AN INSTABILITY ANALYSIS FOR A CRACK GROWTH SITUATION BASED ON THE COMMON FORMAT

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Abstract

The Common Format Equation (CFE) approach, together with a crack growth law that relates crack extension to plastic displacement, are used to develop a relation between load, *P*, and crack extension, Δa, for a situation where there is stable crack growth. The P-Δa relation is then differentiated in order to obtain the condition for which $dP = 0$, i.e., the critical value of crack length for which maximum load is attained. Substitution of the critical crack length on the CFE with crack growth incorporated, and in the crack growth law, will yield the values of maximum load and critical plastic displacement, respectively. The overall *P*-*v* curve with crack growth is obtained by combining the Common and Concise Formats, the latter giving the value of the elastic component of the displacement due to crack growth. Some examples of C(T) specimen *P*-*v*-*a* data are analysed to show the potential of the method.

Introduction

A ductile fracture methodology requires both the fracture toughness of the material as well as the load-displacement relations. The former is usually given by the *J*-*R* curve, i.e., the *J* vs. crack extension curve, whereas the latter is given by the calibration functions. Since the emphasis has usually been placed upon the *J*-*R* curve rather than on the calibration functions, instability has often been dealt with from the standpoint of the *J* vs. stable crack extension behavior.

The calibration functions relate the load, *P*, to the displacement, *v*, and the crack length, *a*, of a fracture toughness specimen. Donoso and Landes developed the Concise Format [1] for the elastic regime ($v = v_{el}$), and the Common Format [2], for the plastic component of the displacement, v_{pl}. These *P*-*v*-*a* relations are based upon separation of the load [3] into a geometry function, which depends upon the stationary crack (or ligament) length, and a hardening function, which depends on v_{pl} in the case of the Common Format.

For a stationary crack, the crack (or ligament) length is constant, and *P* and *v* become the variables of the calibration function. When there is stable crack extension, however, the crack length *a* also becomes a variable, so that a separate relation between *a* and v_{pl} is needed. In this paper, instability for a stable crack growth situation is analysed. At the instability point, the load attains a maximum, then decreases as stable crack extension proceeds further. The load *P* is ultimately expressed as a function of only one of the other two variables (*a* and v_{pl}), through the use of a “crack growth law” [4], that is, the relation between *a* and v_{pl}.

The Common Format Equation

The Common Format Equation, CFE, was originally proposed by Donoso and Landes [2] as an extension of the load separation concept [3] to describe the load-plastic displacement relationship for a blunt-notch fracture specimen. As such, it relates the applied load *P* with two variables representing the deformation of a fracture specimen with a stationary crack: v_{pl}/W, the plastic component of the load-line displacement, normalized by the width *W*, and *b/W*, the normalized ligament size (ligament *b* in lieu of the crack length *a*).

The Common Format also includes a term that denotes the out-of-plane constraint, Ω*. Thus the CFE is written as:

$$P = \Omega^* \cdot B \cdot C \cdot W \cdot (b/W)^m \cdot \sigma^* \cdot (v_{pl}/W)^{1/n} \tag{1}$$

where B is the specimen thickness; *C* and *m* are the geometry function parameters, and σ* and *n* are material properties, which are obtained from a material stress-strain curve [5], or directly from the specimen normalized load-normalized displacement curve.

Recently, a two-parameter power law type of crack growth has been proposed [4], that relates the change in crack length, Δa, with normalized plastic displacement, v_{pl}/W, i.e.,

$$\frac{\Delta a}{W} = l_0 \left(\frac{v_{pl}}{W} \right)^{l_1} \tag{2}$$

The term Δa, crack extension, may also be written in terms of the change in ligament size, that is, $\Delta a = b_o - b$, where b_o is the initial ligament size (equal to *W* minus the initial crack length, a_o). Thus, Eq. (2) gives the following expression for the current ligament size, *b*:

$$\frac{b}{W} = \frac{b_o}{W} - l_o \left(\frac{v_{pl}}{W} \right)^{l_1} \tag{3}$$

Substitution of Eqs. (2) and (3) into Eq (1), gives the CFE for a crack growth situation in terms only of crack extension, Δa:

$$P = DBCW \left(\frac{b_o - \Delta a}{W} \right)^m \left(\frac{1}{l_o} \frac{\Delta a}{W} \right)^{1/n l_1} \tag{4}$$

where *D* is the product of the parameters σ^* and Ω*.

