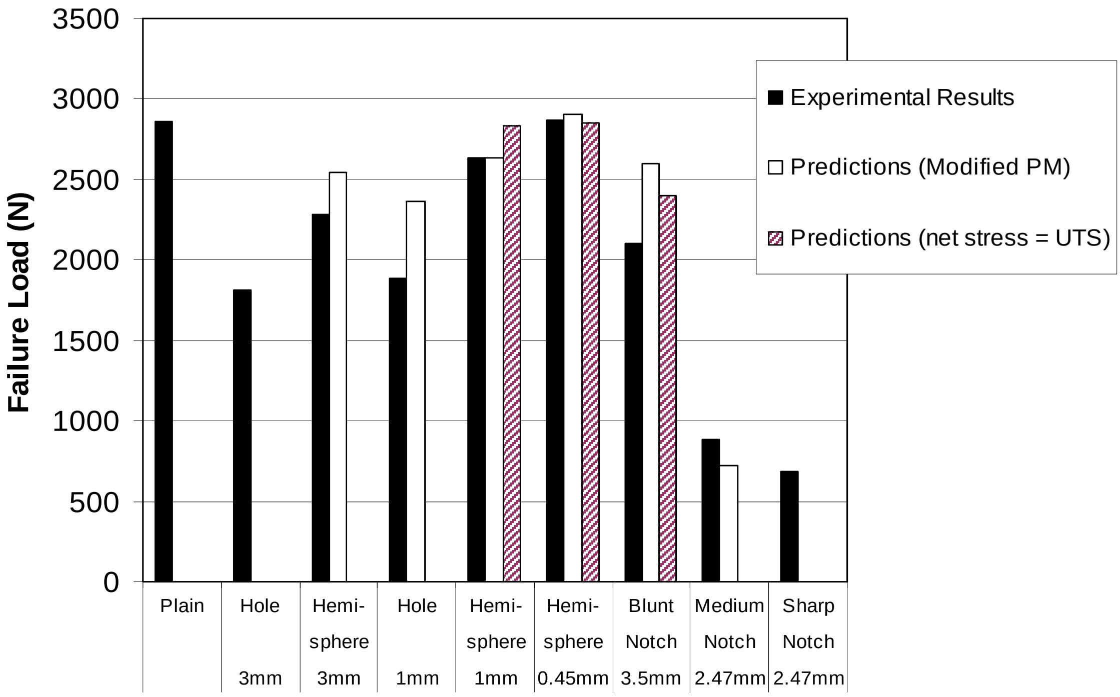

Fig.3: Data and predictions for PMMA

Two different predictions are shown on this figure: a TCD prediction (the PM, using a modified value of σ_o=146MPa) and a prediction based on reaching σ_u (=71.5MPa) on the net cross-section. For some notches (especially a very blunt notch with a K_t factor of 2.25 and two small hemispheres with K_t values of 2.1) the net-strength prediction was similar to the TCD prediction and for some of the smaller hemispheres failure occurred elsewhere in the specimen. This indicates that there are some notches which are 'non-damaging' in that they do not reduce the strength of the specimen (except insofar as they reduce the net cross section). This can be expected for any notch which has a K_t factor less than σ_o/σ_{u}, which in this case was 2.0. This information is very useful for defect assessment in polymers. Likewise there exist non-damaging short cracks (e.g. in PS [9]) whose length can also be predicted.

In PC there is a transition from brittle to ductile behaviour which is dependant both on specimen thickness, t, and notch root radius, ρ. Sharper notches tend to fail in a brittle manner, by sudden crack propagation, whereas blunter notches fail by a gradual spread of plasticity. Nisitani and Hyakutake [10] recorded the stress for brittle fracture and for the onset of ductile fracture. As fig.4 shows, they plotted their results in terms of the maximum stress at the notch root, as a function of 1/ρ. Three different specimen thicknesses were used.

