FRACTURE INITIATION ANGLE IN THE CENTRALLY CRACKED CIRCULAR DISK SPECIMEN

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Abstract

Theoretical predictions based on the maximum tangential stress criterion suggest that mode II brittle fracture initiates along the angle –70.5° relative to the initial crack line. However, a review of the reported experimental results shows that the mode II fracture initiation angle θ_o determined from the centrally cracked circular disk (CCCD) specimen is always significantly higher than –70.5°. The experimentally obtained fracture initiation angle is also different from the angles predicted by other regular mixed mode fracture criteria. It has been shown in this research that a good prediction of the mode II fracture initiation angle can be provided for the CCCD specimen when a modified maximum tangential stress criterion is employed. The modified criterion takes into account the effect of *T*-stress in addition to the conventional singular stresses.

Introduction

Engineering components in practice are usually subjected to complex loading conditions. Therefore, cracks that are found in these components rarely deform according to a pure mode of deformation. There are several theoretical and experimental techniques for evaluating the fracture strength of cracked components in mixed mode loading [see for example 1-3].

Prediction of fracture in mixed mode I/II loading is more complicated than in pure mode I. This is because in mixed mode fracture, in addition to the onset of fracture, the direction of crack growth initiation should also be determined. A few criteria are available for mixed mode fracture in linear elastic materials. For example, Erdogan and Sih [1] suggested the maximum tangential stress (MTS) criterion. According to this criterion crack growth initiates radially from the crack tip along the direction of maximum tangential stress. The onset of fracture occurs when the tangential stress along this direction attains a critical value. The angle of fracture initiation θ_o predicted by the MTS criterion can be calculated from

$$\frac{(1-3\cos\theta_o)}{sin\theta_o}=\frac{K_I}{K_{II}} \tag{1}$$

where K_I and K_{II} are the mode I and mode II stress intensity factors. For pure mode II, the angle of fracture initiation is determined from $(1-3\cos\theta_o)=0$. Therefore, according to the MTS criterion θ_o is always equal to –70.5° when a specimen is subjected to pure mode II.

The MTS criterion was later modified by Williams and Ewing [4] who proposed that the maximum tangential stress is evaluated at a critical distance r_c from the crack tip and the non-singular stress term (called the T-stress) is also accounted in the tangential stress. Williams and Ewing [4] and Ueda et al [5] showed that the predicted fracture results for a square plate containing an angled crack specimen are in better agreement with the experimental results when such a modification is applied to the MTS criterion. Recently, Smith et al [6] have presented a generalized MTS criterion to extend the modification suggested in [4] for general mixed mode loading.

Various specimens have also been suggested to date for conducting fracture tests in mixed modes I and II. These specimens are often designed such that by changing the loading and/or geometry parameters, the full range of mixed mode I/II loading (from pure mode I to pure mode II) can be provided. One of the frequently used mixed mode test specimens is the centrally cracked circular disk (CCCD) specimen (also known as the centrally cracked Brazilian disk specimen). The specimen is cost effective and easy to manufacture. The fracture tests can also be conducted on the CCCD specimen without having to design complicated loading fixtures usually required in mixed mode fracture tests.

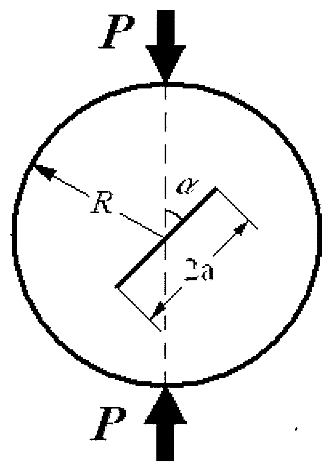

FIGURE 1. Centrally cracked circular disk specimen.

The experimental results available in the papers show that the mode II fracture initiation angle θ_o determined from the centrally cracked circular disk (CCCD) specimen is always significantly different from the angles predicted by the maximum tangential stress criterion and by other criteria. In this paper, the modified maximum tangential stress criterion [6, 7] is employed to predict the angle θ_o for the CCCD specimen in pure mode II. The mode II fracture initiation angles calculated for two different brittle materials are then compared with the corresponding experimental results reported by Shetty et al [8] and Liu et al [9]. A good agreement is shown to exist between the theoretical predictions and the experimental results.