On the other hand, substitution of Eqs. (2) and (3) into Eq (1) may yield the following expression in terms of only the plastic displacement when there is stable crack extension:

$$P = DBCW \left(\frac{b_o}{W} - l_o \left(\frac{v_{pl}}{W} \right)^{l_1} \right)^m \left(\frac{v_{pl}}{W} \right)^{1/n} \tag{5}$$

The fracture toughness test data is usually given as load vs. total displacement, with the crack extension data as a function of either load or displacement, as a complement to the *P-v* output. Eq. (5) shows the relation between load and plastic displacement for a crack growth situation. The elastic component of the displacement may be obtained with the use of the Concise Format, as has been explained elsewhere [4].

Instability Analysis

Equation (4) relates load to crack extension, $\Delta a = a - a_o$. Its generic shape is shown in Figure 1, in the manner of a P vs. crack length curve, for a total crack extension $\Delta a_f = a_f - a_o$. This type of behavior has been reported by several investigators in connection with stable crack extension in ductile materials [6]. The most salient feature of such a curve is the maximum displayed along the load axis. In fact, as Fig. 1 suggests, there is a maximum value of the load, P_{max}, attained at a critical value of the current crack length, a_c.

The value of the maximum load is obtained by differentiation of Eq. (4) with respect to a (or $\Delta a/W$). The solution for the normalized critical crack length is

$$\frac{a_c}{W} = \frac{1 + mnl_1\left({a_o}/{W} \right)}{1 + mnl_1} \tag{6}$$

or, in terms of critical ligament size,

$$\frac{b_c}{W} = \frac{b_o}{W}\left[\frac{mnl_1}{1 + mnl_1} \right] \tag{7}$$

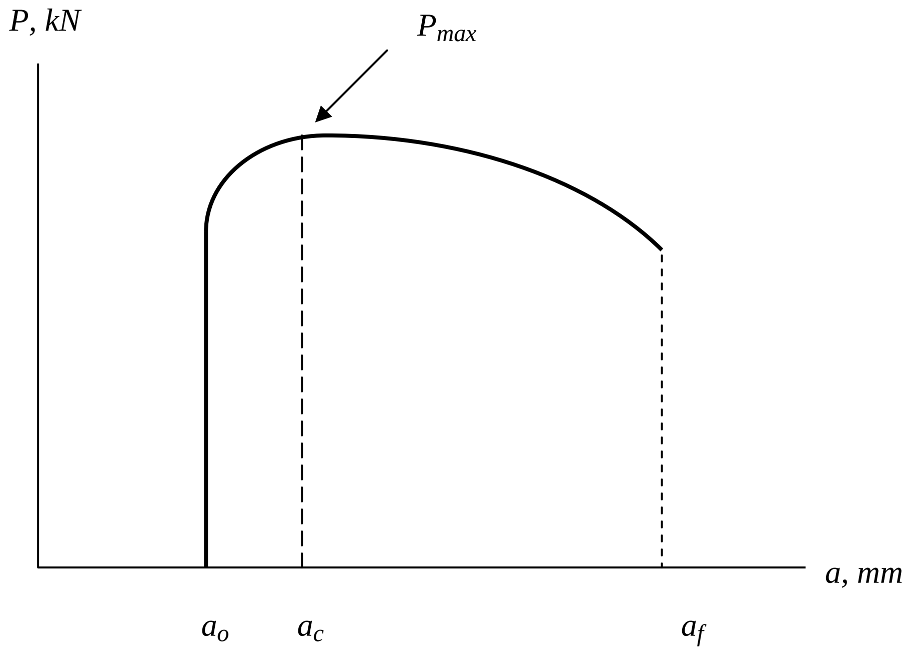

FIGURE 1. Generic shape of a *P-a* curve with crack growth.

