

# A TWO SCALE DAMAGE MODEL FOR THERMOMECHANICAL HIGH CYCLE FATIGUE FAILURE

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## Abstract

Thermal fatigue cracking may be observed in some components of nuclear power plants. The evaluation of mesocrack nucleation in the components subjected to thermomechanical loadings is very important to determine investigation periods and maintenance programmes. On the idea that damage is localized at the microscopic scale, a scale smaller than the mesoscopic scale of the Representative Volume Element (RVE), a three-dimensional model is proposed. It consists in a two scale analysis with quasi-brittle fatigue damage behaviour modelled by plasticity coupled with damage constitutive equations (with thermal stresses) at the microscale. Microscopic failure is assumed to coincide with the RVE failure when the damage at microscale reaches the critical value  $D_c$ .

In order to compute the strains, the stresses and the damage history at the microscale a micro-mechanics based model of a weak micro-inclusion subjected to plasticity and damage embedded in an elastic meso-RVE is considered. A localization law, in the sense of the homogenization theories, is developed to link the two scales and a numerical scheme is proposed to integrate the constitutive equations.

## Introduction

Various components in nuclear power plants are subjected to thermomechanical loading during their service. Thermal fatigue cracking is observed in the mixing zones of the cooling system of nuclear power plants. The evaluation of crack nucleation and their subsequent propagation in a pipe subjected to thermomechanical loading is very important to determine investigation periods and maintenance programmes.

The inception of thermal cracking is due to a biaxial stress state: namely, one-dimensional cyclic thermal stresses and tensile axial or hoop mean stresses. These striping networks are observed in some areas of the residual heat removal system (RHR) of nuclear power plants. Edge cracks located on the internal surface of the pipe are observed (Fig. 1).

High cycle fatigue (HCF) damages are always very localized at a scale much smaller than the plastic strain [1; 5; 7; 4; 8]. Often, the crack initiates on a micro-defect on the surface or in the body of material. The mechanisms of plasticity and damage in a scale smaller than the RVE scale are dominating in HCF. This is the reason to consider a two-scale model in which the damage occurs in a weak micro-inclusion embedded in a meso-RVE, which is elastic (eventually elasto-plastic)

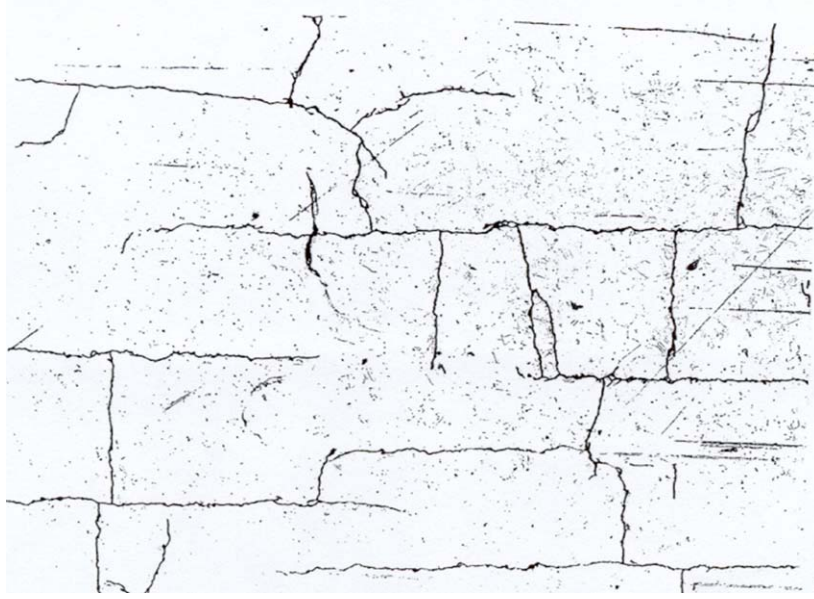


FIGURE 1. Striping network on the internal surface of RHR system.

and free of damage. This hypothesis allows us to obtain the mesostresses and strains by a classical structure calculation performed in elasticity (or elasto-plasticity) with no damage at mesoscale and then to solve the constitutive equations of elasto-plasticity coupled with damage as a post-processor at microscale.

