FATIGUE BEHAVIOUR OF FIBER-REINFORCED CONCRETE BEAMS

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Abstract

The mechanical behaviour of a fiber-reinforced concrete beam under cyclic bending is examined through a theoretical model based on fracture mechanics. The cracked infinitesimal portion of the beam being considered is subjected to an external bending moment and bridging reactions of the fibers. A rigid-perfectly plastic law is assumed for the reinforcements (whose ultimate reactions correspond to either yielding or slippage), whereas the concrete matrix is supposed to behave in a linear-elastic way. The statically indeterminate bridging forces are computed through congruence conditions, and typical phenomena, such as elastic shake-down and plastic shake-down, are described in terms of applied bending moment against beam cross-section rotation. Then, the Paris law is exploited to analyse the fatigue behaviour of the composite beam up to failure.

Introduction

As is well-known, fatigue phenomena related to metallic structures have been analysed for over one century [1], whereas the behaviour of reinforced concrete (RC) structures under cyclic loading has been studied for only a few decades. Some Standards [2,3] give us rules on how to take into account such phenomena when designing RC structures, but additional investigations are needed especially for fiber-reinforced concrete (FRC) structures. If a crack develops in a FRC structure, the overall behaviour is strongly affected by the crack bridging reactions of the fibers intersected by such a defect, and the progressive crack growth under cyclic loading influences the bridging effect, by also causing significant changes in the mechanical properties of the composite material (strength, toughness, stiffness, hysteretic behaviour, etc.). Some theoretical models have been proposed in order to examine FRC structures and to predict fatigue life (for instance, see Zhang and Stang [4], Zhang et al. [5], Matsumoto and Li [6]).

A fracture mechanics-based model proposed in Refs [7-9] for RC beams under cyclic loading has been extended to the case of FRC beams with two reinforcements [10]. In the present paper, the case of FRC beams with multiple reinforcements is analysed, by considering a cracked infinitesimal portion of a composite beam with a rectangular cross-section subjected to an external bending moment M and bridging reactions of the fibers (Fig. 1). Assuming a rigid-perfectly plastic law for the reinforcements (whose reactions present ultimate values corresponding to either yielding or slippage) and a linear-elastic law for the matrix, the statically indeterminate bridging forces are deduced from congruence conditions related to the crack opening translations at the levels of the fibers. Typical phenomena, such