The assessment of instability implies evaluation of the maximum load and critical plastic displacement. Substitution of Eq. (7) into Eq. (3) gives, for the critical plastic displacement:

$$\left(\frac{v_{pl}}{W}\right)_c = \left(\frac{b_o/W}{l_o(1+mnl_1)}\right)^{1/l_1} \tag{8}$$

The value of the maximum, or critical, load, P_{max}, is then obtained by substitution of Eq. (8) into Eq. (5), or of Eq. (7) into Eq. (4). Either way, the solution for the maximum load is:

$$P_{\max} = DBCW \left(\frac{b_o}{W}\right)^{(m+1/nl_1)} \left[\frac{mnl_1}{1+mnl_1}\right]^m \left[\frac{1}{l_o}\left[\frac{1}{1+mnl_1}\right]\right]^{1/nl_1} \tag{9}$$

Equation (9), which predicts the maximum load attained when there is stable crack growth, depends on the material properties (D, n); on the specimen geometry (C, m); on the constraint (Ω^*, included in the term D), on specimen size (B, W) and on the crack growth law behavior (l_o, l_1). The next section will show a couple of examples of instability prediction.

Cases analyzed

Two examples of instability analysis for C(T) specimens are presented. One corresponds to an A508 steel [5], and the other to an A533 steel [7]. Table 1 includes all dimensions and parameters of the two side-grooved C(T) specimens whose data were analyzed to test the instability values predicted by Eqs. (6) through (9).

Table 1: C(T) specimen dimensions and parameters of the CFE with crack growth

Parameter	A508 [5]	A533 [7]
W, mm	50.8	50
B, mm	25.4	25
B_N, mm	20.32	20
a_o, mm	26.19	30.3
a_f, mm	33.12	34.25
D, MPa	372	350
n	5.20	5.65
l_1	2.0	2.0

The values of C and m for the C(T) specimen are 1.553 and 2.236, respectively [2]. The values of D and n for the A508 are slightly different than those reported in [4]. The value of l_o, finally, is not relevant, for it may be set in terms of l_1 and initial and final crack lengths. In fact, since the crack growth law, Eq. (2), has only two parameters, one of them may be expressed in terms of the other, using as a calibration point the final crack length [4]. Thus, the only parameter left that may affect the shape of the generic curve of Fig. 1 is l_1. The values of l_1 reported in Table 1 are those that give the best fit to the overall P-v curves with crack growth [4], shown in Figs. 2 and 3.

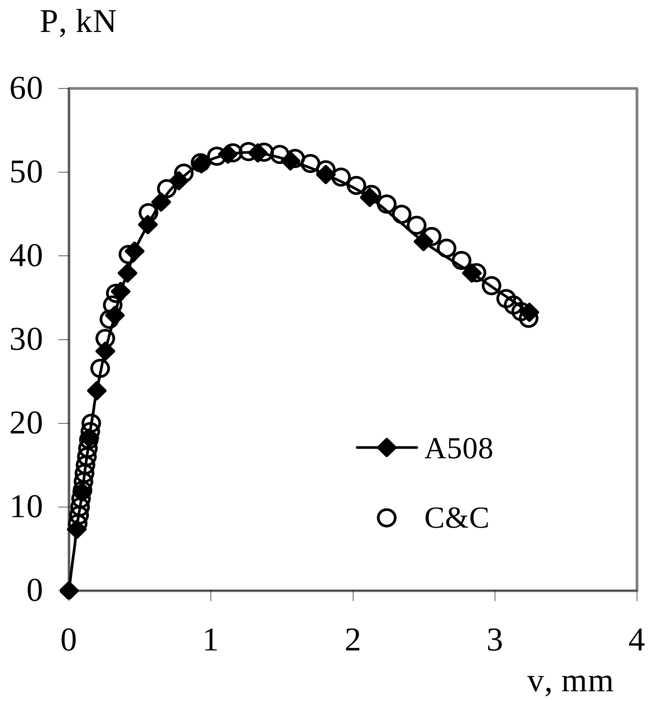

Figure 2.- Experimental and format model fit (C&C) *P*-*v* data for A508 [5].

