Statistical evaluation of composites fatigue results using normalizing techniques

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ABSTRACT: *The adjustment of the S-N curve through a general statistical model and the normalization of the fatigue life of the material permit to establish equivalencies between states of damage for different stress levels. On the other side, the analysis of the stiffness curves of composites under fatigue loading provides an important information about the state of damage accumulation and the failure mechanisms inside of the material. Joining both procedures permits to present the stiffness evolution as a function of a normalized variable, instead of as a function of the number of cycles, supplying in this way the possibility of a unified study for results belonging to various load levels and histories.*

INTRODUCTION

The generally scarce and sparse number of fatigue results available from experimental programmes, especially in composites materials, emphasizes the necessity of efficient procedures for evaluating the statistical parameters defining the S-N field, so that a consistent statistical non-linear regression model for analysing the Wöhler field is needed. In this paper, the statistical fatigue model developed by Castillo et al. will be considered for this purpose [1].

Unlike metals, composites accumulate damage in a general rather than a localised fashion and failure does not occur by the propagation of a single macroscopic crack. Thus, the evolution of the stiffness curve gives an important information about micro-structural mechanisms of damage and their sequence of appearance [2]. Nevertheless, the results from experimental programmes performed with different stress levels and load histories are not feasible to compare. Due to that, representing the material stiffness as a function of the number of cycles, N or log N, is not an adequate procedure for the analysis of these curves.

In this work, the importance of modelling the S-N field and the definition of normalized variables for stiffness curve comparison are presented, together with a study about the influence of load variability in the evolution of these curves.

A MODEL FOR THE S-N FIELD

In the S-N field, two random variables have to be considered − the stress range $\Delta\sigma$ and the number of cycles to failure N − from which two different statistical distributions, F(N; $\Delta\sigma$), representing the number of cycles to failure given the stress range $\Delta\sigma$ or, alternatively, E($\Delta\sigma$; N), representing the stress range given the number of cycles to failure, are envisaged. Both distributions must fulfill physical and statistical conditions for the statistical model to be valid.

Castillo et al. [1,3] have developed a statistical model valid for the S-N field, based on the weakest link principle, arising from a functional equation after setting physical (threshold number of cycles to failure, etc) and statistical requirements (stability, compatibility and limit conditions) to the distributions F(N; $\Delta\sigma$) and E($\Delta\sigma$; N) which prove to be three-parameter Weibull distribution families for minima. It is worthwhile mentioning that the same model has been justified by Bolotin [4] based on micro-structural considerations.

Moreover, using dimensional analysis it can be shown that the variables of the S-N field have to be used in logarithmic scale, i.e. log $\Delta\sigma$ and log N. The cumulative distribution function (c.d.f.) of the logarithm of the lifetime N, given the stress range $\Delta\sigma$, is given by:

$$F(\log N;\Delta\sigma) = 1-\exp\left[-\left(\frac{(\log N - B)(\log \Delta\sigma - C)}{D} + E\right)^{A}\right] \tag{1}$$

from which the percentile curves can be derived:

$$(\log N - B)\ (\log \Delta\sigma - C) = D\left[\ [-\log(1-P)\ ]^{1/A} - E\right]$$

(2)

where N is the lifetime measured in number of cycles to failure, $\Delta\sigma$ is the stress range and A, B, C, D and E are the model parameters to be determined, where:

A = Weibull shape parameter;

B = Threshold value or limit lifetime;

C = Endurance limit;

D = Scale parameter;

E = Parameter determining the position of the zero-percentile curve.

As soon as the five parameters are determined, the analytical expression of the whole S-N field is known, what enables the probabilistic prediction of the fatigue failure under constant amplitude loading. As it can be observed, the percentile curves are represented by equilateral hyperbolas, see Figure1.

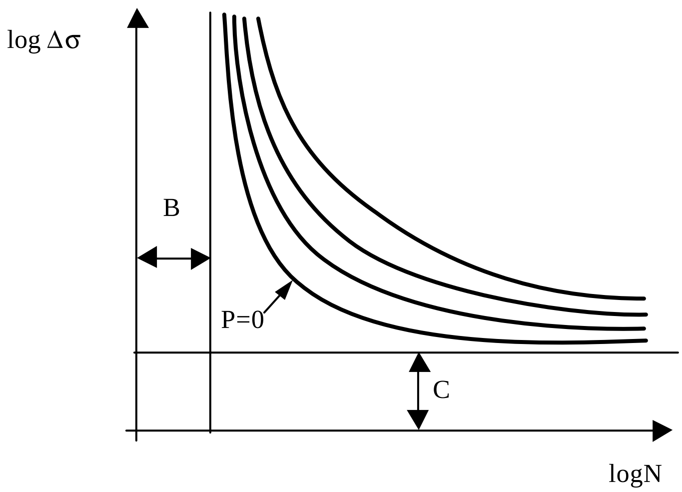

Figure 1 - S-N field with percentiles curves in the fatigue model of Castillo et al. [1,3].