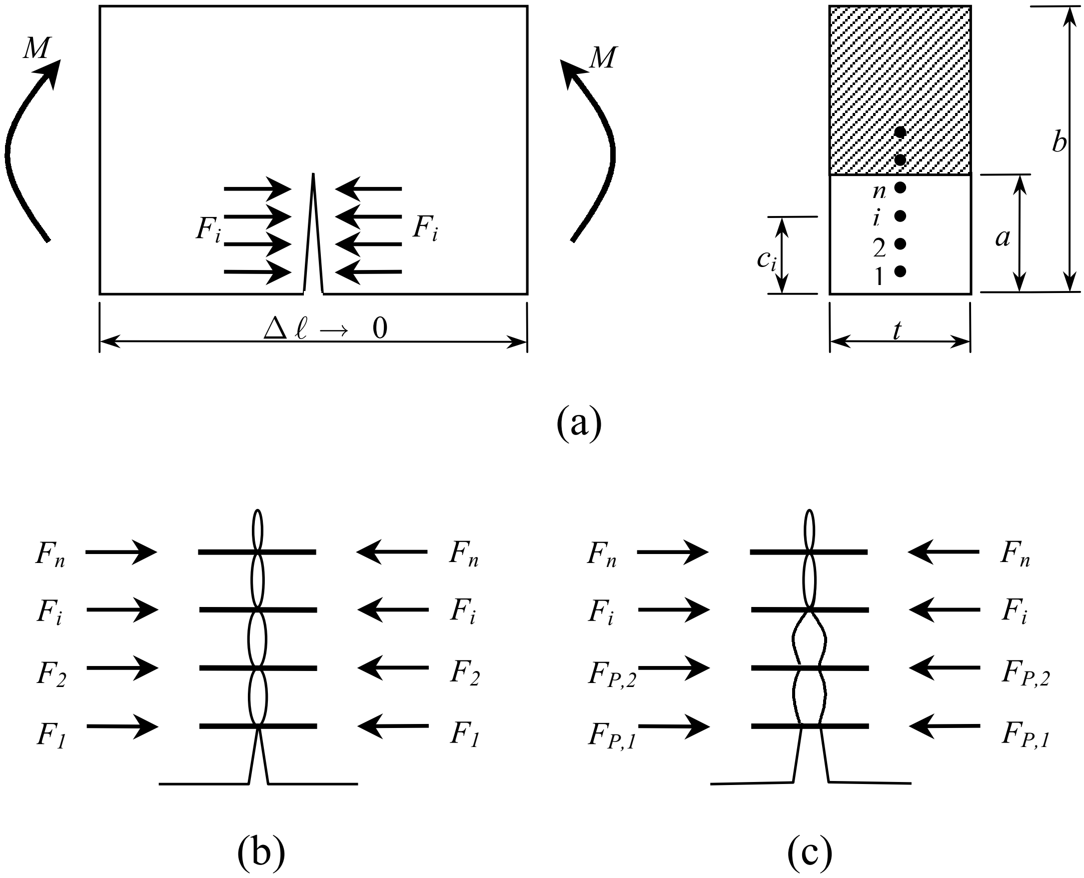

FIGURE 1. (a) Schematic of the model; (b) crack profile for elastic fibers; (c) crack profile for yielded or slipped fibers.

as elastic shake-down and plastic shake-down, are described in terms of applied bending moment against beam cross-section rotation. Then, the flexural behaviour of the composite beam up to failure is captured by applying the well-known fatigue crack growth law by Paris [11].

Theoretical Model

The crack is assumed to be normal to the longitudinal axis of the beam, and the crack depth is equal to a. Reinforcements are discretely distributed across the crack and oriented along the above axis. The position of the i-th fiber ($i = 1, \ldots, n$, where n is the number of fibers intersected by the crack) is described by the distance c_i with respect to the bottom of the beam cross-section (Fig. 1a). The reinforcement numbers are sorted according to the reinforcement positions along the beam height, by assuming that fiber No.1 is the nearest to the bottom of the beam cross-section. The relative crack depth $\xi = a / b$ and the normalised coordinate $\zeta_i = c_i / b$ are also defined.

Fibers act as rigid-perfectly plastic bridging elements (with symmetric behaviour in both tension and compression) which connect together the two surfaces of the crack. The generic (i-th) fiber is characterised by an ultimate force $F_{P,\,i}$ (and $-F_{P,\,i}$ in compression) corresponding to either yielding or slippage of the reinforcement, whichever of them exhibits the minimum absolute value.

Let us consider the above composite beam subjected to a bending moment M (opening the crack) monotonically increasing. The crack opening translation w_i at the i-th fiber level can be obtained through the superposition principle and the localised compliances due to the crack:

$$w_i = w_{iM} + \sum_{j=1}^{n} w_{ij} = \lambda_{iM} M - \sum_{j=1}^{n} \lambda_{ij} F_j \tag{1}$$

where w_{iM} and w_{ij} are the crack opening translations produced by the bending moment M and by the generic reaction F_j (assumed to be positive when the j-th fiber is under tension), respectively. The localised compliances, λ_{iM} and λ_{ij}, due to the crack represent the i-th crack opening translation for $M = 1$ and that for a unit crack opening force, $F_j = 1$, acting at ζ_j, respectively [12].

According to congruence considerations, all the translations w_i ($i = 1, \ldots, n$) are equal to zero until yielding or slippage of at least one of the n fibers is reached (Fig. 1b,c) [13]:

$$\{w\} = \{\lambda_M\} M - [\lambda]\{F\} = \{0\} \tag{2}$$

where $\{w\} = \{w_1, \ldots, w_n\}^T$ is the vector of the crack opening translations at the different fiber levels, and $\{F\} = \{F_1, \ldots, F_n\}^T$ is the vector of the unknown bridging forces. Further, $\{\lambda_M\}$ is the vector of the localised compliances related to the bending moment M, whereas $[\lambda]$ is a symmetric square matrix of order n, whose generic element ij represents the localised compliance λ_{ij}. Hence, the unknown vector $\{F\}$ can be obtained from Eq. 2:

$$\{F\} = [\lambda]^{-1} \{\lambda_M\} M \tag{3}$$

If the generic (i-th) fiber yields or slips, the crack opens at the coordinate ζ_i, and w_i becomes an unknown quantity. Therefore, the number of kinematic conditions in Eq. 2 reduces by one, along with the degree of statical redundancy, F_i being equal to the previously defined maximum bridging force $F_{P,\,i}$. Consequently, the number of equations which can be written (that is, n - 1) continues to be equal to the number of unknowns (that is, n - 1). At the subsequent yielding or slippage of some fiber, the number of kinematic conditions reduces further along with the number of statically indeterminate forces. Therefore, the congruence condition in Eq. 2 can be written as follows:

$$\{w\} = [H]\left(\{\lambda_M\}M - [\lambda]\{\bar{F}\}\right) = \{0\} \tag{4}$$

where $[H]$ is a diagonal matrix whose generic element ii is given by the Heaviside function $H(x)$, with $x = 1 - |F_i|/F_{P,i}$ (that is, $H(x) = 1$ for $x > 0$, and $H(x) = 0$ for $x \leq 0$), and

$$\{\bar{F}\} = [H]\{F\} + \left([I] - [H]\right)\{F_P\} \tag{5}$$

with $\{F_P\} = \{F_{P,1}, \ldots, F_{P,n}\}^T$, whereas $[I]$ is the unit matrix of order n. Note that, since the matrix $[H]$ becomes singular when at least one fiber yields or slips, equation(s) related to $H_{ii} = 0$ must be eliminated for solving Eq. 4. After determing the vectors $\{F\}$ and $\{\bar{F}\}$ from Eqs 4 and 5, the crack opening translations $\{w\}$ and the rotation φ of the cracked beam cross-section are computed as follows :

$$\{w\} = \{\lambda_M\}M - [\lambda]\{\bar{F}\} \tag{6a}$$

$$\varphi = \lambda_{MM}M - \{\lambda_M\}^T\{\bar{F}\} \tag{6b}$$

The collapse of the beam under the applied bending moment might occur because of two possible reasons: (1) unstable fracture of the concrete matrix (when the toughness K_{IC} of concrete is attained, that is, $K_I = K_{IC}$, where the stress intensity factor K_I is obtained by means of the superposition principle: $K_I = K_{IM} - \sum_{i=1}^{n} K_{Ii}$**)**, or (2) crushing of the concrete matrix (when the normal compressive stress σ_c, computed through the classical bending theory applied to the ligament, attains the concrete strength f_c).

Now consider constant amplitude cycles of bending moment M, ranging from M_{min} to M_{max}. Fibers might undergo plastic-to-rigid transitions at load reversals. Therefore, the congruence condition of Eq. 4 is modified in order to consider possible non-zero translations $\{w\}$ at reversals:

$$\{w\} - \{w_0\} = [H]\left(\{\lambda_M\}(M - M_0) - [\lambda]\left(\{\bar{F}\} - \{\bar{F}_0\}\right)\right) = \{0\} \tag{7}$$

with

$$\{\bar{F}\} = [H]\{F\} + q\left([I] - [H]\right)\{F_P\} \tag{8}$$

where $q = 1$ during loading and $q = -1$ during unloading. The subscript '0' refers to the values of some parameters (crack opening translations, bending moment and fiber reactions) at the preceding load reversal. Obviously, the quantities related to the subscript '0' are equal to zero at the beginning of the loading process, i.e. at the first loading halfcycle. After determining the solution vector $\{F\}$ from Eq. 7, $\{\bar{F}\}$ can be computed by Eq. 8, and crack displacements can be deduced by Eqs 6a and 6b.

If the crack is assumed to propagate (under cyclic loading) according to the Paris law ($da/dN = C\,\Delta K_I^m$), increments of crack length due to fatigue crack growth can be determined after every block of a given number of cycles. The above increments imply an increase of localised compliances at constant applied loads (i.e. bending moment M and bridging forces $\{F\}$).

Shake-down Phenomenon

Consider a composite beam with a single fiber. For certain values of loading and mechanical and geometrical parameters, the bending moment vs rotation curve for a single cycle might look like that reported in Fig. 2. The numbers (from 1 to 6) in the graph indicate the sequence of the load steps, while the upwards and downwards triangular symbols refer to tensile and compressive yielding/slippage of the fiber, respectively. It can be noted that the most significant values of the bending moment in a loading cycle are: the plastic bending moment M_P (equal to $M^{(1)}$) which produces yielding or slippage in the reinforcement (subjected to tension) during loading, and the shake-down bending moment M_{SD} (equal to $M^{(5)}$) above which yielding or slippage in the reinforcement (subjected to compression) occurs during unloading (see load step No. 3, represented by the segment 2-3 in Fig. 2).

