Interpretation and inverse analysis of the wedge splitting test

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ABSTRACT *Determination of the stress-crack opening relationship,* $\sigma_w(w)$*, a material parameter in the fictitious crack model by Hillerborg et al [1] has proven to be problematic and is still not a simple task to perform. However, this paper demonstrates that the cracked non-linear hinge model by Olesen [2] may be applied to the wedge splitting test and that it is well suited for the interpretation of test results in terms of* $\sigma_w(w)$*. A fine agreement between the hinge and FEM-models has been found. It has also been found that the test and the hinge model form a solid basis for inverse analysis. The paper also discusses possible three dimensional problems in the experiment as well as the influence of specimen size.*

INTRODUCTION

The use of the fictitious crack model originated by Hillerborg et al [1] for the modelling of crack initiation and propagation in concrete has been widespread and accepted in the research community for the last two decades. This is due to the fact that this model is able to closely describe the fracture behavior of concrete, and because it gives a good understanding for many previously unexplained phenomenas, like e.g. the size effect. However, one reservation against the use of the model is that it requires knowledge of a new material property, the so-called stress-crack opening relationship, $\sigma_w(w)$. Determination of this property has been a major problem until recently. Usually in literature, only the fracture energy, G_f has been determined and from this value a prescribed function for $\sigma_w(w)$ has been calibrated. However, also the shape of $\sigma_w(w)$ is important for the determination of crack propagation.

The wedge splitting test is a very suitable test configuration for determination of fracture mechanical properties of concrete. Unlike the uniaxial test and the three point bending test, this configuration always results in stable crack propagation without snap-back behavior. This is due to the low amount of elastic energy stored in the specimen compared with the energy consumed by crack propagation. The test is also an appropriate choice when a determination of the fracture properties for early age concrete is the goal for the experiments. In this situation, self-weight plays only a minor role compared with the self-weight problems associated with early age testing on e.g. three point bending beams. After approx. 12 hours, depending on mix design, self-weight may be ignored in the analysis. In even earlier ages, say 7-8 hours, simple arrangements can be established to compensate for self-weight. Despite the convenience of this test method, only few papers on the interpretation of the test configuration have been reported in literature. The idea of using the WST-geometry for determination of fracture parameters for concrete was originated by Linsbauer & Tschegg [3], and further developed by Rossi et al [4]. However, these papers do not include a general method for inverse analysis. Such methods may be found in e.g. Wittmann [5].

The present paper explains how the wedge splitting test may be interpreted by application of the cracked hinge model, and how to use the model for inverse analysis. The results are compared with 2D and 3D-FEM models.

HINGE MODEL AND INVERSE ANALYSIS

Figure 1 shows a typical experimental setup. The vertical loading, P, on the wedge and the crack mouth opening displacement (CMOD) (using a clip gage) are recorded during experiments. Closed loop CMOD control may be used, but also a constant rate of displacement of the wedge is sufficient since the experiment is very stable.

The principles of the analytical hinge model are outlined in figures 2 and 3. Figure 2 shows the geometry, loading and deformation of the hinge element. The hinge element models the crack propagation in a short beam segment subject to bending and normal force. As illustrated, it is assumed that the boundary planes are rigid and that they may translate and rotate. Analysis of the hinge allows for determination of the load for any given hinge rotation, 2φ. This P-CMOD curve is a function of the material properties in the pre-crack state and in the cracked state, see figure 3, where the latter is assumed to be a bilinear function. Derivation of the hinge model applied here may be found in Olesen [2], while the adaption of the model to the wedge splitting test was performed by Østergaard et al [6], [7]. The CMOD is dependent on three different contributions, namely the elastic opening, the opening due to

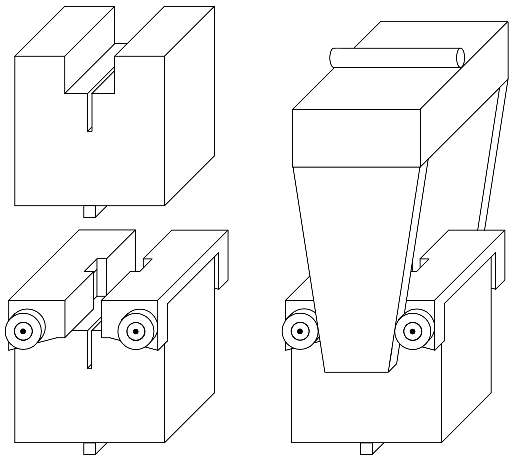


Figure 1: Upper left: Specimen placed on line support; lower left: mounting of two steel loading devices with roller bearings; right: steel wedge in place between roller bearings. Cf. [4].

presence of the crack and the opening due to extrapolation from the bottom of the notch to the line of CMOD-measurement.