The mechanical behaviour of the material is modelled in two different manners related to the two different scales.

- At the mesoscale the material is considered elastic because brittle or HCF failures occur at states of stresses below or close to the yield stress.
- At the microscale the behaviour is modelled by elastoplasticity coupled with damage. For the considered problem, the maximum imposed temperature is less than  $\frac{1}{4}$  of the melting temperature of the material and thus there is no viscosity effect. The weakness of the inclusion is related to its yield stress taken equal to the true fatigue limit  $\sigma_f$  of the material, below which we consider that no damage occurs. The elastoplastic properties at the microscale are those of the material at mesoscale.

By using a damage model, we can calculate the value of damage variable  $D$  for any kind of loading: 1D – 3D, cyclic loading, multiaxial fatigue loading or random fatigue. When  $D$  reaches a critical value  $D_c$ , a mesocrack initiate and the corresponding time or number of cycles is equal to the time or the number of cycles of crack initiation.

### 1. An anisothermal localization law

The use of a two scale model requires a localization law (in the sense of homogenization) to obtain the mechanical parameters at the microscale from the results of a reference calculation at the mesoscale. This scale bridging is performed by using an Eshelby-Kröner localization law. For an isothermal case without any damage, this relationship reads

$$\underline{\varepsilon}^{\mu} = \underline{\varepsilon} + b(\underline{\varepsilon}^{\mu p} - \underline{\varepsilon}^p) \quad (1)$$

where  $\underline{\varepsilon}$  denotes the strain tensor, the exponents  $\mu$  and  $p$  correspond to “micro” and “plastic” strains respectively. The Eshelby coefficient for a spherical inclusion is:

$$b = \frac{2(4-5\nu)}{15(1-\nu)} \quad (2)$$

Equation (1) can be extended to take into account the damage coupled to elastoplasticity in the microscale [2; 6]

$$\underline{\varepsilon}^\mu = \frac{1}{1-bD} \left( \underline{\varepsilon} + \frac{(a-b)D}{3(1-aD)} \varepsilon_{kk} \underline{1} + b((1-D)\underline{\varepsilon}^{\mu p} - \underline{\varepsilon}^p) \right) \text{ with } a = \frac{1+\nu}{3(1-\nu)} \quad (3)$$

In the case of thermo-mechanical fatigue, the localisation law of equation (3) must be extended to take into account the temperature variation effects. Thermal dilatation in “meso” and “micro” scales must be considered as a part of stress free strain for each level. The strain tensors in “micro” and “meso” scales reads

$$\text{At microscale: } \underline{\varepsilon}^\mu = \underline{\underline{E}}^{-1} : \underline{\underline{\sigma}}^\mu + \underline{\varepsilon}^{\mu L*} \quad (4)$$

$$\text{At mesoscale: } \underline{\varepsilon} = \underline{\underline{E}}^{-1} : \underline{\underline{\sigma}} + \underline{\varepsilon}^L \quad (5)$$

where  $\underline{\varepsilon}^L$ ,  $\underline{\varepsilon}^{\mu L*}$  denote the stress free strain tensor at mesoscale and microscale respectively and  $\underline{\underline{E}}$  rigidity tensor. Stress tensor at microscale reads

$$\underline{\underline{\sigma}}^\mu = \underline{\underline{\sigma}} + \underline{\underline{E}} : (\underline{\underline{S}} - \underline{\underline{I}}) : (\underline{\varepsilon}^{L*} - \underline{\varepsilon}^L) \quad (6)$$

where  $\underline{\underline{S}}$  is the Eshelby tensor. By considering equations (4), (5) and (6), deviatoric parts of the strain and the stress at microscale can be obtained as:

$$\underline{\underline{\sigma}}^{\mu d} = \frac{1-D}{1-bD} \underline{\underline{\sigma}}^d - \frac{2G(1-b)(1-D)}{1-bD} (\underline{\varepsilon}^{\mu Ld} - \underline{\varepsilon}^{Ld}) \quad (7)$$