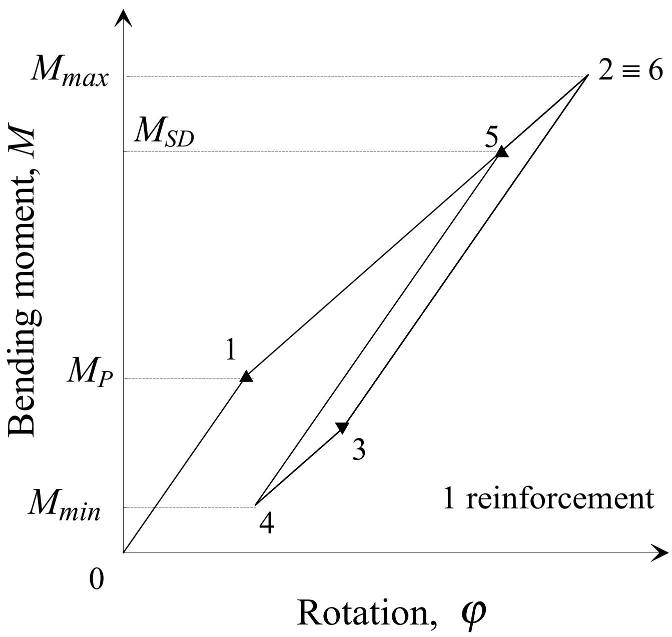

FIGURE 2. Typical bending moment vs rotation diagram in the case of a single fiber.

For $M_{SD} \leq M_{max} < M_F$ (M_F = bending moment of matrix unstable fracture when K_I attains K_{IC}, or bending moment of matrix crushing when the compressive strength f_c of concrete is attained), plastic shake-down with hysteretic loops in the bending moment vs rotation diagram can be observed (for example, the energy dissipated in each cycle is equal to the area 2-3-4-5-6 in Fig. 2).

For a beam with three identical fibers, the bending moment against rotation curve is shown in Fig. 3. As in the previous case, the numbers in the graph indicate the sequence of the load steps, while the upwards and downwards triangular symbols refer to tensile and compressive yielding/slippage of the reinforcements, respectively. The plastic bending moment M_P (equal to $M^{(1)}$) and the shake-down bending moment M_{SD} (equal to $M^{(9)}$) are displayed, and the energy dissipated in each cycle is equal to the area of the hysteretic loop 4-5-6-7-8-9-10-11-12.

The observations made for the cases in Figs 2 and 3 can be extended to the case of n fibers, and the following general relationships can be written:

$$M_{P,i} = M^{(k)} \tag{9}$$

where k refers to the load step for which the i-th fiber is on the verge of its yielding or slippage in tension, and

$$M_{SD,i} = M_{max} - M^{(k)} + M_{min} \tag{10}$$

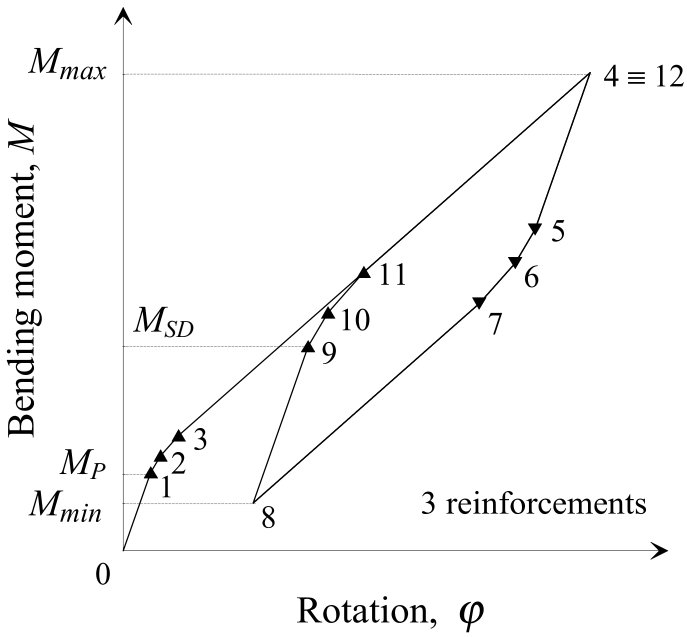

FIGURE 3. Typical bending moment vs rotation diagram in the case of three fibers.

