Microstructural short fatigue crack growth – effect of mixed mode conditions and crack closure

A. Schick[1], C.-P. Fritzen[1], W. Floer[2], U. Krupp[2] and H.-J. Christ[2]

[1] Institute for Mechanics and Control - Mechatronic,
University of Siegen, 57068 Siegen, Germany
[2] Institute for Materials Technology,
University of Siegen, 57068 Siegen, Germany

ABSTRACT: *A slip band crack model based on the boundary element technique is proposed to simulate fatigue crack growth in stage I. The model is based on a yield strip model analogously to that of Bilby et al. [1] and extended in such a way, that the crack is allowed to glide along the ligament and to open simultaneously. Therefore, roughness-induced crack closure can be included and due to a numerical approach no restrictions on slip band geometry are required. An easy but efficient solution procedure to solve the system of equations obtained from the model is presented. The simulation of a crack in a jagged slip band shows the high influence of roughness-induced crack closure. The retardation of the crack increases with decreasing inclination angle of the slip band. A comparison of the simulation with measured short crack data shows the practical importance of crack closure effects and verifies the simulation procedure.*

INTRODUCTION

The interest in short fatigue cracks increased steadily, since Pearsen [2] realized their particular behaviour. Short cracks are known to behave significantly different from long cracks. As proposed by Suresh and Ritchie [3], short cracks can be divided into microstructurally, physically, mechanically and chemically short cracks. Microstructurally short fatigue cracks are characterized by their strong interaction with microstructural features like grain boundaries. Because up to 90% of the fatigue life depends on the initiation and early growth of microsturcturally short cracks, the understanding of their behaviour is important for predicting cyclic life. They often grow in a stage I manner along slip bands and the crack propagation occurs in single slip. Therefore a yield strip model appears to be a reasonable approach for crack simulation in this stage. Numerous models have been proposed dealing with the yield strip concept, see e.g. [1,4,5]. In refs. [4,5] the interaction of the crack with grain boundaries is taken into account. In the present study the approach of Bilby et al. [1] (BCS-model) is applied and extended such as to simulate stage I fatigue in a titanium alloy. The original analytical model is based on the theory of distributed dislocations and is able to describe cracks in pure Mode I, Mode II or Mode III. In contrast to the original BCS model, the model proposed here is treated numerically. Therefore, the ligament is not restricted to lie in one plane and the crack faces are allowed to perform normal and tangential displacements. This treatment allows to take roughness-induced crack closure into account.

THE MODEL

For the numerical treatment, the ligament is divided into elements (Figure 1). The material behaviour inside the slip band is assumed to be linear elastic-ideal plastic while it is linear elastic elsewere. The elements representing the crack are allowed to perform normal and tangential displacements simultaniously. Each part of the ligament can be inclined in arbitary angles to the loading direction.

The distribution of the displacements, which are constant inside each element, is calculated by two mathematical dislocations per element for each deformation direction. Two climb dislocations with Burgers'-vectors b_n parallel to the element's normal direction but different signs represent a constant normal displacement inside a crack element. Two glide dislocations with Burgers'-vectors b_t parallel to the element's tangential direction and different signs have the same function for the tangential direction inside the crack and plastic zone. This approach is analogous to that of Riemelmoser et al. in [6].

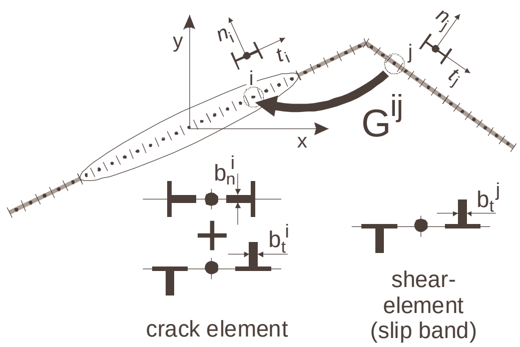

Figure 1: Numerical slip band model with open crack.