$$\underline{\varepsilon}^{\mu d} = \frac{1}{1-bD} (\underline{\varepsilon}^d - b((1-D)\underline{\varepsilon}^{\mu Ld} - \underline{\varepsilon}^{Ld})) \quad (8)$$

where  $G$  is the shear modulus.  $\underline{\varepsilon}^{\mu Ld}$  and  $\underline{\varepsilon}^{Ld}$  are the deviatoric parts of the stress free strain tensor at “micro” and “meso” scale respectively. They correspond to plastic strain at “micro” and “meso” level ( $\underline{\varepsilon}^{\mu Ld} = \underline{\varepsilon}^{\mu p}$  and  $\underline{\varepsilon}^{Ld} = \underline{\varepsilon}^p$ ).

In the same manner, the hydrostatic parts of the stress and strain tensors can be obtained as:

$$\sigma_H^\mu = \frac{1-D}{1-aD} \sigma_H - \frac{3K(1-a)(1-D)}{1-aD} \alpha^\mu \Delta T \quad (9)$$

$$\varepsilon_H^\mu = \frac{1}{1-aD} (\varepsilon_H - (\alpha - a(1-D)\alpha^\mu) \Delta T) \quad (10)$$

where  $K$  is the bulk modulus,  $\alpha^\mu$  and  $\alpha$  thermal expansion coefficient at “micro” and “meso” scale,  $\Delta T$  temperature variation and  $H$  subscripts correspond to hydrostatic values. The localization law can be calculated explicitly in strain form as the sum of equations (8) and (10),

$$\underline{\varepsilon}^{\mu} = \frac{1}{1-bD} \left( \underline{\varepsilon} + \frac{(a-b)D}{3(1-aD)} \varepsilon_{kk} \underline{1} + b((1-D)\underline{\varepsilon}^{tp} - \underline{\varepsilon}^p) \right) + \frac{a(1-D)\alpha^{\mu} - \alpha}{1-aD} \Delta T \underline{1} \quad (11)$$

When there is no damage and when the “micro” expansion is equal to the “meso” one, there is no temperature effect on the localization law. But in presence of damage, the localization law (11) shows an effect (to be quantified) of temperature on the strains in the micro-inclusion, even when  $\alpha^{\mu} = \alpha$ .

## 2. Effect of temperature variation on the “micro” values

In order to investigate the effect of temperature variation on the “micro” values, a set of parametric calculations is performed. The general case of onedimensional tension is considered. The elastic properties of material are:  $E = 210000$  MPa,  $\nu = 0.3$ . The variation of stresses and strains at microscale and of the triaxiality rate are studied as a function of temperature variation for different temperature variations and damage levels.

### Hydrostatic strain

Figure 2-a presents the variation of the normalized hydrostatic strain (recording to hydrostatic strain at mesoscale) as a function of the difference between thermal bulk expansions at “micro” and “meso” scale. In this step, no damage is considered ( $D = 0$ ). The difference between “micro” and “meso” expansions are modelled by considering different values of  $z = \alpha^{\mu}/\alpha$ . This figure shows that  $\alpha^{\mu} > \alpha$  decreases the value of  $\varepsilon_H^{\mu}$  for a given  $\varepsilon_H$  and  $\alpha^{\mu} < \alpha$  increases it.

In practice, it is difficult to measure a different thermal expansion coefficient at microscale. The effect of damage on the hydrostatic strain as function of temperature variation is shown in Fig. 2-b with  $\alpha^{\mu} = \alpha$ . An increase of the damage, increases  $\varepsilon_H^{\mu}$  regarding to  $\varepsilon_H$ .

