Linear and Geometric Nonlinear Finite Element Analysis of Cracked Square Plates Under Mode I and Mode II Loading

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ABSTRACT:

Conventional finite element analyses of cracked specimens assuming linear elastic material behaviour do not take into account geometric nonlinearities in the vicinity of a crack tip. In order to investigate the effect of these nonlinearities simple finite element calculations were carried out, using both a linear solver and an iterative geometric nonlinear solver, for both Mode I and Mode II loading. Under Mode I loading a crack tip blunts. In consequence the effect of using the nonlinear solver was to decrease crack surface displacements in the vicinity of a crack tip. However the effect was small and hence unlikely to be of practical significance. Behaviour under Mode II loading is fundamentally different in that a crack tip remains sharp. Once nodal displacements were taken into account using the nonlinear solver had no effect on results. In view of the results obtained it is concluded that there is no point in carrying out more sophisticated finite element calculations.

INTRODUCTION

Conventional finite element analyses of cracked specimens assuming linear elastic material behaviour do not take into account geometric nonlinearities in the vicinity of a crack tip. In order to investigate whether these nonlinearities are significant some simple finite element calculations were carried out using both a linear solver and an iterative geometric nonlinear solver. Analyses were carried out on cracked 20 mm square 4 mm thick plates under both Mode I and Mode II loading. The STRAND6 finite element program [1] was used. Preliminary two dimensional calculations were also carried out. The crack extended from the middle of one side of a square to its centre. The crack tip region was modelled using elements with midside nodes moved to quarter points [2]. Young's modulus was taken as 205 GPa, and Poisson's ratio as 0.3. Eight noded parabolic plane stress elements were used for two dimensional calculations, and 20 noded parabolic elements for three dimensional calculations. The primary objective was to investigate displacements in the vicinity of a crack tip.

FINITE ELEMENT MODELS

Two dimensional calculations were carried out using the centred fan mesh shown in Figure 1. Half the square was modelled, and the crack tip element size was 0.2 mm. The positions of application of 'primary', 'secondary' and 'tertiary' stresses are shown in the figure. The terms primary, etc are used simply for convenience in descriptions.

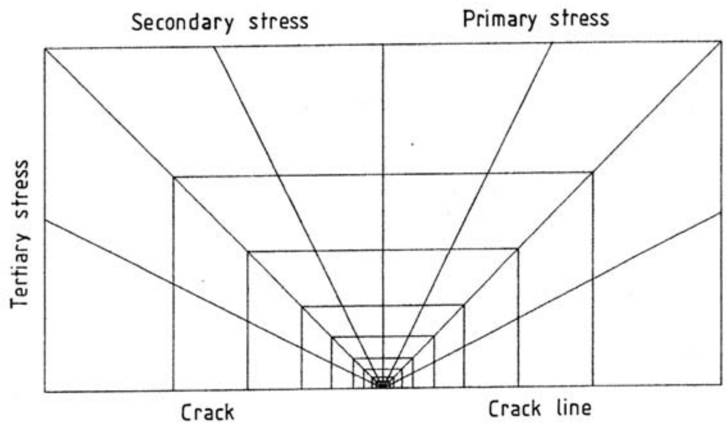

Figure 1: Mesh for two dimensional 20 mm square.

For the Mode I calculations symmetry conditions were applied to the crack line. Primary and secondary stresses were applied perpendicular to the upper edge. The secondary stress was 19 per cent of the primary stress since it is known [3] that this leads to a large stress intensity factor (K) dominated zone at the crack tip. For the Mode II calculations the upper edge was restrained in the vertical (y) direction and antisymmetry conditions were applied to the crack line. A tertiary stress was applied to the left hand edge. This leads to a K-dominated zone of satisfactory size [3].

Three dimensional models were based on the two dimensional models, and contained 20 layers of elements 0.2 mm thick. One quarter of the plate was modelled with symmetry conditions applied to one of the vertical faces. Applied stresses and restraints were otherwise as for the two dimensional models. Models were fixed in space at a node remote from a crack tip.