The stresses on the open crack faces have to disappear and the shear stresses in the slide plane must not overcome the back stress of the dislocations τ^b (local yield stress). The normal displacements of the cracks are restricted to positive values or zero meaning that roughness-induced crack closure occurs. To fulfil the boundary conditions, the stresses on each element are calculated by influence functions $G^{ij}_{kl,m}$ (see [6] for details), which describe the component of the stress tensor σ^i_{kl} in element i due to a unit displacement b^j_m in element j. The stresses σ^i_{nn} and τ^i_{tn} on the midpoint of element i result from the sum of the stresses of all elements caused by the displacements b_t and b_n in these elements and the applied stress $\sigma^{i,\infty}_{nn}$ and $\tau^{i,\infty}_{tn}$. This can be expressed by the following equations:

$$\sigma^i_{nn} = \sum_{j=1}^{p} G^{ij}_{nn,n} b^j_n + \sum_{j=1}^{p+q} G^{ij}_{nn,t} b^j_t + \sigma^{i\infty}_{nn} \leq 0 \qquad i = 1 \dots p \tag{1a}$$

$$\left|\tau^i_{tn}\right| = \left|\sum_{j=1}^{p} G^{ij}_{tn,n} b^j_n + \sum_{j=1}^{p+q} G^{ij}_{tn,t} b^j_t + \tau^{i\infty}_{tn}\right| \begin{cases} = 0 & i = 1 \dots p \\ \leq \tau^b & i = p+1 \dots p+q \end{cases} \tag{1b}$$

The number of elements inside of the crack is p, the number of active elements in the slip band is q. The system of inequalities (1) depends linearly on the displacements. For an open, fully elastic crack (without the q slip band elements) (1) becomes a homogenous system of equations of the order $2p$ for the displacements b_t and b_n of the p crack elements, which can be solved easily. Once these displacements are obtained, the crack tip slide displacement can be derived from b_t of the elements at the crack tip. The condition of an open crack results in the restriction

$$b^i_n \geq 0 \qquad i = 1 \dots p \tag{2}$$

The problem is now to solve efficiently the system of equations (1) taking the restriction (2) into account and to get the number of elements in the slip bands which are active and therefore represent the extension of the plastic zone. For this purpose, an easy-to-use algorithm has been developed which solves the system of inequalities iteratively (see Figure 2):

1. In a first step, yielding is ignored, i.e. the tangential displacements of the elements in the slip bands are set to zero. The stress components σ_{nn} and τ_{tn} in the elements of the crack are also set to zero and the corresponding homogenous linear equation system in (1) is solved.
2. Condition (2) is checked. If negative normal displacements exist, they are set to zero and the corresponding equation is eliminated from the system of equations (1a) by deleting the appropriate lines and rows from the influence matrix **G**. The new system is solved again and step one and two are repeated until condition (2) is fulfilled.
3. The shear stresses of all elements in the slip band are calculated according to eqn. (1) by means of the result from step 2.
4. The elements in the slip band that are overstressed ($\tau_i > \tau^b$) are identified. The shear stresses inside these elements are set equal to the back stress. Therefore, the number of equations is increased by the equations for the overstressed elements. The resulting inhomogeneous system of equations is solved again. Steps 2 to 4 are repeated until no overstressed elements can be found anymore.

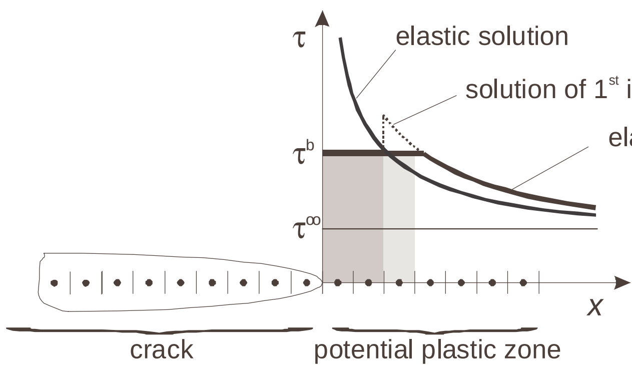

Figure 2: Solution procedure to calculate the extension of the plastic zone.

Usually, less than ten iterations are necessary to solve the above problem providing a fast solution. Once the influence matrix **G** of a certain geometry has been calculated, the required equations are easily obtained by eliminating the lines and rows in (1) of those elements which are redundant.