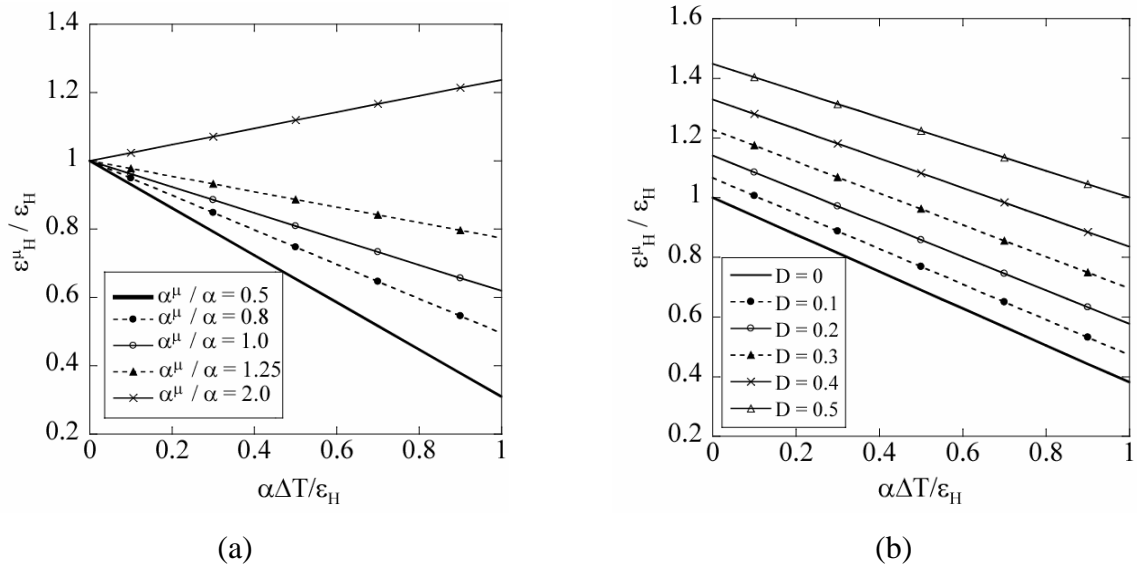
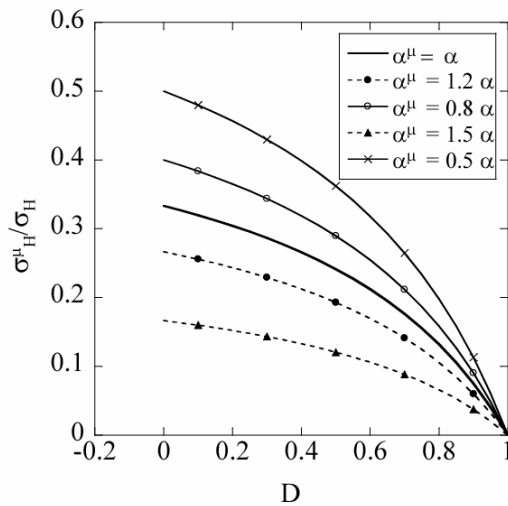
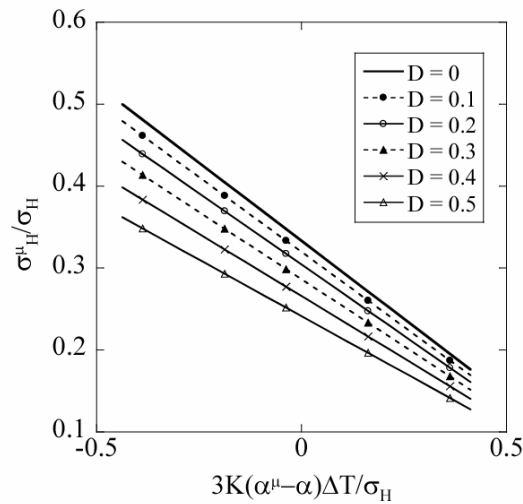


FIGURE 2. Change of normalized hydrostatic strain versus difference between normalized thermal expansions.