ESTIMATION OF MODEL PARAMETERS USING NORMALIZED VARIABLES

To estimate the five parameters of the model, the analyst has pairs of values (N_i , $\Delta\sigma_i$) obtained in experimental tests carried out at several stress ranges. It is possible to estimate the five parameters simultaneously through maximisation of the likelihood function, but this procedure generally encounters convergence and precision problems. Alternatively, a more advantageous two-step method for estimating the model parameters has been proposed by Castillo et al. [3].

Expression (2) reveals that for the proposed fatigue model the probability of failure for an element subject to a stress level $\Delta\sigma$, during N cycles uniquely depends on the product $(\log N - B)\,(\log \Delta\sigma - C)$. As soon as the model parameters B and C are known, the transformation:

$$V = (\log N - B)(\log \Delta\sigma - C) \tag{3}$$

provides an useful normalized variable depending on the stress level and on the number of cycles resulting in the test. This variable follows a Weibull distribution with three parameters λ^*, δ^* and β^*, only related to A, D and E. This fact suggests estimating the five parameters in two steps, first B and C, and then A, D and E.

This procedure is justified by the fact that a Weibull distribution remains stable with respect to location and scale transformations. Thus, if the variable X follows a Weibull distribution for minima $W(\lambda, \delta, \beta)$, expressed as $X \sim W(\lambda, \delta, \beta)$, the normalized variable Z, defined as $Z = (X - a)/b$, will also follow a Weibull distribution for minima (see Figure 2):

$$Z \sim W\left(\frac{\lambda - a}{b}, \frac{\delta}{b}, \beta\right) \tag{4}$$

A suitable choice for “a” and “b” will be crucial in the analysis of the reliability of the evaluation and in its interpretation. In our fatigue analysis it is apparent that log N is the original variable identifiable with X, V is the normalized variable Z and that $a = B$ and $b = 1/(\log \Delta\sigma - C)$.

A normalized variable which fulfills the S-N model is independent of the stress level and its values have associated a probability value, thus, a curve in the S-N field has associated a value of the normalized variable [5,6].

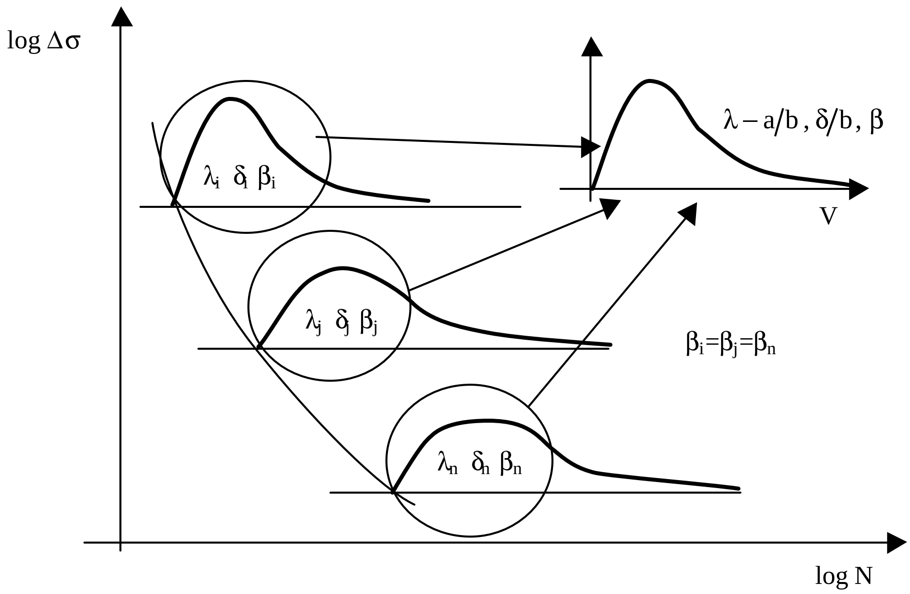

Figure 2 - S-N field with pdfs of log N at different stress ranges and corresponding pdf of the normalized lifetime, V.

STIFFNESS CURVES

In fatigue, the evolution of the stiffness curves indicates the state of damage accumulation of the material [7,8]. The present work analyses the stiffness curves obtained through fatigue experimental programmes carried out under conditions of constant and variable stress, using a carbon fibre and epoxy resin laminate, IM7/8552 $[0/90]_{4S}$.

From the analysis of the experimental results it was observed that for constant stress and low load levels the stiffness degradation of the material follows the description found in the specialized literature, showing three different states, called I, II and III states (Figure 3a), or only two in a semi-logarithmic scale (Figure 3b). The state I is characterized by a quick loss of stiffness due to matrix cracks and fibre breaks, the state II, where inter-laminar matrix cracks occur, is a long period with a small stiffness reduction, and finally, the state III presents a sudden drop of stiffness caused by a joint appearance of delaminations and fibre breaks. It was also observed that for high load levels the state III disappears and the length of state II decreases when the load increases.