Various checks were carried out to confirm the quality of both the two dimensional and the three dimensional models. In all about 60 calculations were carried out. There were two important findings for three dimensional nonlinear calculations. Firstly, it was sufficient to apply the stresses in one increment. Secondly, to minimise numerical problems calculations could be restricted to two iterations. Checks showed that this restriction did not have a significant effect on results. Stress intensity factors were calculated, partly to ensure that they were of the order of magnitude likely to be encountered in practice, and partly to avoid the possibility of gross error.

RESULTS

Stress intensity factors were calculated from nodal stresses and displacements in the vicinity of a crack tip using standard equations [3, 4]. Where appropriate normalisations took into account differences in applied stresses.

Two Dimensional Calculations

A two dimensional linear Mode I calculation was carried out with a primary stress of 100 MPa and a secondary stress of 19 MPa. For a crack surface displacement, v, in the y direction the Mode I stress intensity factor, K_I, is given for plane stress by:

$$K_{\mathrm{I}} = Ev\sqrt{\frac{\pi}{8r}} \tag{1}$$

where E is Young's modulus and r is the distance from the crack tip. Extrapolating from the two nearest nodes to the crack tip gave K_I = 13.95 MPa√m, compared with a value obtained previously for the same configuration [3] of 14.13 MPa√m. For a crack line stress, σ_y, in the y direction K_I is given by:

$$K_{\mathrm{I}} = \sigma_{\mathrm{y}}\sqrt{2\pi r} \tag{2}$$

This equation is valid for both plane stress and plane strain. At a node 0.26 mm from the crack tip it led to K_I = 13.98 MPa√m, which is very close to the extrapolated value.

A two dimensional linear Mode II calculation was carried out with a tertiary stress of 100 MPa. For a crack surface displacement, u, in the x direction the Mode II stress intensity factor, K_{II}, is given for plane stress by:

$$K_{\mathrm{II}} = Eu\sqrt{\frac{\pi}{8r}} \tag{3}$$

Extrapolating from the two nearest nodes to the crack tip gave $K_{\mathrm{II}} = 13.36$ MPa√m, compared with a value obtained previously for the same configuration [3] of 13.30 MPa√m. For a crack surface stress, σ_x, in the x direction K_{II} is given by:

$$K_{\mathrm{II}} = \sigma_{\mathrm{x}}\sqrt{2\pi r} \tag{4}$$

This equation is valid for both plane stress and plane strain. At a node 0.32 mm from the crack tip it led to $K_{\mathrm{II}} = 13.42$ MPa√m, which is close to the extrapolated value.

Geometric nonlinear calculations were also carried, but because of the need to specify either plane stress or plane strain (plane stress was used) the results cannot be regarded as physically realistic. Summarising qualitatively, under Mode I loading it was found that the use of the nonlinear solver reduced crack surface displacements in the vicinity of a crack tip. Under Mode II loading crack surface displacements in the vicinity of a crack tip increased on one side of the crack line and decreased on the other side.

Three Dimensional Calculations

Mode I

A three dimensional Mode I linear calculation was carried out using the same applied stresses as for the corresponding two dimensional calculation. The through thickness distribution of K_{I} was calculated for nodes on the crack plane 0.26 mm from the crack tip using Eq. 2, and normalised using the value 13.98 MPa√m obtained from the two dimensional solution. The results in Figure 2 are very similar to those obtained previously for the same configuration [3]. They show the well known increase in K_{I} at the centre line [5] over the corresponding two dimensional solution, with a decrease towards the surface. At the centre line $K_{\mathrm{I}} = 14.70$ MPa√m.

The nature of the crack tip singularity changes in the vicinity of a corner point where a crack front intersects a free surface. At a corner point it is only possible to define stress intensity factors in an asymptotic sense [4, 6] and K_I for a crack tends to zero as a corner point is approached. Numerically calculated values of K_I for a corner point are extrapolations whose values depend upon the details of the technique used. This does not normally cause difficulties in practice [4] but does account for scatter towards the surface.

