OPTIMISING ENERGY INPUT FOR FRACTURE BY ANALYSIS OF THE ENERGY REQUIRED TO INITIATE DYNAMIC MODE I CRACK GROWTH

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Abstract

A problem for a central crack in a plate subjected to plane strain conditions is investigated. Mode I crack loading is created by a dynamic pressure pulse applied at large distance from the crack. It was found that for a certain combination of amplitude and duration of the pulse applied, energy transmitted to the sample has a strongly marked minimum, meaning that with the pulse amplitude or duration moving away from the optimal values minimum energy required for initiation of crack growth increases rapidly. Results received indicate a possibility to optimise energy consumption of different industrial processes connected with fracture. Much could be gained in for example drilling or rock pounding where energy input accounts for the largest part of the process cost. Presumably further investigation of the effect observed can make it possible to predict optimal energy saving parameters, *i.e.* frequency and amplitude of impacts, for industrial devices, *e.g.* bores, grinding machines, *etc.* and hence significantly reduce the process cost. The prediction can be given based on the parameters of the media fractured (material parameters, prevalent crack length and orientation, *etc.*).

Introduction

A possibility to minimise the amount of energy, required to fracture materials is of a large interest in connection with many applications. Energy input for fracture induced by short impulse loadings are of the major importance in such areas as percussive, explosive, hydraulic, electro-impulse and other means of mining, drilling, pounding etc. In these cases energy input usually accounts for the largest part of the process cost (see for ex. [1]). Taking into consideration the fact that the efficiency of the mentioned processes rarely exceeds a few percent the importance of energy inputs optimization gets evident.

The purpose of the present investigation is to find and explore the amount of energy sufficient to initiate the propagation of a mode I loaded central crack in a plate subjected to plane strain deformation. Two ways to apply the dynamic impulse to the body are studied. In the first case the load is applied at infinity. The study involves the analysis of interaction of the wave package approaching from infinity with an existing central crack in a plane. The existing crack is oriented parallel with the front of the wave package. In the second case the load is applied at the crack. Tractions are normal to the crack faces.

Following the superposition principle these two loading cases should produce identical stress-strain field in the vicinity of the crack tip. It will be shown later that the amount of total energy applied to the body needed to initiate crack growth is depending on the load application manner in different way for the two cases under investigation.

Load applied at infinity

Consider an infinite plane with a central crack (Fig.1). The load is given be the wave, falling on the crack. Displacements of the plane are described by:

$$\rho\, u_{i,tt} = (\lambda + \mu)\ u_{j,ji} + \mu\, u_{i,jj}\,, \tag{1}$$

stresses and strains are coupled with Hooke's law:

$$\sigma_{ij} = \lambda\, \delta_{ij} u_{k,k} + \mu\ (u_{i,j} + u_{j,i})\,, \tag{2}$$

boundary conditions are:

$$\sigma_{22}\Big|_{\substack{|x|<l \\ y=0}} = \sigma_{21}\Big|_{\substack{|x|<l \\ y=0}} = 0\,. \tag{3}$$

The impact is delivered to the crack by the falling wave:

$$\sigma_{22}\Big|_{t<0} = P\ (H(t + \frac{y}{c_1}) + H(t - \frac{y}{c_1}) - H(t + \frac{y}{c_1} - T) - H(t - \frac{y}{c_1} - T))\,, \tag{4}$$

where c_1 is the longitudinal wave speed, H is the Heaviside step function and T is the impact duration. The described problem is solved using a finite element method.

Modeling interaction of the wave coming from infinity with the crack

The process is analyzed utilizing a finite element method. The task for a quarter sample was formulated using the symmetry of the sample about x and y axes. Area adjacent to the crack tip was meshed with triangular isoparametric quarter-point elements. Thus mesh in the vicinity of the crack tip may assume a square root singularity in stress/strain fields.

Computations were performed for granite (E=96.5 GPa, ρ=2810 kg/m^3, ν=0.29, where E is the elasticity modulus, ρ is density and ν the Poisson's ratio). The results of investigation will qualitatively hold for a big variety of quasi-brittle materials. In conditions of the plane strain interaction of the wave approaching from infinity with a central crack was investigated.

