Crack growth in three-dimensional structures

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ABSTRACT: *This paper deals with three-dimensional Mixed-Mode Fracture and Fatigue problems and the simulation of 3D crack growth under Mixed-Mode I+II+III loading. In contrast to pure Mode I loading, where the crack surface develops self-similar (co-planar) from the initial crack, the growth of Mixed-Mode loaded cracks generally leads to non-planar crack surfaces. Furthermore, the paper presents the software system ADAPCRACK3D that has recently been developed at the Institute of Applied Mechanics (FAM) to predict crack growth in complex three-dimensional structures under multiaxial loading. Its main focus is on the determination of 3D-crack paths and the evaluation of components' lifetimes as part of the damage tolerant assessment. A simulation example will show the capability of ADAPCRACK3D's advanced three-dimensional crack growth simulation.*

INTRODUCTION

Traditionally, fracture mechanics as an instrument for the prediction and evaluation of cracks is a domain of safety sensitive branches such as power plants or aircraft industries and their suppliers. However, with the growing interest in lightweight structures and products basically in all fields of mechanical engineering and with the simultaneously rising quality consciousness the interest in fracture mechanical calculations is growing in mass production also. In particular material flaws or pre-cracks, which may have been introduced unintentionally during the manufacturing process, can have an arbitrary orientation with respect to a general type of loading which a machine component or a structure has to carry. However, complex loading and geometries complicate the process of predicting fatigue crack growth in real-world applications. This is especially the case for crack growth prediction in three-dimensional structures.

Three-dimensional Mixed-Mode Fracture and Fatigue

Three-dimensional fracture and fatigue problems might occur in real structure, if they either are subjected to three-dimensional stress states or contain crack geometries, that have an orientation perpendicular or inclined to the maximum principle normal stress.

In the case, that the crack plane is perpendicular to the maximum principle normal stress, a pure Mode I loading of the crack is given. This loading condition results in a self-similar crack propagation, that retains the previous crack orientation. Such crack growth can for example be observed for surface cracks, that are subjected to Mode I loading. If otherwise the crack tip, respectively the crack front, is subjected to a fully three-dimensional Mixed-Mode loading situation (generally a superposition of all three basic crack Modes I, II and III), the orientation changes with the growth of the crack, which means, that the crack is kinked and/or twisted. The new orientation of the crack can be described by two angles φ_0 and ψ_0 ([1], Eq. 1, Eq. 2,)

$$\varphi_0 = \mp\left[140^\circ \frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|} - 70^\circ \left(\frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|}\right)^2\right] \tag{1}$$

$$\psi_0 = \mp\left[78^\circ \frac{|K_{III}|}{K_I + |K_{II}| + |K_{III}|} - 33^\circ \left(\frac{|K_{III}|}{K_I + |K_{II}| + |K_{III}|}\right)^2\right] \tag{2}$$

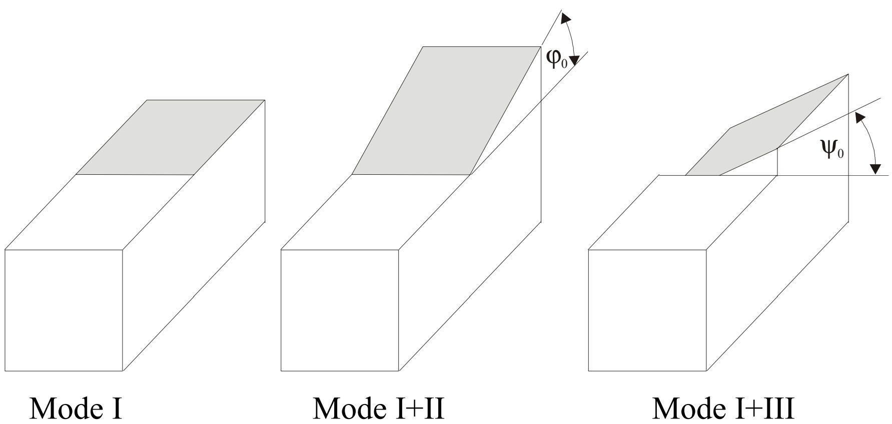

Figure 1: New orientation of the propagating crack due to Mixed-Mode-loading

Both angles φ_0 and ψ_0 only depend on the three stress intensity factors K_I, K_{II} and K_{III} at the crack front. Eqs. 1 and 2 are approximations [1], that only differ very little from the exact solution of the σ_1'-criterion [2,3], but are much easier to handle.