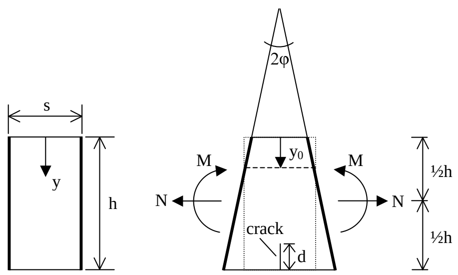


Figure 2: Geometry, loading and deformation of the hinge element

The material parameters entering the hinge model may be established through an inverse analysis of test results by minimizing the difference between the test result and hinge model predictions.

The inverse analysis uses a phased approach, where only the parameters governing the response of a particular phase are free, while the other parameters are fixed. For example, in the linear elastic phase, only the modulus of elasticity influences on the P-CMOD curve, and thus, only this parameter is free. In the first phase of crack propagation, see Figure 3, only tensile strength, f_t, initial slope a_1 and modulus of elasticity, E, influence the response. But since E is known, the optimization may be performed for f_t and

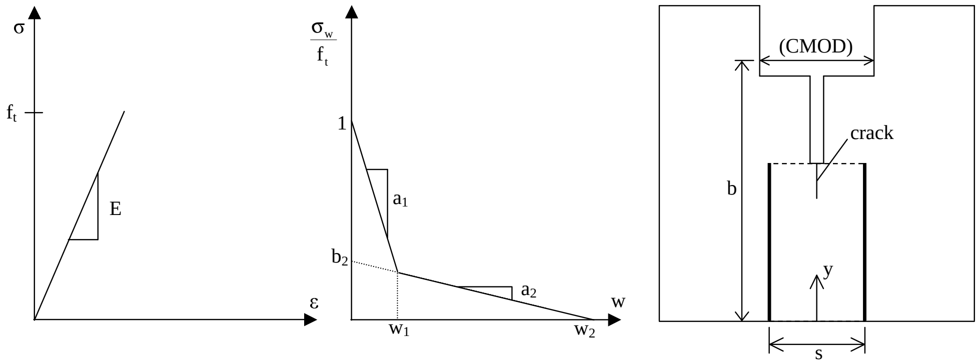


Figure 3: Stress-strain relationship (left), $\sigma_w(w)$ (middle) and upside down incorporation of the hinge element (right)

a_1 only. Finally, for phase II and III, all parameters are fixed except a_2 and b_2. This method depends on good initial values for the fixed parameters when the optimization for a particular parameter is performed. If such values are unknown, the method will still converge if a few reiterations are performed.

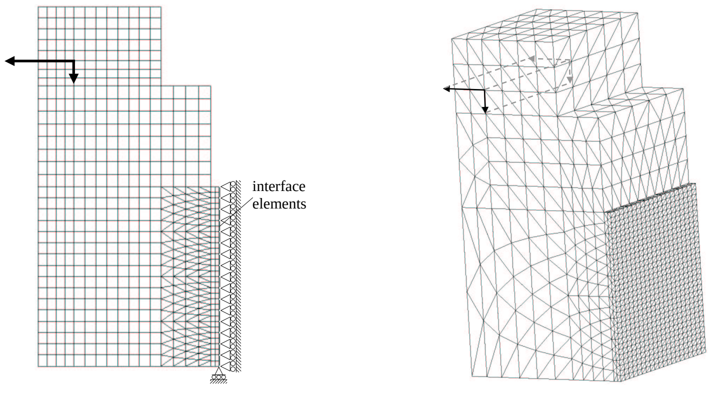
FINITE ELEMENT MODELS

Figure 4 shows the meshes used for FEM-modelling of the WST-specimen. The 3D-mesh was used to investigate if the 2D-mesh is sufficiently accurate and to determine the effects of uneven load distributions throughout the specimen thickness. The meshes were found to perform satisfactorily in a convergence analysis.

