A FRACTURE MECHANICS APPROACH FOR THE CRACK GROWTH IN WELDED JOINTS WITH REFERENCE TO BS 7910

Philippe Darcis[1], Diego Santarosa[1], Naman Recho[1] and Tom Lassen[2]
[1] LERMES, IUT de Montluçon, B.P. 2235, Av. Aristide Briand, 03100 Montluçon, France
[2] Agder Univ. College, Faculty of Engineering, Grooseveien 36, N-4876 Grimstad, Norway
darcis@moniut.univ-bpclermont.fr

Abstract

The fatigue process in welded joints is discussed and modeled. A fracture mechanics model has been proposed to describe the entire fatigue process. The model is calibrated to fit the crack growth measurements carried out on fillet welded joints. The objective is to establish a unified approach which is consistent with rules and regulation both based on the *S-N* approach (Eurocode 3) and applied fracture mechanics (BS 7910). Emphasis is put on how to choose growth parameters in conjunction with a fictitious initial crack size to obtain both reliable crack growth paths and predictions of the entire fatigue life. If the growth rate parameters given in BS 7910 for a single linear relationship between log da/dN and log ΔK are used in conjunction with initial crack depths near 0.015 mm the model fits both measured crack histories and *S-N* fatigue life estimates at various stress levels.

Introduction and objectifs

In the present paper the fatigue process in fillet-welded joints where cracks emanate from the weld toe is studied and modelled. The objective is to establish a model that strikes the balance between simplicity and accuracy in order to provide a tool for the practising engineer. The model should be useful for both durability analysis and inspection planning.

Although several investigations have proven the existence of a crack initiation phase in welded joints, it may be argued that a fracture mechanics model, although lacking a firm footing for the early crack growth, is good enough for all practical purposes.

The information needed is usually:

- Prediction of time to failure.
- Predictions of likely crack growth histories leading to the failure.

The first criterion is obvious from a fatigue durability point of view; we require a reliable estimate for the entire fatigue life. This estimate must be compared with planned service life and proper dimensions for the joint must be chosen to obtain a safety margin. Hence, our model should be corroborated by *S-N* data for the joint in question when these are available. The second criterion is essential if in-service inspections are to be planned; we must know what crack sizes to look for at different times before final failure. This will make the scheduled inspection more efficient and economical. Hence, our model should predict a crack evolution that coincides with measured crack growth histories before failure. On this background we shall in the following develop, elaborate and calibrate a fracture mechanics model.

Rules and regulations

The S-N approach based on Eurocode 3

The *S-N* approach is based entirely on Constant Amplitude (CA) experimental fatigue life. A linear relationship between log *N* and log ΔS is assumed and a regression analysis is carried out. The mean curve with its standard deviation is obtained. In the present work, the *S-N* curve taken from Eurocode 3 [1], reads:

$$N = \begin{cases} A\Delta S^{-m} & \Delta S > \Delta S_0 \\ \infty & \Delta S \le \Delta S_0 \end{cases} \tag{1}$$

where m is the fatigue exponent, A is the fatigue strength constant and ΔS is the nominal stress range. ΔS_0 is the endurance limit (at 5×10^6 cycles). The *S-N* curve for a fillet-welded joint is designated class 71 with $A = 1.9\times10^{12}$ (mean value), $m = 3$ and $\Delta S_0 = 52$ MPa.

The fracture mechanics approach, guidance given in BS 7910

The guidance given in the former PD 6493 [2] and the more recent BS 7910 [3] is based on applied fracture mechanics and the simple version of the Paris law:

$$\begin{aligned} &\frac{da}{dN} = C(\Delta K)^m = C(\Delta S\sqrt{\pi\, a}\; F(a))^m, \qquad \Delta K > \Delta K_0 \\ &N = \frac{1}{C}\int_{a_0}^{a_C} \frac{da}{\left(\Delta S\sqrt{\pi\, a}\; F(a)\right)^m} \end{aligned} \tag{2}$$

where C and m are treated as material parameters for a given mean stress and environmental condition. ΔK is the stress intensity factor range (SIFR) at crack tip. a_0 is the initial crack depth and a_C is the critical crack depth. *F(a)* is a dimensionless geometry function accounting for loading mode, crack and joint geometry. In the present work, a_c is set to half of the plate thickness for test specimens. The *F(a)* geometrical function is based on the work by Gurney [4] who derived numerical values for *F(a)* for an edge crack with average weld toe profile. Joint and crack geometry together with *F(a)* are shown in Fig. 1.