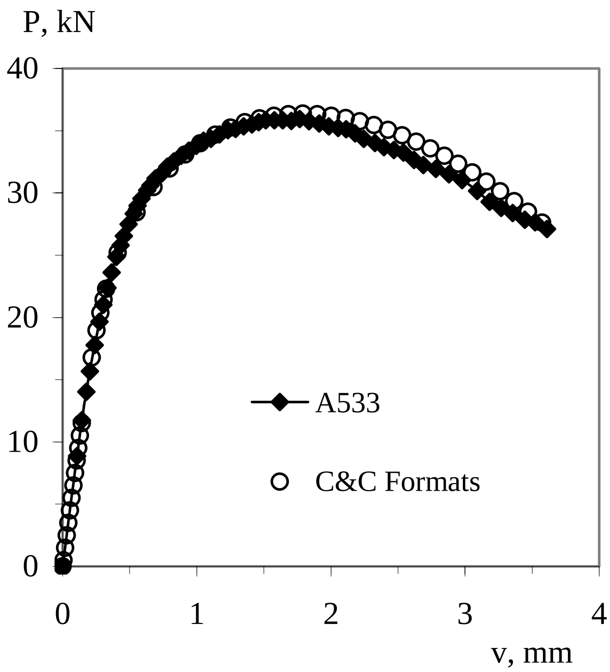

Figure 3.- Experimental and format model fit (C&C) *P*-*v* data for A533 [7].

Figures 2 and 3 show the goodness of the C&C (Concise & Common) Format model fit. The experimental points (black symbols) show all the irregularity of the actual *P*-*v* data; the C&C data points (open symbols) show, necessarily, a regular pattern. This is to say that the C&C model fit will never overlap completely the experimental points. However, l_1 = 2.0 gives a very good match between experiment and model data points.

Table 2 summarizes the critical values of *P*, *v*, and *a*. The first two may be read from the experimental points, or from *P*-*v* curves like those of Figs. 2 and 3; the critical value of *a* may only be obtained from Eq. (6), or from a curve like that of Fig. 1. Figures 4 and 5 show the respective *P*-*a* curves for the two cases considered here.

Table 2: Critical values of load, displacement, and crack length

Parameter	A508 Experiment	A508 C&C Model	A533 Experiment	A533 C&C Model	% error A508	A533
P_{max}, kN	52.35	54.10	35.93	36.37	3.34	1.22
v_c, mm	1.33	1.46	1.77	1.79	9.77	1.13
a_c, mm	26.92	27.18	31.0	31.05	0.97	0.16

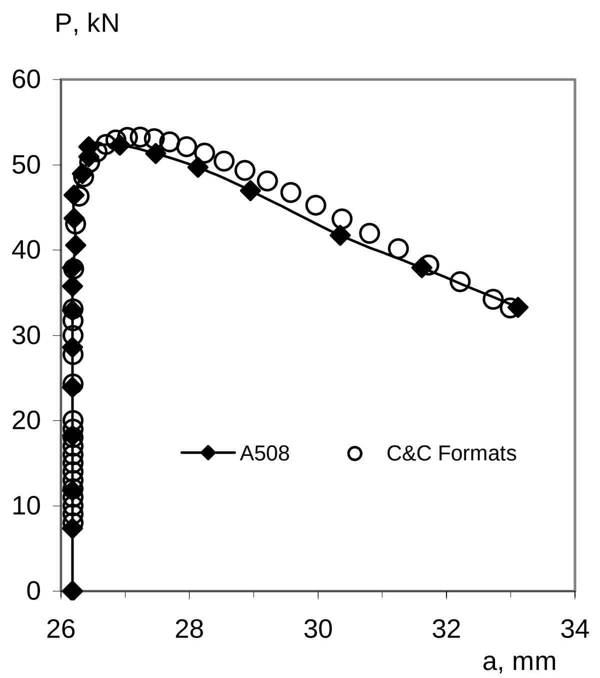

Figure 4.- Experimental and format model fit (C&C) *P*-*a* data for A508 [5].