Crack tip parameters for CCCD specimen

In this section the crack tip parameters K_I, K_{II} and T are presented for the CCCD specimen. As shown in Fig. 1 the specimen is a circular disc with a through crack along a diameter of the disc. Fracture test is conducted by applying a diametral compressive load. Different combinations of mode I and mode II can be provided by changing the angle α that is the angle between the crack line and the loading direction. When α is zero, the specimen is subjected to pure mode I. Depending on the crack length and the disc radius, mode II is achieved at different angles ($\alpha \approx 20^o$-30^o). The mode I and mode II stress intensity factors K_I and K_{II} for the CBD specimen are often written as:

$$K_I = \frac{PN_I}{RB}\sqrt{\frac{a}{\pi}} \tag{2}$$

$$K_{II} = \frac{PN_{II}}{RB}\sqrt{\frac{a}{\pi}} \tag{3}$$

where P is the applied load, a is the semi- crack length , and B and R are the disc thickness and radius, respectively. The geometry factors $\mathrm{N_I}$ and $\mathrm{N_{II}}$ are functions of the crack length ratio a/R and the crack angle α.

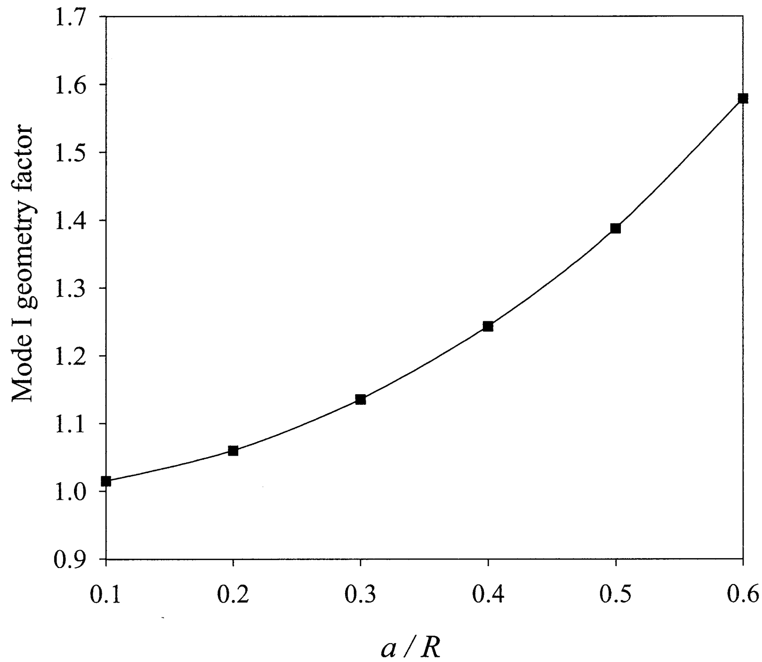

FIGURE 2. Variations of the mode I geometry factor $\mathrm{N_I}$ with a/R for pure mode I [10].

Fig. 2 shows the values of $\mathrm{N_I}$ versus the crack length ratio a/R for pure mode I that is when $\mathrm{N_{II}} = 0$ [10]. While the angle α is always zero for pure mode I, it varies with a/R for pure mode II (i.e. when $\mathrm{N_I} = 0$). Thus for each value of a/R, first the related mode II angle α_{II} should be calculated from the results given by Atkinson et al [10]. Then $\mathrm{N_{II}}$ is determined for pure mode II using the assumed a/R and the corresponding calculated α_{II}. Fig. 3 shows the variations of $\mathrm{N_{II}}$ with a/R in pure mode II.