As can be seen the solution differ very little form the 3D solution found in BS 7910.

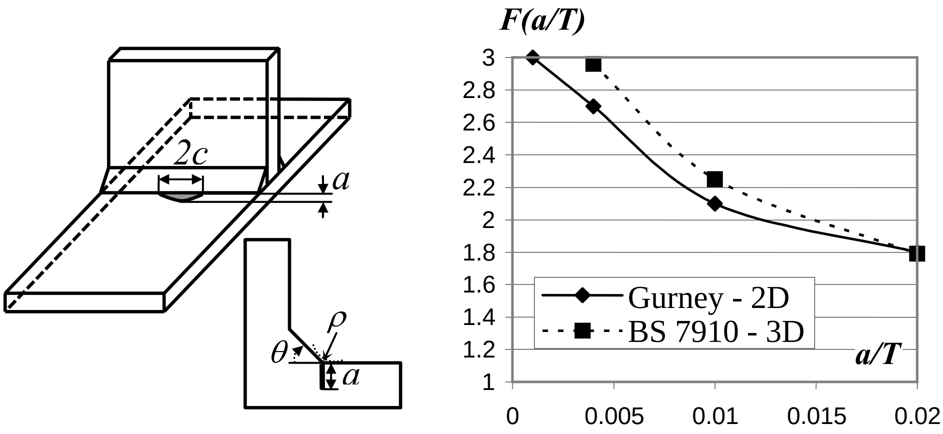

FIGURE 1. Definition of joint and crack geometry - Geometry function $F(a/T)$ for small crack depths.

Recommendations are given in BS 7910 for the growth rate parameters C and m and the threshold value ΔK_0 for the stress intensity factor range. Two alternatives are suggested for the relationship between the growth rate da/dN and the SIFR for a log-log scale.

The first alternative is based on a single linear relationship, whereas alternative two proposes a bi-linear relationship. The main difference between the two models is that the bi-linear one models the gradual decrease in the growth rate for low values of the SIFR before the threshold value is reached. For the simple linear relationship m is set to 3.0 and only the upper bound for the C is given. The former document PD 6493 recommended $C = 3.0\times10^{-13}$, whereas BS 7910 recommends as high as 5.21×10^{-13}, see Table 1. Hence, the value is increased as much as by 80% from the first document. The mean values are not given, and we have listed the mean value found by Johnston [5] in Table 1. This is done because the figure for the mean plus two standard deviations (mean + 2SD) given by Johnston [5] coincides with the upper bound PD 6493.

TABLE 1. Growth rate parameters for the linear relationship.

Mean curve [5]		Mean + 2SD [2]		Upper bound [3]	
C	*M*	*C*	*m*	*C*	*M*
1.85×10^{-13}	3.0	3.00×10^{-13}	3.0	5.21×10^{-13}	3.0

For da/dN in mm/cycle and ΔK in $N/mm^{3/2}$.

Data for the bi-linear relationship are given in Table 2. The stage A / stage B transition point is 363 $N/mm^{3/2}$ for the mean curve and 315 $N/mm^{3/2}$ for the mean plus two standard deviations curve. For shallow surface cracks the threshold value of SIFR is given as 63 $N/mm^{3/2}$ as a lower limit regardless of applied stress ratio.

TABLE 2. Growth rate parameters for the bi-linear relationship [3].

R	Stage A				Stage B			
	Mean curve		Mean + 2SD		Mean curve		Mean + 2SD	
	C	M	C	M	C	m	C	m
< 0.5	1.21×10^{-26}	8.16	4.37×10^{-26}	8.16	3.98×10^{-13}	2.88	6.77×10^{-13}	2.88

For da/dN in mm/cycle and ΔK in $N/mm^{3/2}$.

Discussion and calibration of the fracture mechanics models

The models are fitted to experimental crack growth curves derived from extensive testing on fillet welded joints were cracks are emanating from the weld toe [6]. The database contains measurements from the first measurable crack size (0.1 mm) to the final critical crack size (half plate thickness 12.5 mm) carried out under accelerated laboratory conditions. The specimens were tested under constant amplitude axial loading at ΔS =150 MPa with a loading ratio of R = 0.3. Experimental details are found in Ref. [6].