At spatial Mixed-Mode loading cases instable crack growth can – according to linear elastic fracture mechanics – be expected, if the comparative stress intensity factor K_v reaches the fracture toughness K_{Ic}:

$$K_v = K_{Ic}, \tag{3}$$

whereas K_v can be obtained from the stress intensity factors K_I, K_{II} and K_{III} at the crack front according to following equation [1]:

$$K_{V_{I,II,III}} = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 4(\alpha_1 K_{II})^2 + 4(\alpha_2 K_{III})^2} \leq K_{IC} \tag{4}$$

For α_1=1.155 and α_2=1 excellent agreement with the σ_1'-criterion by Schöllmann et. al [2] can be found.

The combination of Eq. 3 and Eq. 4 describes a fracture surface within a K_I-K_{II}-K_{III}-space (Figure 2).

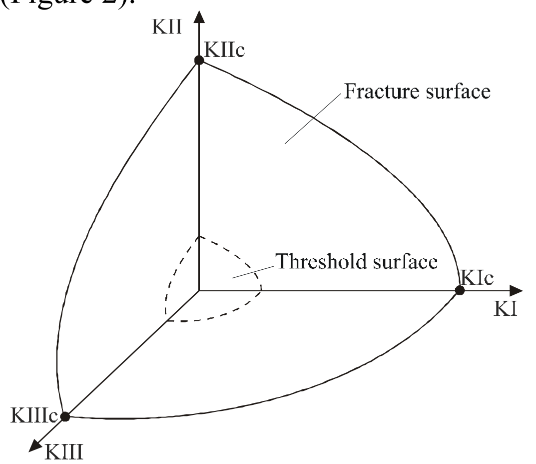

Figure 2: Fracture limit surface and threshold surface for three-dimensional Mixed-Mode Fracture and Fatigue problems

For single Mode loading cases instable crack growth can be observed according to Eqs. 3 and 4, if following conditions are fulfilled:

Mode I: $K_I = K_{Ic}$

Mode II: $K_{II} = 0.87 K_{Ic} = K_{IIc}$

Mode III: $K_{III} = K_{Ic} = K_{IIIc}$

If a component or structure is subjected to oscillating load a crack might grow under certain circumstances. Especially under general Mixed-Mode loading conditions following questions have to be taken into consideration:

a) What are the necessary conditions for a crack to grow?
b) How will the path of the growing crack look like?
c) What remaining lifetime until failure does the cracked structure have?

In order to be able to answer question a) in a quantitative manner, it is necessary to know the fatigue Threshold value ΔK_{th}. Under Mode I loading conditions the crack is able to grow, if the cyclic stress intensity factor $\Delta K=\Delta K_I$ exceeds the experimentally determined Threshold value ΔK_{th}. With increasing cyclic stress intensity stable crack growth will be observed, until ΔK reaches the fracture limit $\Delta K_c=(1-R)K_{Ic}$. If a general Mixed-Mode loading condition at the crack front is to be considered, fatigue crack growth will occur for

$$\Delta K_{th} \leq \Delta K_v \leq \Delta K_c \,. \tag{5}$$

In this equation ΔK_v is the cyclic comparative stress intensity factor, that can be derived from Eq. 4 [1]:

$$\Delta K_{V_{I,II,III}} = \frac{\Delta K_I}{2} + \frac{1}{2}\sqrt{\Delta K_I^2 + 4(\alpha_1 \Delta K_{II})^2 + 4(\alpha_2 \Delta K_{III})^2} \tag{6}$$

As before the parameters should be set as α_1=1.155 and α_2=1.

With the use of

$$\Delta K_v=\Delta K_{th} \tag{7}$$

the Threshold-surface is defined in the K_I-K_{II}-K_{III}-space (Figure 2). Fatigue crack growth, i.e. stable crack growth, is consequently possible, if the stress intensity at the crack front –given by the stress intensity factors K_I, K_{II} and K_{III}– is located within the K_I-K_{II}-K_{III}-space in between the Threshold and the Fracture limit surface. An existing crack, that begins to grow under fatigue conditions, propagates under Mixed-Mode loading into a new direction. This means, that the new developing crack surface is inclined with respect to the already existing one (Figure 1). The kinking angles φ_0 and ψ_0 might be calculated by Eqs. 1 and 2 also in the "fatigue case".

For the purpose of estimating the crack growth lifetime an iterative approach, that e.g. is realized by the program system ADAPCRACK3D, is reasonable for those complicated three-dimensional problems. In order to perform such a calculation, the experimentally investigated crack growth curve (for Mode I loading) is of special interest. This crack growth curve has to be determined for the particular material as well as the particular R-ratio of the loading, that is under consideration. When performing a lifetime analysis for Mixed-Mode loading, the cyclic stress intensity factor ΔK_I is replaced by ΔK_v:

$$\frac{da}{dN} = f(\Delta K_v) \,, \tag{8}$$

with ΔK_v defined as in Eq. 6.

Calculation of fatigue crack growth with ADAPCRACK3D

The three-dimensional crack simulation program ADAPCRACK3D consists of three independent modules, that in conjunction perform a fully automatic crack growth simulation. This is done by iterative adaptation of the underlying FE-models according to the change of geometry due to the growth of the crack in every simulation step.