The non-singular stress term T-stress has also been recently calculated for the CCCD specimen by Fett [11]. He used the boundary collocation method with an appropriate green's function to determine the dimensionless parameter

$$T^* = T\frac{\pi RB}{P}\left(1-\frac{a}{R}\right) \tag{4}$$

for different combinations of modes I and II . Fig. 4, shows the variations of T^* versus the crack length ratio a/R for pure mode II, extracted from the results given by Fett [11].

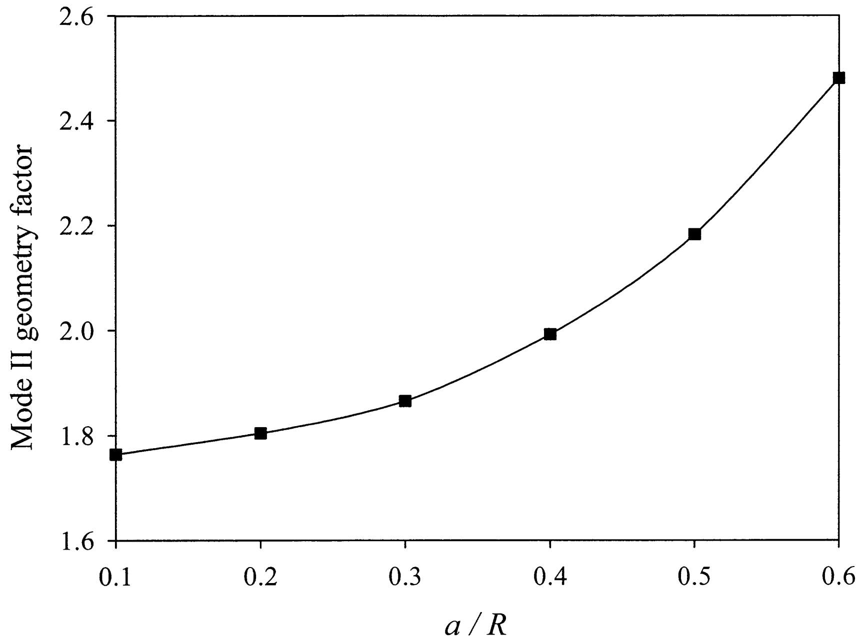

FIGURE 3. Variations of the mode II geometry factor N_{II} with *a/R* for pure mode II.

Fracture initiation angle

The MTS criterion is used in this research only for investigating pure mode II fracture. It is also modified such that both the singular term and the T–stress are considered in the tangential stress. For pure mode II, the tangential stress near the rack tip is written as

$$\sigma_{\theta\theta} = \frac{-3K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \cos^2\frac{\theta}{2} + T \sin^2\theta + O\left(r^{1/2}\right) \tag{5}$$

where r and θ are the conventional crack tip coordinates. The higher order terms $O\left(r^{1/2}\right)$ are negligible near the crack tip. Thus, the angle of maximum tangential stress $\theta_\circ$ is calculated from

$$\left.\frac{\partial\sigma_{\theta\theta}}{\partial\theta}\right|_{\theta=\theta_\circ} = 0 \tag{6}$$

which gives

$$1 - 3\cos\theta_\circ + 5.33\frac{T\sqrt{2\pi r_c}}{K_{II}} \sin\frac{\theta_\circ}{2} \cos\theta_\circ = 0 \tag{7}$$

where r_c is the radius of critical distance from the crack tip. By solving Eq (7), the angle θ_o can be determined in terms of the dimensionless parameter $\overline{T} = T\sqrt{2\pi r_c}/K_{II}$. According to Eq. (7) the angle of maximum tangential stress in mode II depends on the T-stress. The variations of θ_o with $\overline{T}$ is shown in Fig. 5.