In principle it is possible to model corner point singularities by moving quarter point nodes slightly, or by using very small crack tip elements, but in practice it is difficult to devise satisfactory meshes.

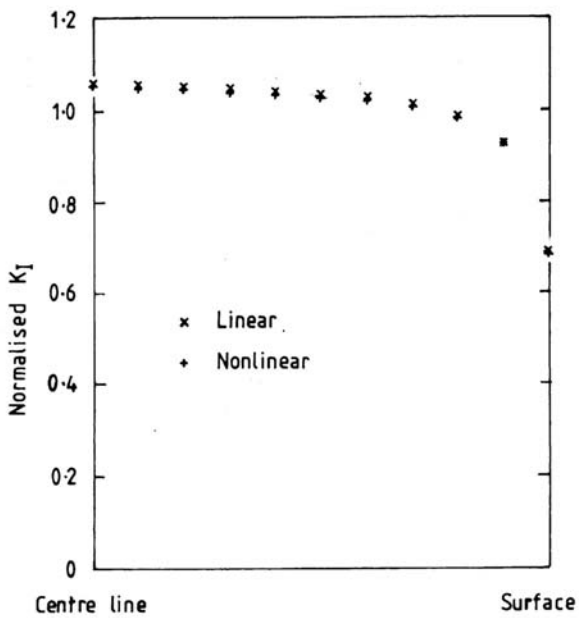

Figure 2: Through thickness distribution of K_I for Mode I models.

In the vicinity of a corner point [4, 6] components of the crack tip stress field are proportional to K_λ/d^{λ} where K_λ is the stress intensity measure for the corner point singularity, d is distance from the corner point, and λ is a coefficient characterising the singularity. Crack surface displacements are proportional to $K_\lambda d^{1-\lambda}$. Theoretical values of λ [6] are 0.452 for Mode I and 0.598 for Modes II and III. For stress intensity factors displacements are proportional to $Kr^{0.5}$ and $\lambda = 0.5$ for all three modes. The value of λ was estimated, as described by Pook [3], from crack surface displacements at the model surface. It was found to be 0.463, close to the theoretical value.

Nonlinear calculations were carried out at several different load levels. At the highest load level the primary stress was 600 MPa and the secondary stress was 114 MPa. The general effect on crack surface displacements was to decrease them in the vicinity of the crack tip, with a smaller increase further away. Typical behaviour is shown in Figure 3, where crack surface displacements for the highest load level at the model centre line are normalised by corresponding displacements for the linear model. The effect of this crack surface displacement pattern was to increase the estimated value of λ to 0.463. Estimated values of K_I calculated using the linear equations as an approximation were slightly lower. The same nodes were used as for the linear model. The normalised through thickness distribution is shown in Figure 2. At the centre line $K_I = 87.58$ MPa√m.

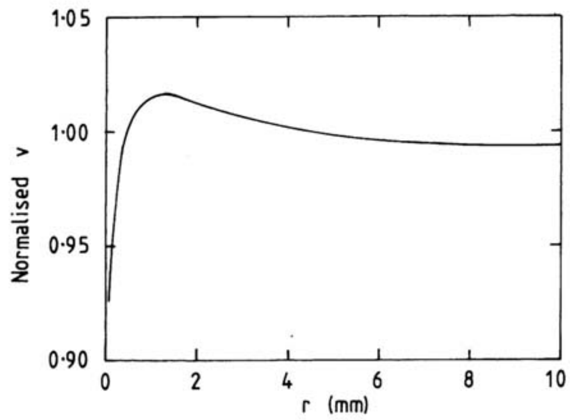

Figure 3: Normalised crack surface centre line displacements, Mode I nonlinear, highest load level.