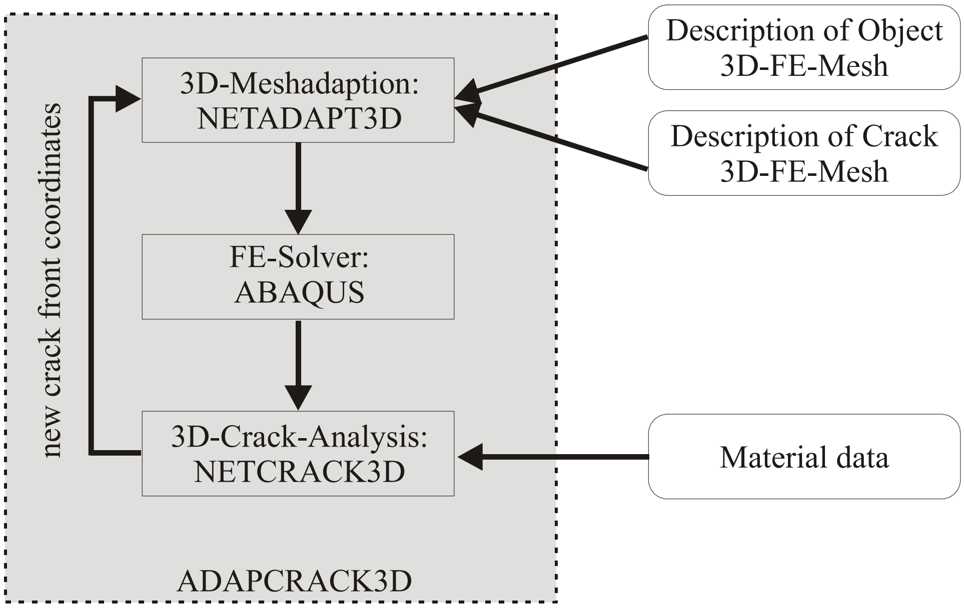

Figure 3: Simplified functionality scheme of ADAPCRACK3D

Figure 3 shows the simplified functionality scheme of ADAPCRACK3D. In order to run a simulation three major external input files are needed. The geometrical description of the object and of the original crack have to be provided in terms of 3D-FE-meshes. In the first simulation step these both geometries are combined to a description of the cracked object by the module NETADAPT3D. The created FE-system is then solved by the commercial FE-code ABAQUS. Afterwards the 3D-Crack-Analysis is performed by the module NETCRACK3D. Therefore additional material data is inevitable, which especially includes the crack growth rate curve, as already mentioned in the previous section. Besides several fracture mechanical parameters the main output are new crack front coordinates, which are sent back to the first module for the next simulation step.

NETADAPT3D

The module NETADAPT3D provides all mesh manipulating work, that is necessary throughout the automatic simulation. This includes two major tasks, which are the creation of the geometrical correct description of the object in terms of an FE-model in each simulation step and the assertion of a sufficiently good mesh quality especially near the crack front in order to get reliable results from the fracture mechanical evaluation. The first task is realized by manipulating the existing mesh in a way, that new FE-faces and –edges are inserted, that represent the additional crack surface and that can easily be debonded in a second step. The necesssary good mesh quality is especially asserted by the use of the *submodeling technique,* that allows to create a special ideally shaped mesh around the crack front in each simulation step (Figure 4), without any need to create a physical connection to the global mesh. Despite this simplification by using the submodeling technique, enormous efforts regarding mesh improvement algorithms have to be taken in order to keep a full 3D-simulation running [5, 6].

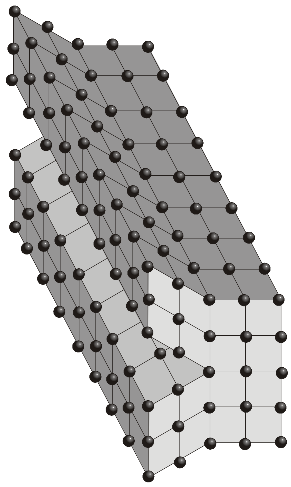

Figure 4: Submodel for straight crack front

NETCRACK3D

The main purpose of NETCRACK3D is the fracture mechanical evaluation along the crack front. Therefore at first the stress intensity factors K_I, K_{II} and K_{III} are determined, which is done by the MVCCI-method [4]. The application of this method requires the evaluation of forces and displacements of the nodes of the submodel, but totally renounces to any values from fields, that become singular at the crack front. The determination of the crack deflection angles φ_0 and ψ_0 and the verification of the fracture mechanical limits (Threshold value, Fracture toughness) is performed with the application of the σ_1'-criterion and its approximations mentioned before. The user-defined maximum crack growth increment Δa is applied at that particular node of the crack front, that holds the maximum comparative stress intensity. The length propagation of all other nodes of the crack front calculates from their stress intensity and the belonging crack growth rate (which is naturally smaller than Δa). Together with the calculated crack deflection angles the propagation length at each crack front node uniquely defines the new position of the crack front for the next simulation step [3,5,6].