where k now refers to the load step for which the i-th fiber is on the verge of its yielding or slippage in compression. Then, the overall plastic bending moments and shake-down bending moment are given by:

$$M_P = \min\left\{M_{P,i}\right\} \tag{11a}$$

$$M_{SD} = \min\left\{M_{SD,i}\right\} \tag{11b}$$

Now assume a crack propagating according to the Paris law. The case of three identical fibers, analogous to the case in Fig. 3, is analysed. Figure 4 shows bending moment vs rotation hysteretic loops at different numbers of loading cycles (the first cycle, the generic N-th cycle, and the final N_f-th cycle). As the crack propagates, the values of the local compliances increase and, hence, the slopes of the linear segments in the diagram M vs φ decrease. Further, by increasing the crack length ξ, the shake-down bending moment values decrease (see $M_{SD}^{(1)}$, $M_{SD}^{(N)}$ and $M_{SD}^{(N_f)}$ in Fig. 4). Consequently, the energy dissipated at every hysteretic loop varies as the number of loading cycles increases. Finally, the fatigue collapse of the beam can occur due to either matrix unstable fracture, when K_I attains K_{IC}, or matrix crushing, when σ_c attains f_c.

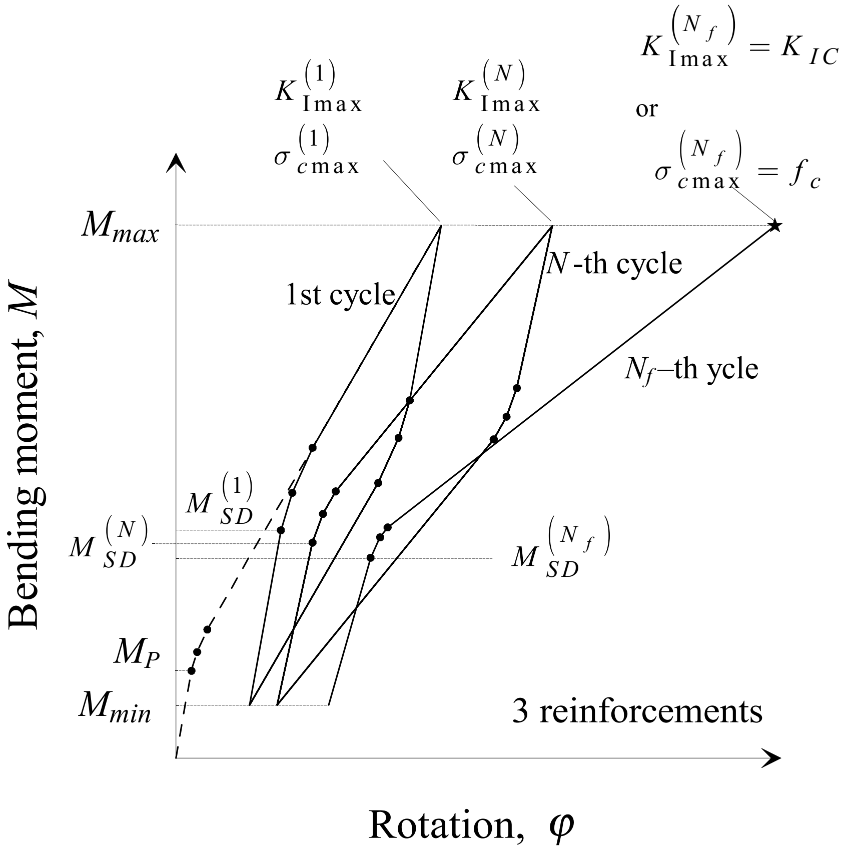

FIGURE 4. Typical bending moment vs rotation hysteretic loops at different numbers of loading cycles, in the case of three fibers.

Conclusions

A fracture mechanics-based theoretical model proposed to analyse the hysteretic behaviour of a RC beam subjected to cyclic bending is here extended to the case of a FRC beam with multiple reinforcements. Accordingly, an infinitesimal portion of a cracked beam with an elastic concrete matrix and rigid-perfectly plastic fibers has been examined, subjected to an external bending moment and bridging reactions of the fibers. The capabilities of the model are presented in terms of applied bending moment against beam cross-section rotation. In spite of the simple assumptions adopted, typical cyclic phenomena, including elastic shake-down and plastic shake-down, can be described, and fatigue life can be predicted.

Acknowledgements

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