Firstly infinite impulse durations were supposed, i.e. $T = \infty$. Time dependence of the stress intensity factor K_1 was studied. Modeling of different amplitudes of the loading impulse was performed. Typical dependence of K_1 on time is presented on Fig.2.

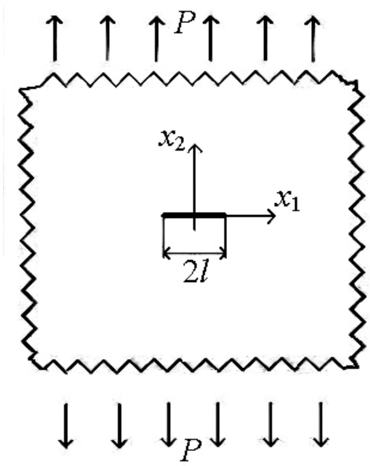

FIGURE 1. Experiment scheme

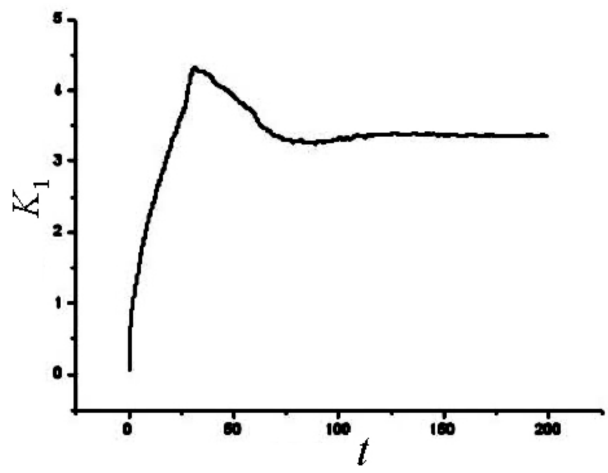

FIGURE 2. Typical K_1 time dependence.

Apparently K_1 is rapidly approaching the statical level. Thus the time to approach a steady-state situation in the vicinity of a crack tip can be estimated as 5-10 times more than the time needed for the wave to travel along the crack half-length.

Fracture criterion fulfillment was checked for different load amplitudes and durations. Dependence of time-to-fracture T^* on the amplitude of the impulse applied was investigated. Time-to-fracture is the time from the beginning of interaction between the wave package and the crack to the crack start. An incubation time criterion of fracture (Morozov and Petrov [2]) was chosen.

Incubation time criterion of fracture

For the mode I loaded crack incubation time (or structural time) criterion [2] can be written as:

$$\int_{t-\tau}^{t} K_1(t')dt' < K_{1C} \ . \qquad (5)$$

where τ is the microstructural time of fracture – assumed to be constant for a given material. As follows from the criterion proposed, fracture depends not only on a stress field in a vicinity of a point, but also on a history of a stress field development. In an extreme case when the load is applied quasistatically crack propagation starts at time $t+\tau$ where t is the moment when K_1 had reached the value of K_{1C}.

Using criterion (5) dependence of time-to-fracture on the amplitude of the load impulse applied was studied. Values of $K_{1C} = 2.4 \ MPa \ \sqrt{m}$ and $\tau = 72 \ \mu s$ typical for granite under investigation were used. Integration of the temporary dependence of stress intensity factor was done numerically. On Fig.3 horizontal axis represents the time from the beginning of interaction of the wave coming from infinity with the crack to the fracture start. Vertical axis represents the corresponding amplitude of the load applied at infinity. Point on Fig.3 marked with X corresponds to the maximum possible time-to-fracture. As follows for investigated granite and experiment conditions fracture is possible for times less than 92 μs.

At the same time the critical (threshold) amplitude of the applied load was found. This amplitude corresponds to the maximum time-to-fracture possible. Loads with amplitudes less then the critical one, does not increment the crack length.