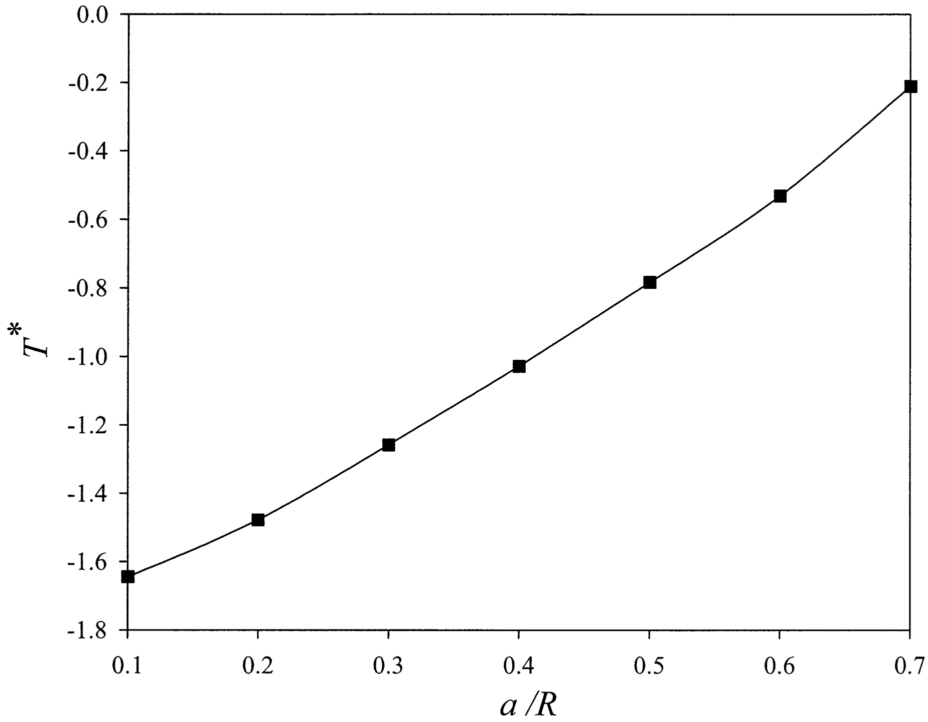

FIGURE 4. Variations of the normalized T-stress with *a/R* for pure mode II.

Discussion

Similar to predictions by the conventional MTS criterion, Fig 5 shows that the angle between the crack line and the direction of fracture initiation is -70.5° when the T-stress is zero. The absolute value of angle θ_o becomes larger for positive values of T and smaller for negative values of T.

Although the critical fracture load is often the main concern in mixed mode crack problems, the fracture initiation angle is also used for validating mixed mode fracture criteria. Since the fracture angle can be measured readily after mixed mode fracture tests, it has been used by several researchers for evaluating the proposed fracture criteria [see for example 4,5, 12].

The fracture initiation angles experimentally determined for two different materials [8, 9] are used here to validate the theoretical results presented in the previous section. Using the CCCD specimen, Shetty et al [8] and Liu et.al. [9] conducted a series of fracture test on soda lime glass and epoxy resin, respectively. According to their tests results for pure mode II, the average fracture initiation angle was found to be -63° for soda lime glass and -58° for epoxy resin. To calculate this angle from Fig 5 the dimensionless parameter $\overline{T} = T\sqrt{2\pi r_c}/K_{II}$ or the critical radius r_c is needed. The parameter $\overline{T}$ can be estimated from the ratio K_{IIc}/K_{Ic} as described by Ayatollahi et al [7]. The average of ratio K_{IIc}/K_{Ic} has been reported to be 1.232 for soda lime glass [8] and 1.588 for epoxy resin [9]. Using a relation between K_{IIc}/K_{Ic} and $\overline{T}$ the values of $\overline{T}$ is found to be -0.39 for soda lime glass and -0.64 for the epoxy resin. Using these values, the fracture initiation angle can be predicted from Fig 5, which is -60° for soda lime glass and -51° for epoxy resin. It is seen that there is a good agreement between the experimental and predicted results. In particular, the calculated fracture angle for the soda lime glass consists very well with the experimentally observed angle.