DISCUSSION

Figure 5a demonstrates the performance of the hinge model compared with the 2D-FEM model. The figure shows the agreement for different $\frac{f_t}{E}$-ratios, and except for some dependency of the $\frac{f_t}{E}$-ratio on the optimal value of s/h, the results are good. Even if s/h is fixed once and for all, the error introduced on the results is small. A dependency of the material parameters on the optimal s/h has also been found on the other material parameters, but again, ignoring this effect will result in small errors. However, if these errors must be minimized, a calibration for the optimal s/h-value must be made.

Figure 5b shows the results of the inverse analysis algorithm, where the



(a) (b)

Figure 4: 2D- and 3D-meshes used in the FEM-modelling. The 3D-mesh models one-quarter of the specimen

2D-FEM model has been used to generate P-CMOD curves with known $\sigma_w(w)$ parameters. Then, the algorithm has been applied and as the examples indicate, the result is very promising. Except for $\sigma_w(w)$ relationships with high b_2 values the method always yields the correct solution within a few percent, depending on the calibration of the s/h-parameter. The algorithm was also tested on $\sigma_w(w)$-curves with varying f_t or a_2 values, and in these cases, the global minimum was always found. The only restriction on the start guess that must be obeyed stems from the fact that local minima exist in the phase where f_t and a_1 are optimized for. However, by requiring that the guesses on these parameters are below the correct values, the method always converges to the global minimum.

The application of load on the specimen from the roller bearings, see Figure 1, is different from the original setup proposed by Linsbauer & Tschegg, [3]. The problem is that the experimental results might change if the load is not distributed uniformly over the specimen depth. The 3D-FEM model was created to investigate the relevance of such concerns and to check if the 2D-model is sufficiently accurate, see Figure 6a. The P-CMOD graphs show a comparison between the 2D-FEM, the 3D-FEM (with concentrated load) and the hinge model for a $\sigma_w(w)$ with a value of b_2=0.7. Both the CMOD value obtained at the free surface and at the symmetry surface from the 3D-model are shown. It is clearly seen that the differences are minor. This picture is repeated on the small inserted figures where the crack fronts in the two dif-

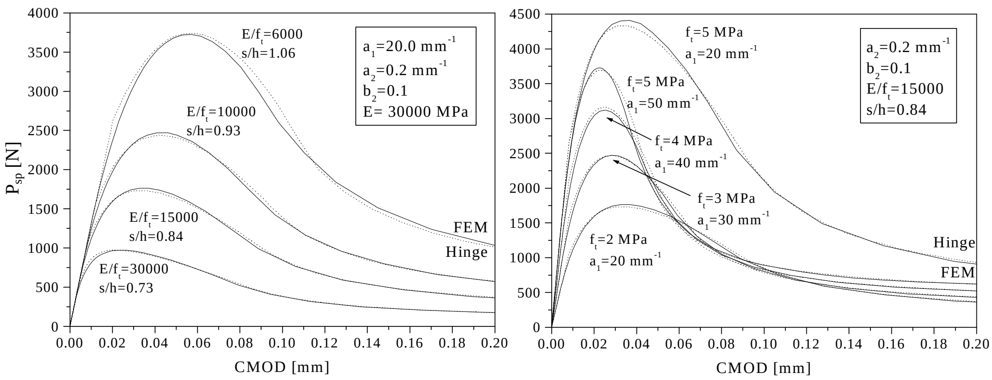


Figure 5: Hinge model compared with 2D-FEM model for different $\frac{f_t}{E}$-ratios (left) and inverse analysis results for different values of f_t, a_1 and $\frac{f_t}{a_1}$-ratio (right)

ferent cases are shown. In the real experiment, the load is applied with some unknown distribution between the extremes modelled in Figure 6a, where the distribution will be dependent on the stiffness of the loading devices. However this is irrelevant as demonstrated, and both the 2D-FEM model and the experimental setup illustrated on Figure 1 are sufficiently accurate.

Figure 6a also shows the depth of the crack as a function of CMOD. The crack depth is shown normalized, where $\alpha = d/h$, cf. Figure 3. In general these curves follow each other with the difference that the FEM-model has a lower CMOD for a given crack depth. This is probably associated with a different stress distribution in the hinge model compared with the FEM model and the fact that the hinge element has no shear stiffness.