Calibration of the model

In the present analysis we will examine both the linear and the bi-linear relationships in BS 7910 as we try to fit the experimental results in the database. We will adopt the m values given in Tables 1 and 2 and see if the corresponding C values are compatible with the statistics given in the same tables. The procedure is as follows:

1) The slope parameter m of the growth rate curve is chosen in accordance with BS 7910.
2) The parameter C is determined so the model life calculation coincides with the measured experimental life. This life is defined between a crack depth of 0.1 mm to a final crack depth of 12.5 mm.
3) With the C and m values derived above for a given sample, a_0 is determined so that the number of cycles from a_0 to the first measured crack depth of 0.1 mm coincides with the experimental results. Hence, for each of the 34 samples a set of the variables m, C and a_0 is derived.

The results for one specimen are given in Fig. 2. As can be seen the two approaches describes the growth history equally well, the difference between them is not significant. The fatigue lives are close to the S-N estimates if the number of cycles spent before 0.1 mm crack depth is added. The discrepancy is less than 10 %.

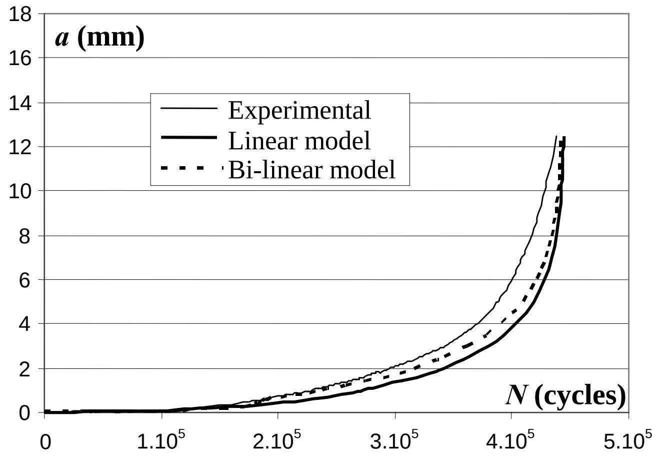

FIGURE 2. Comparison between BS 7910 relationships and experimental results.

Crack growth rate parameters

For the single linear relationship, the growth rates found (see statistics for the parameter *C* in Table 4) have a mean value close to the mean value given in BS 7910, Table 1. Furthermore, all the rates are well below the upper bound given in BS 7910. In fact, the mean plus two standard deviations is quite close to the values found in the former document PD 6493 [2].

TABLE 4. Statistics for the parameter *C* (linear relationship).

Median	Median + 2SD	COV
1.67×10^{-13}	2.48×10^{-13}	0.20

For *da/dN* in mm/cycle and ΔK in $N/mm^{3/2}$.

For the bi-linear relationship, Table 5 shows statistics for the parameters C_1 and C_2.

TABLE 5. Statistics for the parameter C_1 and C_2 (bi-linear relationship).

Stage	Median	Median + 2SD	COV
C_1	4.06×10^{-26}	5.80×10^{-26}	0.18
C_2	3.58×10^{-13}	5.24×10^{-13}	0.19

For *da/dN* in mm/cycle and ΔK in $N/mm^{3/2}$.

Figure 3 on the left shows the derived figures plotted for an arbitrary value of SIFR together with the BS 7910 mean and upper bound curves. For the single line relationship, none of the results obtained were in the vicinity of the upper bound. The data derived for the bi-linear relationship do not fit the lower line as shown to the right on Fig. 3. The growth rates are close to 3 times higher than the values given in BS 7910.