**Hydrostatic stress**

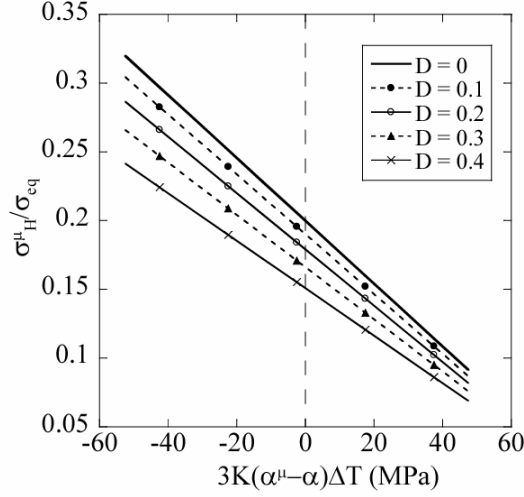
The sensitivity of hydrostatic stress and triaxiality rate to damage variation and the ratio of “micro” and “meso” expansion coefficients are studied in this paragraph. Figure 3-a shows the change of normalized hydrostatic stress versus temperature variation. This figure exhibits that the hydrostatic stress at microscale is independent from the temperature variation when “micro” and “meso” thermal expansion are equivalent. Furthermore, an increase of the damage decreases the hydrostatic stress value at microscale.

Figure 3-b presents the change of  $\sigma_H^{\mu}$  with respect to the damage for different ratios  $\alpha^{\mu}/\alpha$  and for  $\Delta T = 100^{\circ}\text{C}$ . At microscale, the von Mises equivalent stress do not change notably and it is almost equal to the fatigue limit of material [2]. Figure 3-c shows the change of the stress triaxiality rate with respect to the temperature variation. The stress triaxiality is independent of the temperature when  $\alpha^{\mu} = \alpha$ .



(a)

(b)



(c)

FIGURE 3. Change of normalized hydrostatic stress versus  $\sigma_H^\mu / \sigma$  and of the stress triaxiality  $\sigma_H^\mu / \sigma_{eq}^\mu$  for a mesoscopic tensile stress  $\sigma$ .

### 3. Anisothermal constitutive equations at microscale

To build a thermomechanical fatigue model, the effect of the temperature variation on the material behaviour must be taken into account. Temperature variation can influence the behaviour of the structure in two manners. First, thermal expansion must be considered in the elasticity law. Second, the possible dependence of material properties with respect to the temperature imposes us to consider this effect in the time discretization of the equations.

Let us consider equation (11) as the localization law that binds the “micro” values to the “meso” ones under thermomechanical loading. If we neglect the temporal derivatives of the material parameters, the incremental form of the constitutive equations reads:

$$\begin{aligned} \underline{\dot{\varepsilon}}^{\mu e} + \frac{1-b}{1-bD} \dot{p} \underline{m}^\mu + \frac{1-a}{1-aD} \alpha^\mu(T) \dot{T} \underline{1} - \frac{1}{1-bD} \underline{\dot{\varepsilon}} - \frac{(a-b)D}{(1-bD)(1-aD)} \text{tr} \underline{\dot{\varepsilon}} \underline{1} + \frac{a\alpha(T)}{1-aD} \dot{T} \underline{1} &= 0 \\ f^\mu = (\underline{\tilde{\sigma}}^\mu - \underline{X}^\mu)_{eq} - \sigma_f^\infty &= 0 \\ \dot{D} - \left( \frac{Y^\mu}{S} \right)^s \dot{p}^\mu = 0 & \quad \text{if} \quad p^\mu > p_D \end{aligned} \quad (12)$$

where  $f^\mu$  is the yield criterion at microscale,  $\dot{D} = (Y^\mu / S)^s \dot{p}^\mu$  Lemaitre damage evolution law for damage governed by the micro-plastic accumulated strain  $p^\mu$  and:

$$\begin{aligned} \underline{m}^\mu &= \frac{3}{2} \frac{\underline{\tilde{\sigma}}^\mu - \underline{X}^\mu}{(\underline{\tilde{\sigma}}^\mu - \underline{X}^\mu)_{eq}} \\ \underline{\tilde{\sigma}}^\mu &= \frac{\underline{\sigma}^\mu}{1-D} = \underline{E}(T) : \underline{\varepsilon}^{\mu e} \\ \underline{X}^\mu &= \frac{2}{3} C_y(T)(1-D) \underline{\varepsilon}^{\mu p} \end{aligned} \quad (13)$$

with  $X^\mu$  the kinematics hardening and  $C_y$  the plastic modulus of the material. The strain energy release rate  $Y^\mu$ , taking into account the different damage behaviours in tension and compression, is defined as [3]