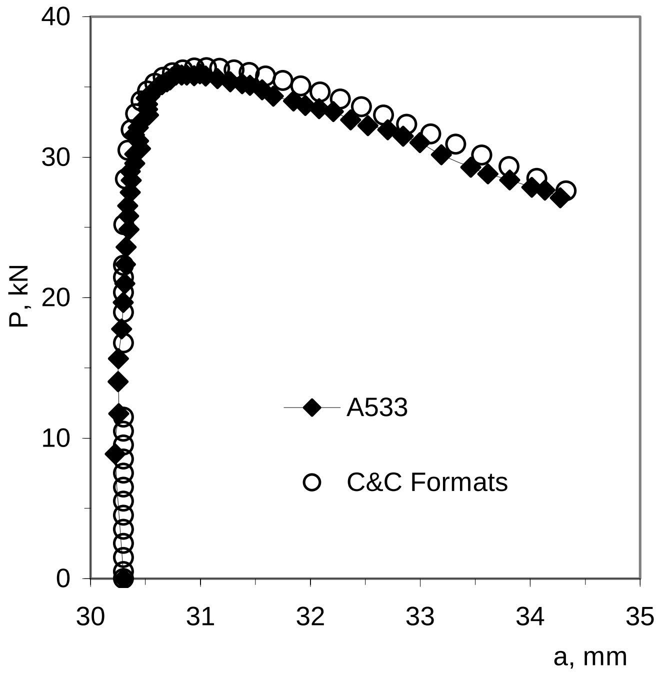

Figure 5.- Experimental and format model fit (C&C) *P-a* data for A533 [7].

As it may be noted from the results of Figs. 2-5, and the summary of Table 2, the model gives a fairly accurate prediction of the critical values of load, displacement, and crack length for instability.

Discussion and Concluding Remarks

An algorithm used to predict the maximum load capacity in a specimen geometry has been derived from the Common and Concise Formats, and is based upon the assumption of a crack growth law, Eq. (2). The assumption of a relation between crack extension and plastic displacement is not new. Gioelli and Landes [8] postulated that initiation of stable crack growth could not be possible before there was some plasticity developed, i.e., initiation should occur at a load larger than yield load. They also suggested that initiation should occur before maximum load.

Our results show that at maximum load, Δa is not zero, but there is already a finite crack extension, which is size-dependent, confirming Gioelli and Landes´s proposal. In fact, from Eq. (6), the critical crack extension is:

$$\frac{\Delta a_c}{W} = \frac{1-\left(a_o / W \right)}{1+mnl_1} = \frac{b_o / W}{1+mnl_1} \tag{10}$$

This result is quite relevant. It states that the critical crack extension value depends on the material behavior (n), on size (W), on specimen geometry (m), on initial crack length (a_o/W), and, finally, on the crack growth law exponent, (l_1).

For any given material, specimen size, initial crack length, and assuming that l_1 = 2, Eq. (10) predicts that the crack extension amount at instability should be the smallest for the C(T) geometry (m = 2.236), increase somewhat for the SE(B) specimen (m = 2), and be the largest for the M(T) geometry (m = 1).

In fact, using in Eq. (10) n = 5 and l_1 = 2, the critical crack extension values for M(T), are practically twice as large as those of the C(T) and SE(B) specimens at any given value of a_o/W . By contrast, the critical crack extension values for the SE(B) specimen are merely 11% larger than those of the C(T) specimen. The values for maximum load may then be obtained from Eq. (9), and compared under the condition of similar crack growth behavior, since the parameter l_o, included in the relation for P_{max} but not in that for Δa_c, has to be obtained by calibration with the final plastic displacement and crack length data point.

In summary, the approach used here predicts the maximum load bearing capacity for a specimen or structure based on a set of equations developed from the Common and Concise Formats. Here it was applied only to the C(T) specimen geometry, predicting the maximum load within a few percent. A goal of this work would be to expand this analysis to see if the forms of Eqs. (2), (4) and (5) apply to, and may be used for, other component geometry models, like the predominantly tension-type specimens M(T), SE(T) and DE(T).

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