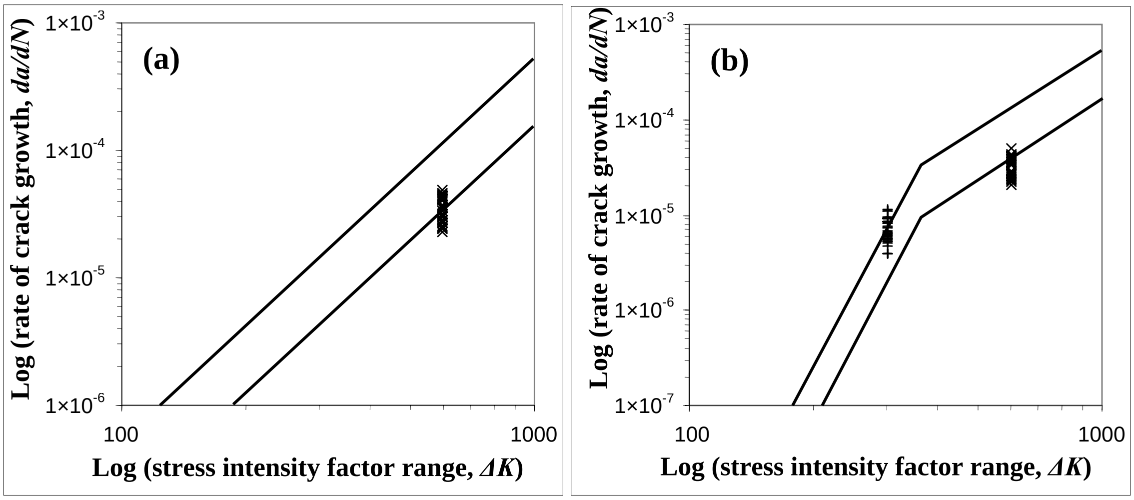

FIGURE 3. Experimental results plotted within the BS 7910 scatterband, (a) Linear model, (b) Bi-linear model.

The conclusion to be drawn from this is that small surface breaking cracks in the weld toe with SIFR less than 363 N/mm$^{3/2}$ (crack depth less than 0.6 mm) grow considerably faster than the lower rate curve given in BS 7910 prescribes. Hence, care should be taken not to use the bi-linear growth curve in BS 7910 for such small cracks. This curve should be used for what it probably is intended for, prediction of the behavior of larger cracks found after production or during service.

Determination of the initial crack depth

For the linear model, the statistics for the derived initial crack depths are given in Table 6.

TABLE 6. Statistical values of a_0 for the linear model.

Mean	Standard Deviation	COV
0.0151	0.0045	0.30

As can be seen from the Table, the mean value is 0.0151 mm and the upper bound is close to 0.03 mm. These crack are much smaller than the recommendations given in rules and regulations. In these recommendations the cracks are often as deep as 0.5 mm, [7]. With such deep cracks the fracture mechanics approach will not coincide with the S-N estimates with reasonable growth parameters.

For the bi-linear model, the statistics for the derived initial crack depths are given in Table 7.

TABLE 7. Statistical values of a_0 for the bi-linear model.

Mean	Standard Deviation	COV
0.0600	0.0056	0.09

It should be kept in mind that the model does not take into account the variability of the local toe geometry when determining the initial crack depth, i.e. we have held the geometry function *F(a)* constant at its mean value (θ = 45 degrees, ρ = 0.1 mm, see Fig. 1). It should also be emphasized that this initial crack depth distribution is a purely theoretical concept, i.e. it cannot be proven that the crack depths are related to initial flaws created by the welding process.

Constructing crack growth histories and S-N curves from the fracture mechanics model

The derived mean values for the a_0, *C* and *m* are substituted into the Paris Law to calculate both crack evolution and fatigue life at various constant amplitude stress levels. Both the linear and bi-linear relationship is used. Crack evolutions at stress ranges equal 150 MPa and 100 MPa are shown in Fig. 4. The first stress range is the one used during the laboratory experiments, whereas the lower stress level is more typical for in service conditions.