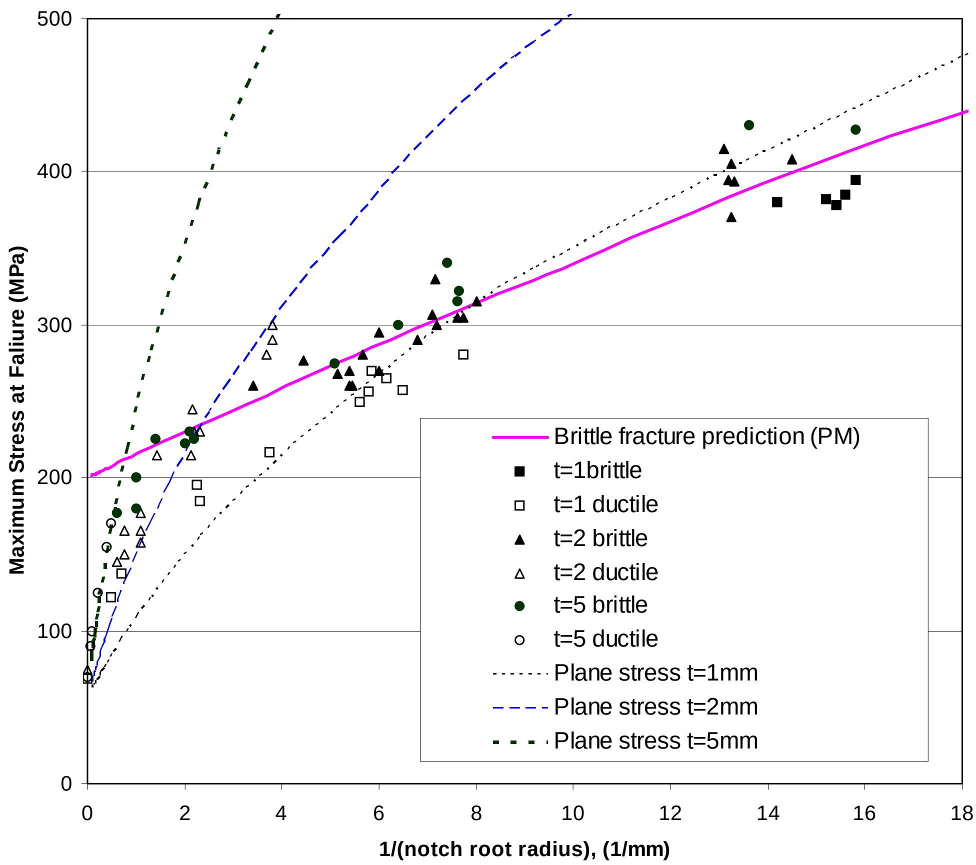

Fig.4: Experimental data on notches in PC specimens of three different thicknesses (from [10]). Brittle fracture (solid symbols) can be predicted using the PM; the onset of ductile fracture (open symbols) coincides with the onset of plane stress conditions.

The brittle-to-ductile transition in polymers is generally associated with a loss of constraint: i.e. a plane strain to plane stress transition. The brittle fracture strength was found to be independent of thickness: the data (solid symbols on fig.4) could be accurately predicted using TCD (the PM). The onset of plane stress conditions was estimated (after Irwin [1]) using the condition that the plane stress plastic zone size is equal to the thickness. To find the plastic zone size for the notches from the elastic stress analysis we used a variation of the point method in which the critical stress was the yield strength and the critical distance was the thickness t. This gives a series of prediction lines on the figure which correspond quite closely to the experimental data on ductile failure (open symbols). The intersections between these lines and the brittle-fracture line give predictions of the brittle/ductile transition for each specimen thickness.

Metals

It was found that the prediction of brittle fracture in metals could be accomplished using a similar approach to that used for polymers: the Modified TCD with allowance made for constraint effects. Fig.5 shows the successful use of the Line Method (LM) for notched aluminium alloy specimens tested at different temperatures [11].