Up to this point the calculation is based on the assumption that the stress is applied from zero to its maximum value. However, during fatigue, the current load results always from the last load peak (Figure 3). In order to take the crack history into account, equations (1) and (2) have to be extended. For this purpose the displacements b_t^i and b_n^i are linked to a vector of displacements **b**. According to Figure 3 the solution of a certain point of applied stress in the load history, e.g. point *c*, is obtained by superpositioning the solution for the prior step (point *b*) and the solution for the applied stress interval $\Delta\sigma^{bc}$ between those two points *b* and *c*. Therefore, the vector of displacements is divided into a sum of two parts,

$$\mathbf{b}^{c} = \mathbf{b}^{b} + \Delta\mathbf{b}^{bc} \tag{3}$$

The first part $\mathbf{b}^{b}$ describes the solution of the displacements at the point *b* and the second one is the solution of the crack problem loaded by $\Delta\sigma^{bc}$. Inserting equation (3) into the primary equations (1) and (2), the problem can be solved analogously. In that way one obtains a solution which depends on the previous history of the crack. This is of high significance when calculating the cyclic plastic displacement at the crack tip, which is generally not the same as the static one.

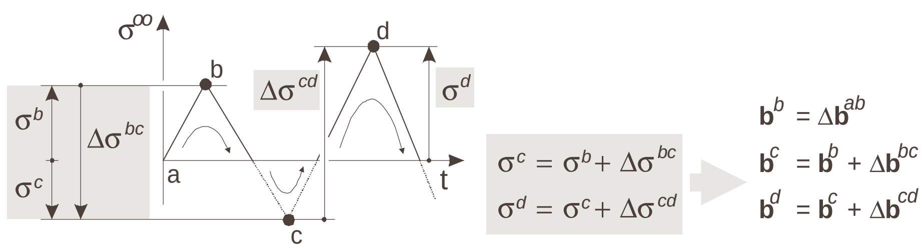

Figure 3: Principle of superposition applied to the simulation of fatigue.

The crack growth in the simulation is - analogously to the model of Navarro and de los Rios [4] - correlated with twice the amplitude of the crack tip plastic displacement. In the present study, the crack advance is calculated by equation (4):

$$\frac{da}{dN} = A \cdot \Delta CTSD^{n} \tag{4}$$

in which A is a constant of the order magnitude of one and $\Delta CTSD$ is twice the amplitude of the crack tip slide displacement. The value of $CTSD$ can be calculated by the tangential displacement at the crack tip. The exponent n is set to one as a first approach. For crack growth, the crack tip displacement has to fulfil a complete loop according to the range-pair counting method. That means, a positive displacement has to be followed by a negative one of the same magnitude. The crack growth is set proportional to these oscillation amplitudes.

SIMULATION EXAMPLES

Influence of roughness-induced crack closure on jagged cracks

To investigate the influence of the roughness-induced crack closure described above, the simulation algorithm is applied to a model with a jagged geometry consiting of a sequence of slip band parts (Figure 4). The load axis is vertical and the stress amplitude is 400MPa. About 400 elements where used within seven slip band parts with a length of 100µm each. 21 elements in the center of the ligament where defined as the starter crack. The back stress is set to 500MPa.

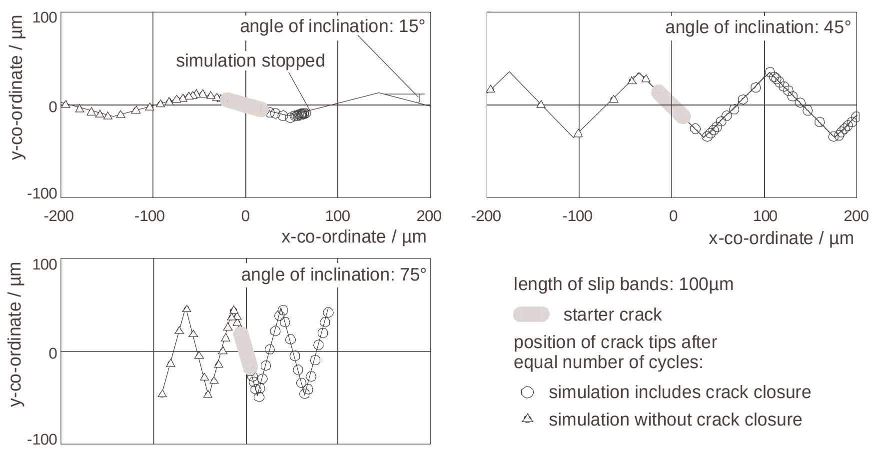

Figure 4: Influence of roughness-induced crack closure on crack extension in cracks growing along inclined slip bands with different inclination angles.