Application of the hinge model to different specimen sizes is exemplified in Figure 7b. Here, three different specimen sizes, expressed as a variation of the side length, L, have been modelled. The s/h-parameter has been kept constant in all cases. The curves have been normalized pairwise with respect to the peak load for the actual FEM-model, thus the peak load of the FEM-model always reaches 1.00. It is seen from these results that the hinge can be applied to different specimen sizes without problems and that no problems with stability of the experiment will be encountered for large specimens. However, due to the analytical expressions for the hinge model and depending on the material input for the $\sigma_w(w)$-relationship, an upper limit on the absolute size of the specimen exists, see [2]. However, this limit is far beyond the size range of normal specimens.

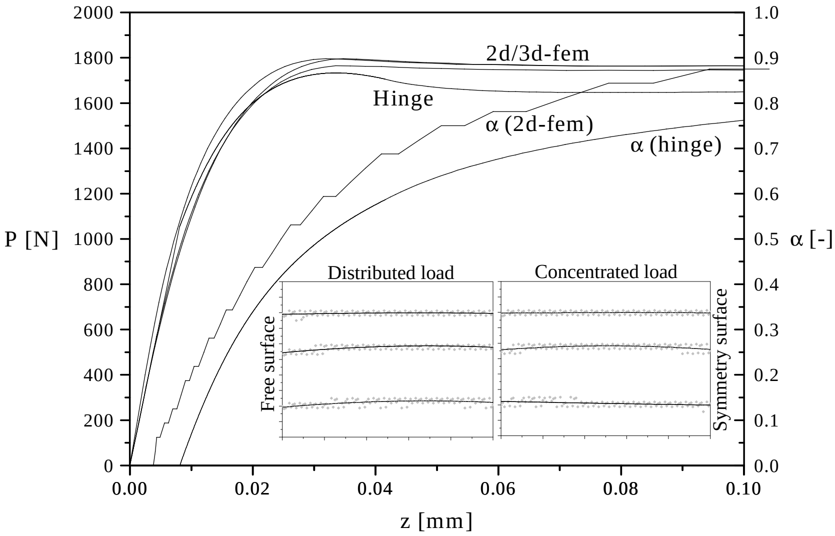


Figure 6: Results from the 3D-FEM analysis compared with the 2D-FEM analysis and the hinge model. Small graphs show the shape of the crack front for a load distributed over depth of the specimen and for a concentrated load applied at the free surface.

CONCLUSION

This paper shows that the cracked hinge model is well suited as an analytical tool for the interpretation of the wedge splitting test. It has also been demonstrated that it is possible to perform inverse analysis on results from such experiments by applying the hinge model together with a simple and robust inverse analysis algorithm, which is able to yield all material parameters.

It has also been found that the s/h-parameter is dependent on the choice of material parameters and specimen geometry. However, for most applications, s/h may be set to at fixed value.

Finally, the FEM-models show that the hinge closely describes the behavior of the test setup and also that a 2D FEM-modelling is sufficiently accurate.

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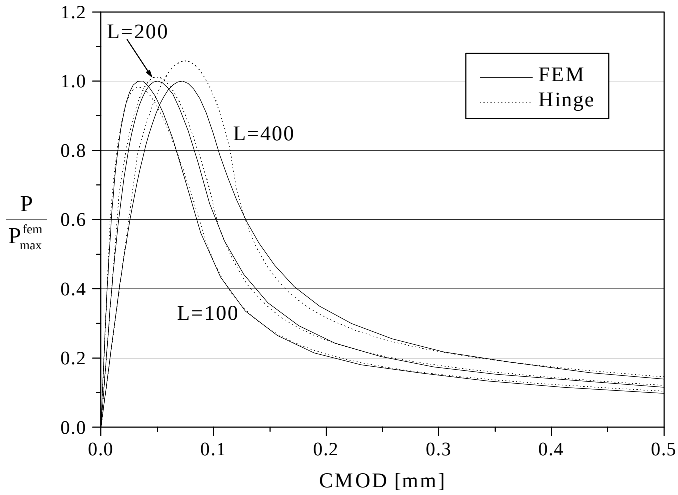

Figure 7: Influence of specimen size. The peak load error at *L*=400 is 5%

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