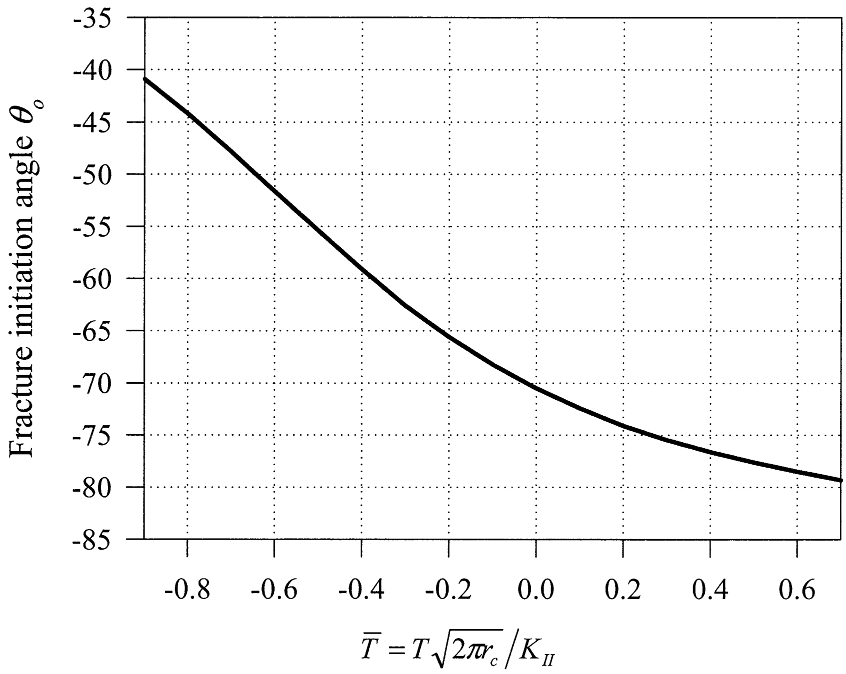

FIGURE 5. Variations of fracture initiation angle with T-stress in pure mode II.

Conclusions

- The maximum tangential stress criterion was extended to account for the effects of T-stress on the angle of fracture initiation in mixed mode loading.
- The extended criterion was used to predict the angle of fracture initiation in the CCCD specimen for pure mode II. It was shown that the fracture angle depends on the ratio T/K_{II} and also on the critical radius r_c.
- Theoretical predictions for mode II fracture angle in the CCCD specimen were compared with earlier experimental results for two different brittle materials. It was shown that a good agreement exists between the theoretical and experimental results.

References

1. Erdogan, F. and Sih, G.C., *J. Basic Engng, Tran. of ASME.*, vol. **85**, 519-527, 1963.
2. Hussain, M.A., Pu, S.L. and Underwood, J. *Fracture Analysis, ASTM STP 560.* American Society for Testing and Materials, Philadelphia, 1974, 2-28.
3. Sih, G.C. *International Journal of Fracture.* Vol. **10**, 305 321, 1974.
4. Williams, J.G. and Ewing, P.D., *Intl. J. of Frac*, vol. **8**, 441-446, 1972.
5. Ueda, Y. , Ikeda, K. , Yao, T. and Aoki, M. *Engng Fract Mech.* vol. **18**, 1131-1158, 1983.
6. Smith, D.J., Ayatollahi, M. R. and Pavier, M.J., *Fatigue Fract Engng Mater Struct*, vol. **24**, 137-150, 2001.

7. Ayatollahi, M.R., Pavier, M.J. and Smith D.J., In *Proceedings of 14th European Conference on Fracture*, EMAS Publications, 2002; vol. 1, 161-168.

8. Shetty, D.K., Rosenfield, A.R. and Duckworth, W.H., *Engng Fract Mech*; vol. **26**, 825-840, 1987.

9. Liu, C., Huang, Y. and Stout, M.G., *Acta Mater*; vol. **46**, 5647-5661, 1998.

10. Atkinson, C., Smelser and R.E., Sanchez, J. *Intl J. Fract.*; vol. **18**, 279-291, 1982.

11. Fett, T., *Engng Fract Mech*; vol. **68**, 1119-1136, 2001.

12. Maccagno, T.M. and Knott, J.F. *Engng Fract Mech*. Vol. **34**, 65-86, 1989.