The inclination angles of the individual slip planes are varied between 15° and 75°. According to Mohr’s transformation rule the resolved shear stress on the slip plane is equal for the inclination angles 15° and 75°. In Figure 4 the positions of the crack tips are depicted which were achieved after a constant number of load cycles. On the left hand side of the starter crack the positions of the crack tips are displayed which where determined by a simulation ignoring the crack closure i.e. negative normal displacements were allowed. The corresponding results on the right hand side of the starter crack are obtained by the same simulations but including the roughness-induced crack closure.

In all cases, the crack growth rate decreases immediately after passing an edge (in a real microstructure this should be usually a grain or phase boundary) and subsequently increases. The crack velocity generally increases with increasing crack length. The simulation with the highest resolved shear stress in the slip band (inclination angle is 45°) which ignores crack closure shows the highest crack growth rate.

Figure 4 demonstrates the influence of the inclination angle on the relevance of roughness-induced crack closure: the smaller the angle the stronger is the crack retardation due to crack closure. This oberservation can be explained by the stronger influence of the crack normal displacements which are larger at smaller inclination angles due to the higher normal stress in the slip planes. Therefore the crack with an inclination angle of 15° propagates with lower speed than the crack with an inclination angle of 75°, although the resolved shear stresses due to the applied stress are the same. This theoretical approach indicates a strong effect of roughness-induced crack closure on crack extension in stage I.

Application of the model to a observed crack geometry

To verify the behaviour of the model by comparison with a real naturally grown crack, which was experimentally observed in the β-Ti alloy LCB [7], the geometry of such a crack is put into the slip band model including a starter crack (Figure 5). Subsequently, the simulation is carried out with and without including crack closure. The constant A in eqn. (4) was set to one, the back stress was assumed to be 500MPa and the applied stress amplitude was again 400MPa.

Comparison of the crack propagation curves in Figure 5 shows that according to the results reported above the resolved shear stress in the slip band has a high influence on the crack growth rate. The higher the local Schmid factor at the crack tip the higher is the crack growth rate. Of course, the simulation which accounts for crack closure yields lower da/dN values than the one without crack closure, for which the amplitude of effective plastic crack tip displacement is naturally higher. Therefore, the stress amplitude in the simulation without crack closure should be set to 50% of that including crack closure. Furthermore, the simulation including crack closure fits the shape of the measured curve in a better way as seen for the crack length between about 70µm to 120µm. The contours of the two simulated curves are not congruent. Therefore, the resolved shear stress seems not to be the only parameter responsible for the driving force at the crack tip. Moreover, crack closure has to be taken into account.

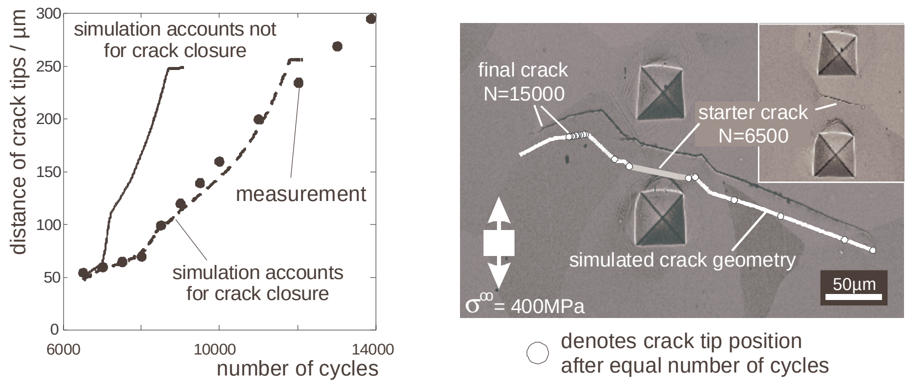

Figure 5: Comparison of the model's behaviour with that of a real naturally grown crack.

CONCLUSIONS

A numerical slip band model has been proposed based on the theory of distributed dislocations, which is able to account for roughness-induced crack closure. An easy and effective solution procedure allows to calculate crack growth cycle by cycle. The simulations of microstrucutrally short cracks in stage I show the high significance of roughness-induced crack closure, which has to be taken into account to describe the crack growth rate of microstructural short fatigue cracks. A comparison with experimental data verifies the results obtained from the simulations.

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