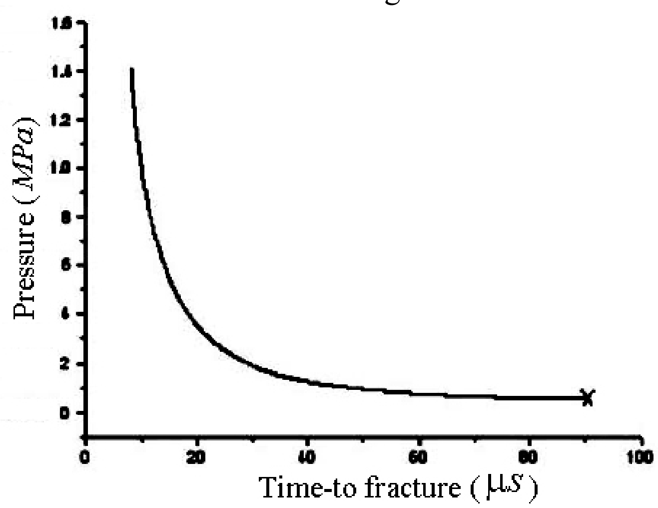

FIGURE 3. Curve limiting the pulses leading to crack propagation

Dependence of the energy inputs for fracture on the impulse amplitude and duration

Now we examine the specific momentum transferred to the plane under investigation by a loading device. In our case

$$P(t) = P(H(t) - H(t-T)), \tag{6}$$

so the specific momentum of the impact will be:

$$R = PT\,. \tag{7}$$

The area filled on Fig.4 corresponds to a set of momentum values causing a fracture. For the values out of this area crack propagation does not occur. The minimum value for the momentum incrementing the crack length is reached at impulse with duration of 72 μs while the amplitude of the load exceeds the minimal one by more than 10%.

Now we come to examination of the energy transmitted to the sample by a loading device in the process of impact. The shape of the load applied is given by (6). A specific (for the unit of length) energy transmitted to the stripe can be calculated using the solution for the uniformly distributed load acting on a half plane. This problem can be easily solved using D'Lembert method (see for ex. Bratov *et al.* [3]). The solution for a specific energy transmitted to the half plane appears to be:

$$\varepsilon_{spec} = \frac{1}{c\ \rho}\int_0^T P^2(t)dt.. \tag{8}$$

This solution can be used for the problem under investigation as interaction of the loading device and the sample is finished before the waves reflected from the crack come back. Substitution of (6) into (8) gives $\varepsilon_{spec} = \dfrac{P^2 T}{c\ \rho}$.

Analogously to Fig.4 we plot a limiting curve for a set of energies that, being transmitted to the sample, cause the crack propagation (Fig.5)

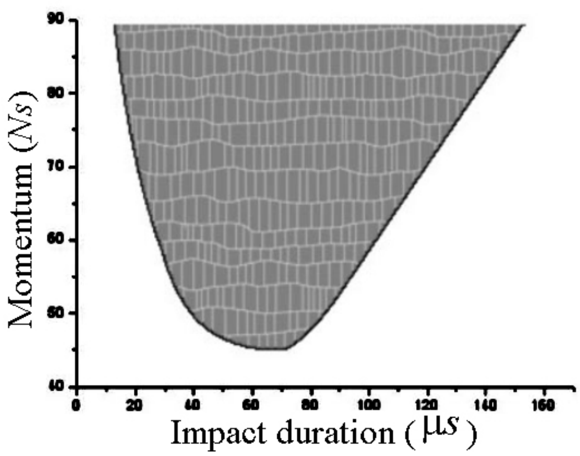

FIGURE 4. Momentum minimisation

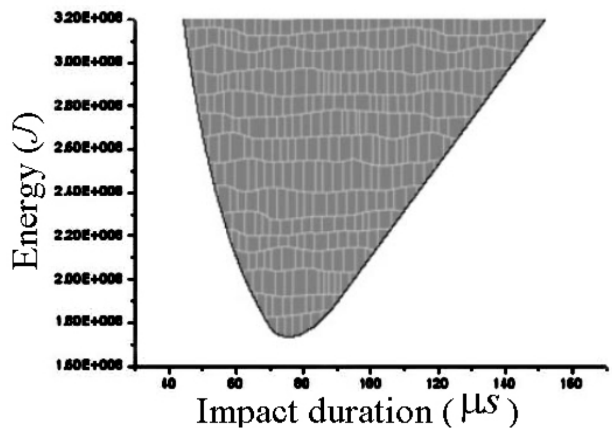

FIGURE 5. Energy minimisation

Minimum energy able to increment the crack length is reached at impulses with duration of 78 μs. As it is evident from Fig.5, minimal energy, required to propagate the crack by impacts with durations differing much from the optimal duration, significantly exceeds the minimal value possible. Thus, minimum energy, incrementing the crack for the load with duration of 90 μs (at this impact duration crack propagation is possible with the impact of threshold amplitude), will exceed minimal energy possible by 10%, and at duration of 40 μs it will be more than two times bigger.