$$Y^\mu = \frac{1+\nu}{2E} \left[ \langle \underline{\tilde{\sigma}}^\mu \rangle_+ : \langle \underline{\tilde{\sigma}}^\mu \rangle_+ + h \left( \frac{1-D}{1-hD} \right)^2 \langle \underline{\tilde{\sigma}}^\mu \rangle_- : \langle \underline{\tilde{\sigma}}^\mu \rangle_- \right] - \frac{\nu}{2E} \left[ \langle \text{tr} \underline{\tilde{\sigma}}^\mu \rangle_+^2 + h \left( \frac{1-D}{1-hD} \right)^2 \langle \text{tr} \underline{\tilde{\sigma}}^\mu \rangle_-^2 \right] \quad (14)$$

#### 4. Numerical scheme for post-processing a reference structure calculation

The damage can be determined by post-processing a reference structure calculation, performed in elasticity for quasi-brittle and high cycle fatigue applications and in elasto-plasticity for more ductile conditions. The reference calculation gives the stresses, strains and the plastic strains history with no damage at the mesolevel of classical continuum mechanics. Considered altogether with the localization law (11) and isotropic elasticity, they are the inputs for the time integration of elasto-plasticity fully coupled with damage constitutive equations at the microscale. By integration of the equation set (12), we can obtain the number cycles corresponding to initiation of a mesocrack ( $D = D_c$ ). The integration method used is governed by strains: for each time increment  $t_n$  and for an incremental value of the strains at the mesolevel  $\Delta \underline{\varepsilon} = \underline{\varepsilon}_{n+1} - \underline{\varepsilon}_n$ , the post-processor evaluates the stresses and the state variables at the microlevel at time  $t_{n+1}$ . The different steps of the numerical scheme are as follows:

##### *Elastic predictor*

A local elastic prediction, which assumes an elastic behaviour with constant plastic strain  $\underline{\varepsilon}_{n+1}^{lp} = \underline{\varepsilon}_n^{lp}$ , constant kinematic hardening  $\underline{X}_{n+1}^\mu = \underline{X}_n^\mu$  and constant damage  $D = D_n$ , gives a first estimation of the strains, elastic strains and effective stresses at microscale at time  $t_{n+1}$ :

$$\begin{aligned} \underline{\varepsilon}_{n+1}^\mu &= \frac{1}{1-bD} \left( \underline{\varepsilon}_{n+1} + \frac{(a-b)D_n}{3(1-aD_n)} \underline{\varepsilon}_{kk,n+1} \underline{1} + b((1-D_n)\underline{\varepsilon}_n^{lp} - \underline{\varepsilon}_{n+1}^p) \right) + \frac{a(1-D_n)\alpha^\mu - \alpha}{1-aD_n} (T_{n+1} - T_{ref}) \underline{1} \\ \underline{\varepsilon}_{n+1}^{\mu e} &= \underline{\varepsilon}_{n+1}^\mu - \underline{\varepsilon}_n^{lp} - \alpha^\mu (T_{n+1} - T_{ref}) \underline{1} \\ \underline{\tilde{\sigma}}_{n+1}^\mu &= \underline{\underline{E}} : \underline{\varepsilon}_{n+1}^{\mu e} \end{aligned} \quad (15)$$

##### *Local plastic correction*

A local plastic correction of the state and internal variables gives us their corresponding values at time step  $t_{n+1}$ . To detail this step, consider that the elastic predictor has given  $\underline{\tilde{\sigma}}_{n+1}^\mu$ ,  $\underline{\varepsilon}_{n+1}^\mu$  et  $\underline{\varepsilon}_{n+1}^{\mu e}$  as initial estimates of the stresses, strains and plastic strains. If the yield condition  $f_{n+1}^\mu \leq 0$  is satisfied, then the calculation for this step is finished. If not, Newton iterative process starts. For simplicity, the damage is assumed to remain constant over a time increment.