Mode II

A three dimensional Mode II linear calculation was carried out using the same applied stresses as for the corresponding two dimensional calculation. The through thickness distribution of K_{II} was calculated for nodes on the crack surface 0.32 mm from the crack tip using Eq. 4, and normalised using the value 13.42 MPa√m obtained from the two dimensional solution. The results in Figure 4 are very similar to previous results [3], with K_{II} nearly constant through most of the thickness. Because of extrapolation effects values fell towards the surface rather than the theoretical increase towards infinity [3, 6]. At the centre line $K_{II} = 13.81$ MPa√m.

Under nominal Mode II loading Mode III is induced in the vicinity of a corner point [3,6]. K_{III} is given by:

$$K_{\text{III}} = \frac{wE}{1+\nu}\sqrt{\frac{\pi}{8r}} \tag{5}$$

where w is the crack surface displacement in the through thickness (z) direction. Values were calculated for the two sets of nodes nearest the crack tip. They were then extrapolated to the crack tip and normalised by the two dimensional value of K_{II} = 13.42 MPa√m. By symmetry K_{III} is zero at the centre line, where it changes sign. The through thickness distribution of induced Mode III stress intensity factors, K_{III}, is shown in Figure 4. Results are similar to those obtained previously [3]. The value of λ was estimated [3] from crack surface displacements at the model surface. It was found to be 0.551, lower than the theoretical value, but close to values obtained previously [3].

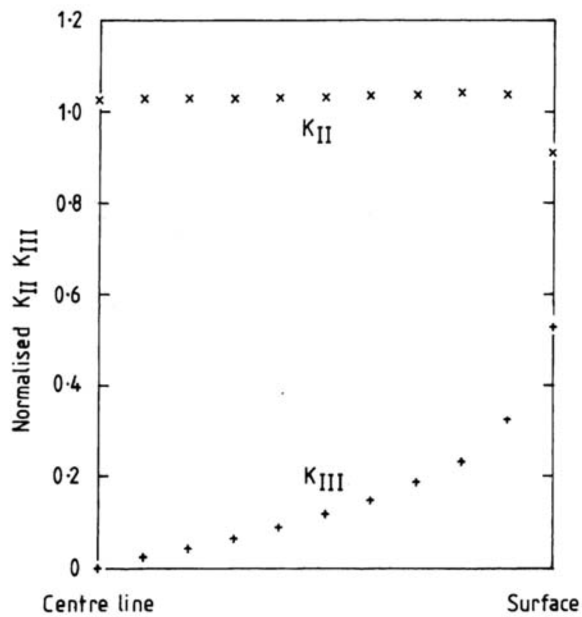

Figure 4: Through thickness distributions of K_{II} and K_{III} for Mode II linear model.

Geometric nonlinear calculations were carried out at several different load levels At the highest load the tertiary stress was 300 MPa. In the linear model stresses and displacements are antisymmetric about the crack plane so it is only necessary to apply the stress in one direction. For the nonlinear model exact antisymmetry cannot be assumed. Two calculations were therefore carried out for the highest load level, one with a tensile tertiary stress, and the other with a compressive tertiary stress. Initial examination of the results suggested that crack surface displacements were not precisely antisymmetric. However, detailed examination showed that, once nodal displacements were taken into account, results were antisymmetric, and also that the use of the nonlinear solver had no effect on results. In particular, values of normalised stress intensity factors agreed to at least three decimal places.

CONCLUDING REMARKS

Under Mode I loading a crack tip blunts. In consequence the effect of using the geometric nonlinear solver was to decrease crack surface displacements in the vicinity of a crack tip. Calculated stress intensity factors were also reduced, but the effect was small and hence unlikely to be of practical significance.

Behaviour under Mode II loading is fundamentally different in that a crack tip remains sharp. Once nodal displacements were taken into account using the geometric nonlinear solver had no effect on results.

For realistic values of stress intensity factors geometric nonlinear effects are relatively unimportant. There is therefore no point in carrying out more sophisticated finite element calculations.

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