Load applied at the crack faces

We consider a problem similar to the previous one, but now the load is applied not at infinity but on the crack faces. The problem is solved numerically utilizing the finite element method. Obviously, according to the superposition principle, the solution will coincide with one for the strip starched by a load applied at infinity. Thus all the consequences of the previous solution are applicable, except for estimations of energy. Obviously, momentum transmitted to the sample will be the same as the one of the previous problem.

It is not possible to estimate energy transmitted to the sample analytically for the problem, when the load is applied at the crack faces. However the finite element solution can be used in this case to estimate this energy. Fig. 6 represents time dependence of full, kinetic and potential energies of deformation contained in a loaded sample.

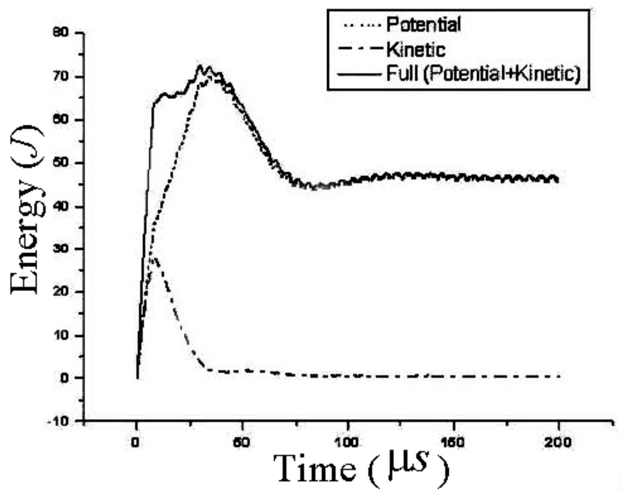

FIGURE 6. Transmitted energy time dependence

Firstly the kinetic energy is growing linearly along with the potential energy, in the same manner as in the case with the loaded half-plane. However at the moment of time equal to the time sufficient for a wave to travel along the crack length, kinetic energy is starting to transform into potential energy of deformation. Some part of the energy is returned to the loading device.

Limiting curve for the set of energies incrementing the crack length is presented on Fig.7. As it can be noticed in the case of the load applied at the crack faces, the energy input to increment the crack length has no marked minimum. Minimum energy needed to produce fracture in this case is decreasing with the growth of impulse duration. When the duration is equal to maximal time-to-fracture possible, energy reaches the minimal value.

Fig.8 enrages the area adjacent to the point where the minimal energy is firstly reached on Fig. 7.