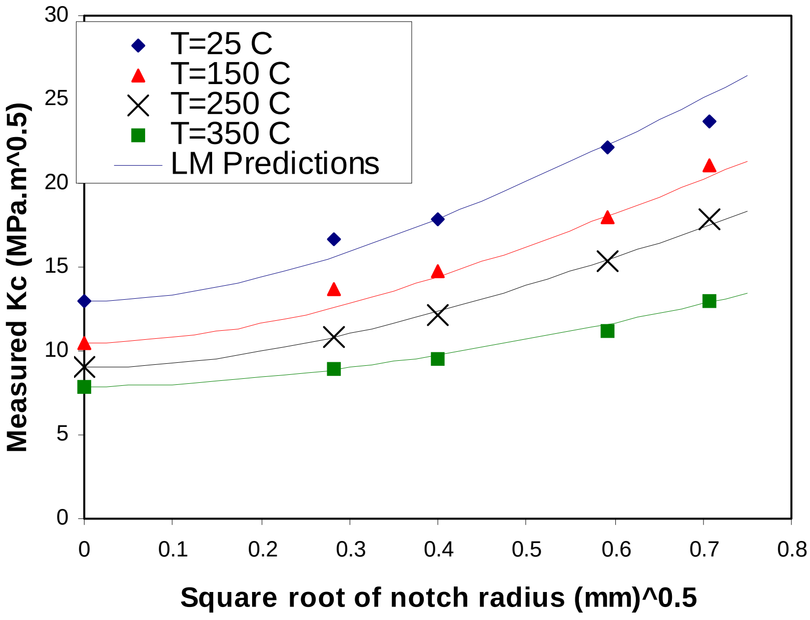

Fig.5: LM predictions of fracture data on notched aluminium alloy [11] at various temperatures (all in plane strain conditions)

In this case the specimens were all under plane strain conditions. In some cases a loss of constraint can occur as notch radius is increased, due to the higher applied stress before failure. Fig.6 shows data on steel failing by low-temperature cleavage [12]: lines are drawn on the figure showing the onset of plane stress and of general yield on the net section. In this case cleavage fractures continue to occur throughout but it is clear that, once plane stress conditions arise, the fracture stress suddenly increases: failure occurs after general yield and no longer corresponds to the TCD prediction line derived for plane strain. From other data (not shown here) we found that a TCD prediction can also be made for plane stress conditions, so that the effect of loss of constraint can be incorporated into the TCD.

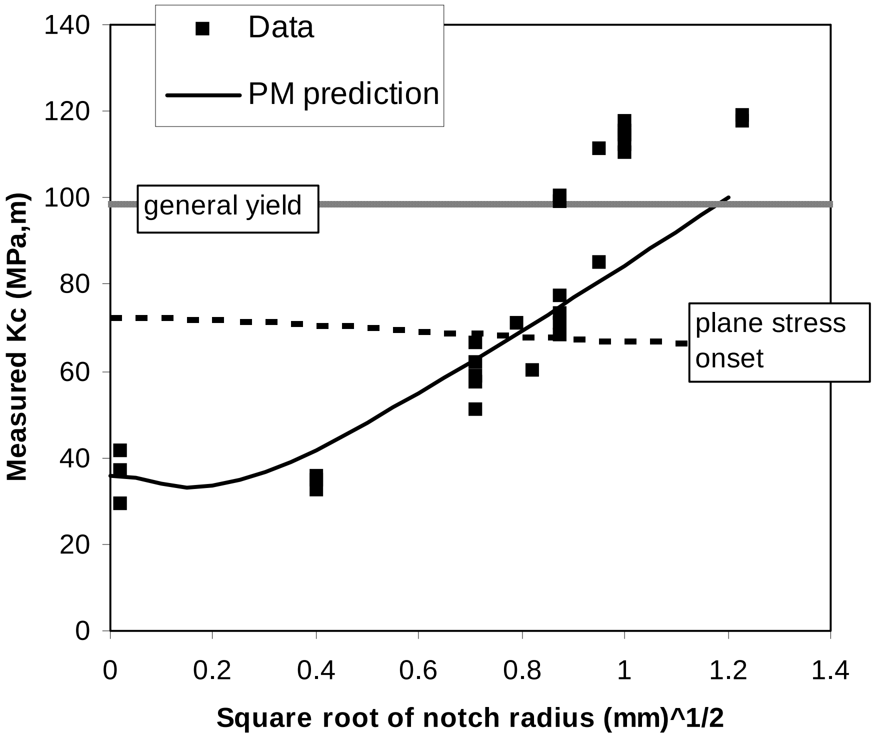

Fig.6: Data on cleavage fracture in steel at low temperatures [12]. Predictions using TCD (the PM) for plane strain conditions, plus estimates of the onset of plane stress and general yield.