Simulation example

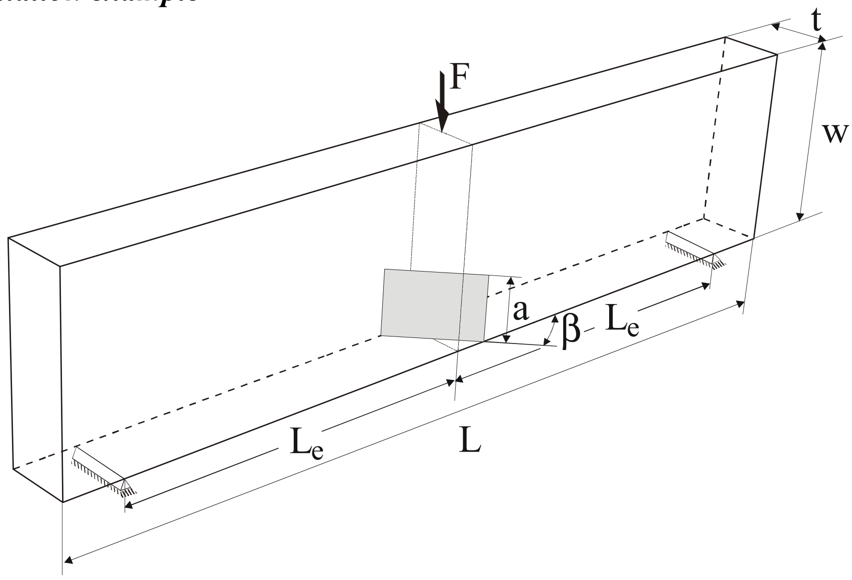

Figure 5: 3point bending specimen with inclined crack

The following simulation example presents a 3-point bending specimen with an inclined crack (Figure 5). The dimensions are L=260mm, L_e=120mm, a=t=20mm, w=60mm, b=45° and F=2000N. The simulation was performed for a 7075 T651 aluminium alloy with a Threshold value ΔK_{th}=99.5 N/mm$^{3/2}$ and K_{Ic}=1041.8 N/mm$^{3/2}$. The assumed R-ratio for the external force F was R=0.1. The graphical results of this simulation are presented in Figure 6. Due to the inclination of the original crack the crack front is subjected to a Mixed Mode loading condition consisting of all three crack opening modes K_I, K_{II} and K_{III}.

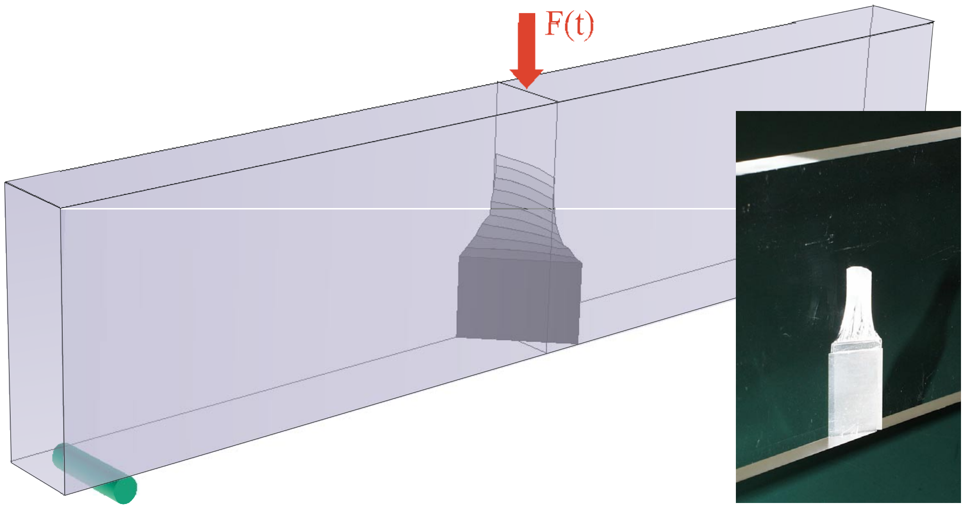

Figure 6: Simulation of crack growth in 3PBS with inclined crack

The determination of the crack angles along each crack front are based on the σ_1'-criterion. The outcome of the simulation is a step-by-step "back-bending" of the crack front until in the last simulation step before instable crack growth a crack front nearly perpendicular to the surface can be found. This result shows qualitatively excellent agreement with experimental findings (comp. Figure 6).

References

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