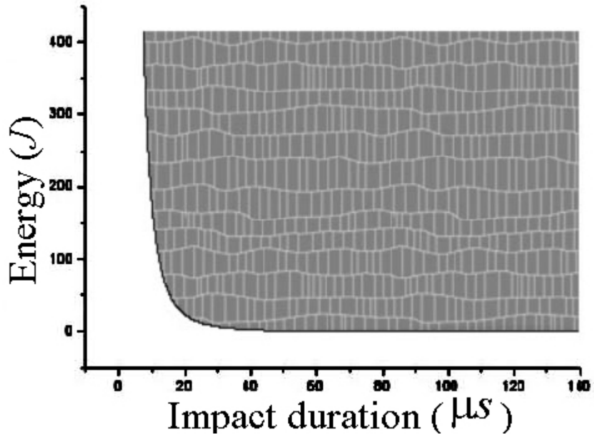

FIGURE 7. Energy minimization

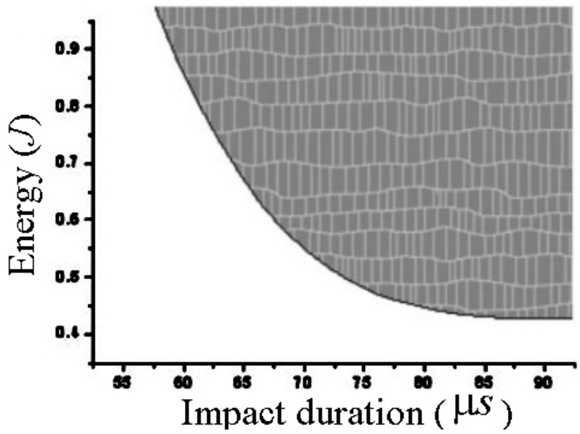

FIGURE 8. Energy minimisation

As follows from Fig. 8 for the impulse durations close to the maximal possible time-to-fracture, minimal energy inputs to increment the crack length grow slightly.

Optimization of the impulse parameters to minimize energy cost for the crack growth

With the majority of non explosive methods used to fracture materials (drilling, grinding etc.) it is possible to control amplitude and frequency of impacts from the side of a rupture machine. The performed modeling shows that at a certain load duration (at impact fracture of big volumes of material impulse duration is connected to the frequency of the machine impacts) energy inputs for crack propagation has a marked minimum.

Analogously to Fig.5 it is possible to plot the limiting curve for the set of energy values leading to propagation of a crack in the sample at different load amplitudes.

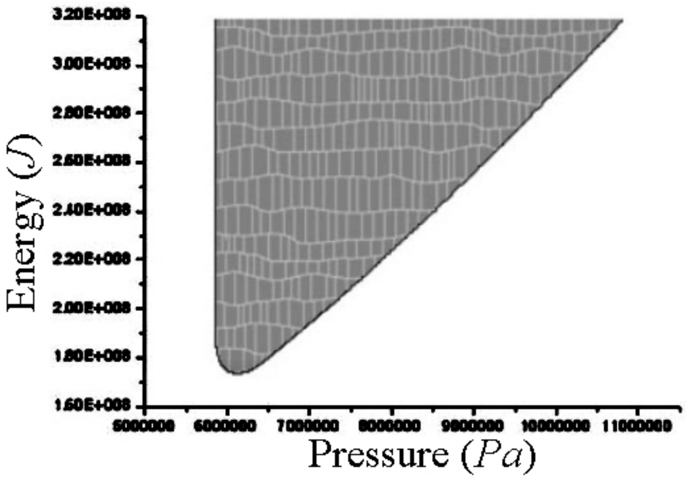

FIGURE 9. Finding optimal pulse amplitude

Thus it is possible to establish ranges of amplitudes and frequencies of load, when energy costs for fracture of the material are minimal. These ranges are dependent on parameters of fractured material, predominant length of existing material cracks and a way a load is applied.

Dependence of the parameters of load minimizing the energy for fracture on the length of the existing crack

Dependence of the optimal load parameters on the crack length was studied. The received results are represented on Fig. 10. As follows from Fig. 10 duration of load, that minimizes energy, and momentum inputs are linearly dependent on the existing crack length. With the crack length approaching zero the duration of the load minimizing momentum needed to move the crack tends to zero. At the same time the duration optimal for the energy inputs tends to the microstructural time of the fracture process. Maximum possible time-to-fracture also tends to the microstructural time of fracture.

Thus, considering intact media as the extreme case of media with cracks when the crack length tends to zero, we find that the maximum possible time-to-fracture is the same as the microstructural time of the fracture process. Durations for the loads being optimal for the energy inputs for the fracture of intact media are also equal to the microstructure time of the fracture process.

Amplitudes of loads, that minimize energy and momentum sufficient to increment the crack length, are presented on Fig. 11.