Discussion

The Theory of Critical Distances has had a chequered history, having been discovered and re-discovered many times by workers in different fields. It is a very easy technique which requires only an elastic stress analysis and some simple material properties. So it is surprising that it is only used extensively in this explicit form in one area: the prediction of fracture in fibre composite materials. Indeed it is also used quite frequently in high-cycle fatigue, but only in the form of empirical equations for notch-sensitivity factors, devised by Peterson and Neuber, rather than in the explicit form described here. This is a pity because it is very easy to use the method in conjunction with finite element analysis of components, as we have shown previously [13]. The use of TCD for predicting brittle fracture in metals and ceramics is almost unheard of, and its use in polymers is limited to a few research papers (e.g. [8]): we have not been able to find any reference to the use of the technique in industrial design for any of these classes of materials.

This paper has focussed on two issues which caused difficulties for previous researchers. The first of these is the issue of non-damaging notches and non-damaging cracks. It is clear that, in cases where $\sigma_o > \sigma_u$, there are going to be situations where the TCD approach will be invalid. Plain specimens are the obvious case, plus any cracks or notches for which the TCD predicts failure at a nominal stress higher than the plain-specimen strength. The significance of this theoretically is that σ_u is not relevant to the failure of notched specimens because there is a change in the mechanism of failure (from ductile to brittle) or because the failure of plain specimens is determined by some other factor (for example by inherent defects in the material). We found by examining experimental data that, for all the notches which were invalid for TCD, the measured net-section strength of the specimen was invariably close to σ_u, i.e. the notch had no effect on strength once the net cross-section was taken into account. However it is not clear that this conclusion would be true for all materials: interaction effects could conceivably lower the strength still further. The second issue that we focussed on was the effect of constraint. It is well known that, in conventional fracture mechanics, the value of K_c changes with constraint, owing to the relative difficulty of crack propagation under plane stress conditions. Since the TCD is essentially an extension of LEFM, it could be expected that the values of the constants L and σ_o would also change with constraint, and this indeed was shown to be the case. To our knowledge there is currently no theoretical method, either in LEFM or in TCD, for predicting how the material properties will change: currently the only solution is to obtain experimental data at different constraint levels. In practice in a component of complex, three-dimensional shape it is not a trivial matter to define the level of constraint at a given location.

The physical significance of the parameters L and σ_o is unclear. Some workers have suggested that L is related to the size of microstructural features, and in many cases it does take values similar to, for example, the grain size in metals. In this respect there is a link between TCD and the RKR model [14] which predicts cleavage fracture in steels using a mechanism based on the cracking of a carbide particle in a grain boundary ahead of the notch. Others propose that L measures the size of the process zone or damage zone ahead of the notch when fracture occurs, though it is not clear why this should be a constant. However it is an advantage of TCD that, like LEFM, it does not require information about the mechanism of failure and indeed can be used to predict failures that happen from different micromechanisms. On the other hand, TCD shares some of the limitations of LEFM: it is an elastic approach which is not expected to work under conditions of general plasticity, or indeed any conditions which violate the ‘contained yielding’ criterion which requires the plastic zone to be small compared to the surrounding elastic region of the rest of the body.

Conclusions

1. The theory of critical distances (TCD) can be used to predict brittle fracture in bodies containing cracks and notches, in a range of material types: ceramics, metals and polymers. In particular it can predict the deviations from LEFM behaviour that happen with short cracks and with notches of finite root radius.

2. In ceramics the characteristic strength parameter σ_o is equal to the plain-specimen strength σ_u. In polymers and metals σ_o is larger, typically by a factor of 2-3. The consequence of this larger value is the existence of non-damaging notches (below a critical K_t factor) and non-damaging cracks (below a critical length).

3. Constraint effects play a significant role, altering the values of the constants L and σ_o and determining a ductile/brittle transition in polymers. These effects can be incorporated into the TCD by introducing a measure of the constraint level.

4. The method is relatively simple and easy to use in industrial design, since it requires only an elastic finite element analysis of the component.

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