The nonlinear equations to be solved in a coupled manner are:

$$\begin{aligned} \Delta \underline{\varepsilon}^{\mu e} + \frac{1-b}{1-bD} \Delta \underline{\varepsilon}^{\mu p} + \frac{1-a}{1-aD} \alpha^\mu \Delta T \underline{1} - \frac{1}{1-bD} \Delta \underline{\varepsilon} - \frac{(a-b)D}{(1-bD)(1-aD)} \text{tr} \Delta \underline{\varepsilon} \underline{1} + \frac{a\alpha}{1-aD} \Delta T \underline{1} &= 0 \\ f_{n+1}^\mu = (\tilde{\sigma}_{n+1}^\mu - \underline{X}_{n+1}^\mu)_{eq} - \sigma_f^\infty & \end{aligned} \quad (16)$$

They may straightforwardly be solved by use of Newton iterative scheme but it is advantageous here to write them in terms of the plastic strain  $p^\mu$  and of the variable  $\underline{s}^\mu = \tilde{\sigma} - \underline{X}^\mu$  as a set of two equations:

$$\begin{cases} \underline{R}_s = \frac{1}{E(T_{n+1})} \underline{s}_{n+1} - \frac{1}{E(T_n)} \tilde{\sigma}_n^\mu + \frac{2}{3} \Gamma^* m_{n+1}^\mu \Delta p^\mu - \\ \frac{1}{1-bD} \left( \underline{E}^* : \Delta \underline{\varepsilon} + K^* \frac{(a-b)D}{1-aD} \Delta \varepsilon_{kk} \underline{1} - 2G^* b \Delta \underline{\varepsilon}^p \right) \underline{1} + \frac{3K^*}{1-aD} ((1-a)\alpha^\mu + \alpha) \Delta T \underline{1} = 0 \\ \underline{R}_p = (\underline{s}_{n+1}^\mu)_{eq} - \sigma_f^\infty = 0 \end{cases} \quad (17)$$

where  $\Gamma^* = 3 \frac{G(1-b)}{E(1-bD_n)} + \frac{C_y(1-D_n)}{E}$ ,  $\underline{E}^* = \frac{1}{E} \underline{E}$ ,  $K^* = \frac{K}{E}$  and  $G^* = \frac{G}{E}$ . For each iteration  $q$  of Newton scheme the solution  $\underline{s}^{\mu(q+1)}$ ,  $p^{\mu(q+1)}$  or in an equivalent manner the ‘‘corrections’’  $\underline{C}_s = \underline{s}^{\mu(q+1)} - \underline{s}^{\mu(q)}$  and  $C_p = p^{\mu(q+1)} - p^{\mu(q)}$  to apply at each step to the previous iterated values are given by

$$\begin{cases} \underline{R}_s + \frac{\partial \underline{R}_s}{\partial \underline{s}^\mu} : \underline{C}_s + \frac{\partial \underline{R}_s}{\partial p} C_p = 0 \\ \underline{R}_p + \frac{\partial \underline{R}_p}{\partial \underline{s}^\mu} : \underline{C}_s = 0 \end{cases} \quad (18)$$

where  $\underline{R}_s$ ,  $\underline{R}_p$  and their derivates are taken at time  $t_{n+1}$  and at the iteration  $q$ . The procedure is then an implicit scheme with the advantages of explicit one as to finish any system of equations has to be solved: the unknowns are updated explicitly by use of the closed-form solutions of  $C_p$  and  $\underline{C}_s$ .

## 5. Computation of the damage

The damage is then calculated as  $D_{n+1} = D_n + \Delta D$

$$\Delta D = \left( \frac{Y_{n+1}}{S} \right)^s \Delta p \quad (19)$$

$Y$  being given by Eq. (14) in which  $D = D_n$  and  $\tilde{\sigma} = \tilde{\sigma}_{n+1}$  is set.

## 6. Conclusion

A two-scale damage model for thermomechanical fatigue is proposed. An anisothermal localisation law is developed to link up the micro values to the ‘‘meso’’ ones obtained by a reference structure calculation. An implicit numerical scheme with closed-form solutions for local plastic corrections, is developed for integration of constitutive equations in microlevel. The model is programmed as a post-processor of a reference structure calculation to determine the number cycles corresponding to mesocrack initiation in the studied RVE.



## 7. References

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