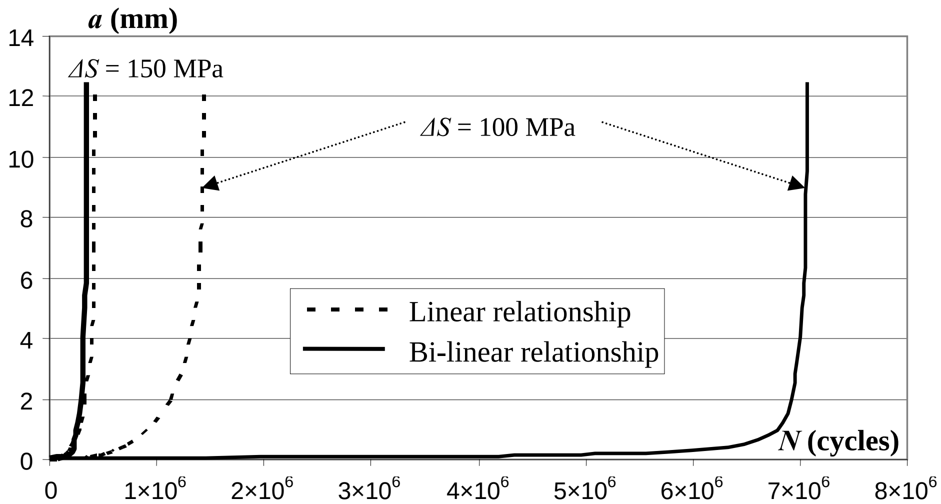

FIGURE 4. Crack evolution from a_0 to a_c at stress levels 150 and 100 MPa.

As can be seen from the Figure the two relationships give slightly different paths to arrive at the same end point for ΔS = 150 MPa, but the fatigue lives predicted at ΔS = 100 MPa are very different. The linear relationship gives a fatigue life close to 1.5×10^6 cycles that is again 10 % less than the class 71 *S-N* prediction, whereas the bi-linear relationship gives a fatigue life close to 7×10^6 cycles, which is far too long. This is due to the fact that close to half the fatigue life (from the initial crack depth up to 0.6 mm) is scaled relative to the stress level with a power of *m* = 8.16 (Table 2). As a result, the predictions made by the linear relationship will correspond to the predictions made by the *S-N* curve class 71, whereas the predictions made by the bi-linear curve are overly optimistic.

Conclusions

The fatigue process in fillet welded joints has been discussed. A pure fracture mechanics model has been calibrated to describe the entire fatigue process. The simple version of the Paris law has been adopted. The initial crack depth and growth rate parameters have been determined to fit experimental *a-N* curves and the class 71 life predictions.

The single linear relationship between da/dN and SIFR for a log-log scale with slope $m = 3.0$ gives a good fit between the calculated *a-N* curves and the curves measured at a stress range of 150 MPa. The growth rates found have a mean value close to the mean value given in BS 7910. Furthermore, all the rates are well below the upper bound given in BS 7910. The predicted fatigue life is very close to the predictions given by the class 71 *S-N* curve at any stress level above the endurance limit.

The initial cracks are in the range between 0.005 and 0.03 mm with a mean value of 0.015 mm. The initial crack distribution applied for the model is consistent with the experimental findings, but cannot be verified by measurement. The concept of a threshold value does not apply for these shallow cracks.

The bi-linear relationship between da/dN and SIFR for a log-log scale also gives good agreement with experimental *a-N* curves. However, the derived growth rates are higher than the upper bound given in BS 7910 for the lower line segment. If the parameters proposed for the bi-linear relationship between the log da/dN and log ΔK are used the fatigue life estimates are far too optimistic at low stress levels. Hence, the latter approach should not be applied for small surface breaking cracks at weld toes.

References

1. Eurocode 3: Design of steel structures, Part 1-9: *Fatigue strength of steel structures*, European Norm EN 1993-1-9, 1993.
2. PD 6493, *Guidance on Methods for Assessing the Acceptability of Flaws in Fusion Welded Structures*, London, British Standard Institution (BSI), 1991.
3. BS 7910, *Guidance on Methods for Assessing the Acceptability of Flaws in Fusion Welded Structures*, London, British Standards Institution (BSI), 2000.
4. Gurney, T. R. 1976. Finite element analysis of some joints with the welds transverse to the direction of stress, *Welding Research International*, 6: 40-72.
5. Johnston, G.O., Statistical scatter in fracture toughness and fatigue crack growth rates, ASTM STP 798: *Probabilistic fracture mechanics and fatigue methods: application for structural design and maintenance*, 42-66, 1983.
6. Lassen, T., The effect of the welding process on the fatigue crack growth in welded joint, *Welding Journal*, vol. **69**, 75- 85, 1990.
7. ABS: Guide fro the Fatigue Assessment of offshore structures, *American Bureau of Shipping*, 2003.