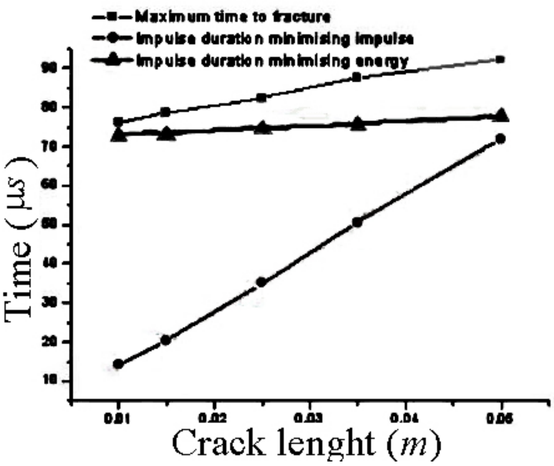

FIGURE 10. Optimal time dependence

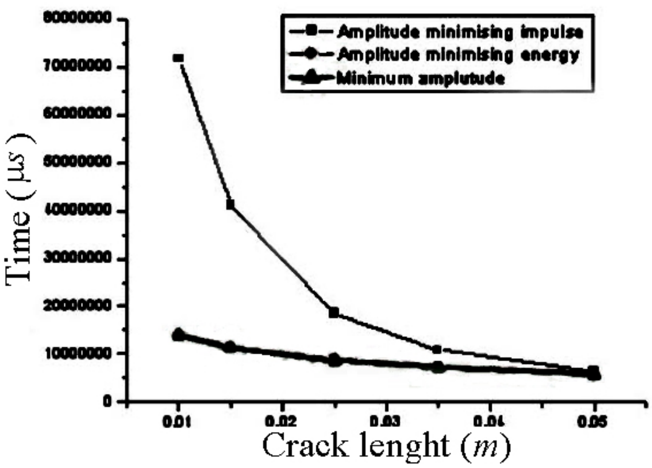

FIGURE 11. Optimal pressure dependence

As expected, the amplitude of the threshold impulse is inversely dependent on $\sqrt{l}$, where l is the crack length. Dependence of the amplitude, minimizing the energy inputs, is close to$1/\sqrt{l}$. The amplitude, minimizing the momentum, is inverely proportional to the crack length. When the crack length is close to zero, the amplitude of the load, that minimises the energy cost of the crack growth, is close to threshold. However, the amplitude, minimising the energy inputs, deviates from the threshold amplitude more and more with the growing crack length (Fig. 12).

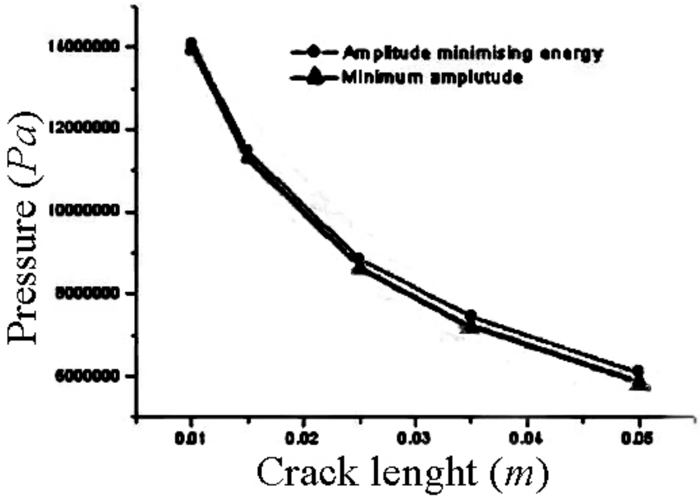

FIGURE 12. Optimal pressure dependence

Conclusions

The results received stand for a possibility to optimize energy consumption of different fracture connected industrial processes. It is shown that the energy required for crack propagation strongly depends on the amplitude and frequency of the applied load. In the studied problem when the frequency of the load differs from the optimal by 10%, energy required for crack initiation is exceeding the minimal value by more then 10%.

The obtained dependencies of the optimal characteristics of a load impulse on the existing crack length can help predicting the energy saving parameters for the fracture processes by the investigation of the predominant crack size in a fractured material.

References

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2. Morozov, N., Petrov, Y., *Dynamics of Fracture*, Springer-Velrag, Berlin-Hidelberg-New York, 2000
3. Bratov, V., Gruzdkov, A., Krivosheev, S., Petrov, Y., *Reports of the Russian Academy of Sciences*, vol. **395**, № **4**, 1-4, 2004, (in Russian)