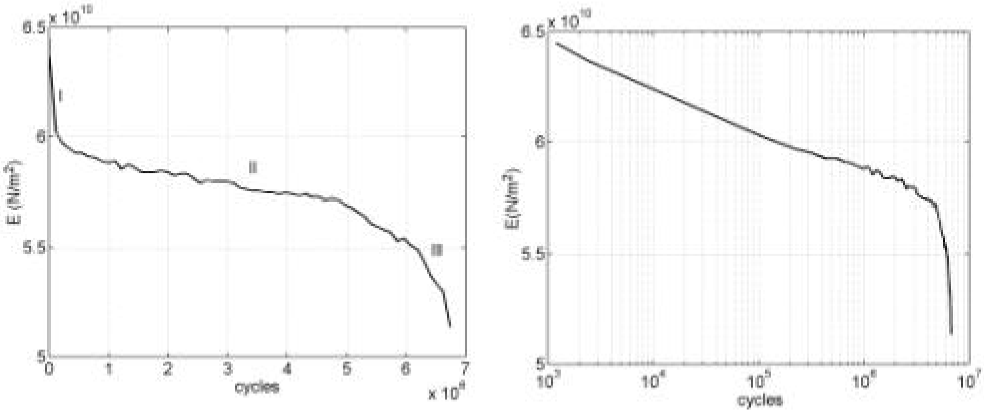

Figure 3. Stiffness curves for constant stress and low load levels a) in natural scale and b) in semi-logarithmic scale.

To compare the stiffness curves obtained by testing programmes under different load levels and histories it is compelled the use of a normalized variable which permits to convert all the curves to an unique reference unit. Using the normalized variable, V, defined before, Figure 4a shows how for high load levels the failure of the composite laminate occurs before reaching a stiffness reduction of 10% approximately, whereas for low load levels the stiffness reduction of 10% marks a change of the slope between the two existing states of the curve.

In this way, first of all, plotting the stiffness reduction data as a function of the normalized variable, V, permits to distinguish the fatigue failure of high and low number of cycles and, on the other hand, to fix a threshold of 10% that determines the presence of delamination inside the material.

Related to fatigue under conditions of variable stress, such as block and random loading, Figure 4b shows similar curves to that obtained under constant stress conditions, but in this case the threshold is located around the

5% or 6% of the initial value and the total loss of stiffness is much lower. Apart from that, it can be also observed that the curve pertaining to random loading has got a significant smaller slope and a less total stiffness reduction. This denotes that for random loading the failure process takes place with an apparent minor damage degradation, what agrees with the prediction results of the fatigue life obtained for this kind of testing.

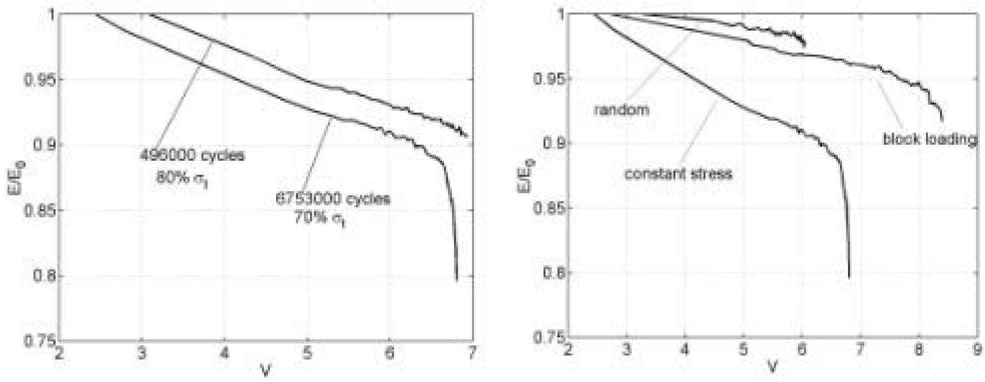

Figure 4. Stiffness curves of the IM7/8552 $[0/90]_{4s}$ laminate in function of V a) for constant stress fatigue and b)for different fatigue load histories.

CONCLUSIONS

1.- Plotting the stiffness curves as a function of a normalized variable permits to make a complete analysis of the data evolution obtained under different load histories.

2.- The analysis of the stiffness curves allows to exclude wrong results in the adjustment of fatigue data, as well as to redefine the testing strategy.

3.- The study of the fatigue stiffness curves under constant stress reveals a threshold value of stiffness reduction of 10% of the initial value, which is independent of the load level.

4.- The total stiffness reduction and the slope of the first part of the curve decrease with the loading variability. Due to that, it is not valid to extrapolate results under constant to variable stress.

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