Limit Equilibrium of a Piecewise-Homogeneous Cylindrical Shell with Cracks

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ABSTRACT*: The problem of the limit equilibrium of a closed circular piecewise-homogeneous cylindrical shell with a longitudinal crack, which is located in one of the parts, ends at the interface or crosses it, is reduced to a system of singular integral equations. The numerical analysis of the problem, utilizing the method of mechanical quadratures, is carried out for a shell welded from two different semi-infinite shells. The effect of mechanical characteristics of the shell parts, crack location and its length on the value and character of distribution of both the force-intensity and moment-intensity factors is investigated for the normal forces of constant intensity which are applied at the crack faces. The obtained results are compared with the known ones for a plate made of the same materials and weakened by the similar crack. The redistribution of residual stresses in such a shell caused by a crack is determined and analyzed too.*

INTRODUCTION

The approach utilizing the distribution technique for determination of the stressed state and the limit equilibrium of a piecewise-homogeneous cylindrical shell with cracks is suggested in [1-3]. The approach is realized by mathematical statement of the generalized coupling problem for equations of thin homogeneous cylindrical shell with inherent stresses. Besides, the presence of cracks in the considered shell is modelled by the inner sources of such stresses (dislocations and disclinations with the unknown densities) which are distributed along the lines of cracks location [2-6]. To determine stresses in cracked shells it is convenient to reduce the problem to solving a system of integral equations using the scheme proposed in [3-5]. The solutions of these equations are constructed using numerical methods (in particular, a method of mechanical quadratures), and the forces and moments intensity factors are found.

Often, elements of shell structures are subject to the action of residual stresses appearing as a result of some technological operation. The presence of crack-like defects in these structures results in the redistribution of such stresses, which should be taken into account in the analysis of the stressed-strained state and limit equilibrium of structures. For determining the residual stresses we apply nondestructive theoretical-experimental method based on the statement and solution of conditionally correct inverse problem of mechanics with the use of the available experimental data [5,6]. For the concrete technological conditions of shell manufacture, taking into account *a priori* information about the distribution of the residual free strain field which incompatibility causes the residual stress field, the residual strains are described by some function which belongs to certain compact set and depends on some arbitrary parameters. To find these parameters the experimental information about the residual strain field is used and a functional, which minimization provides the least deviation of theoretically calculated stress fields from experimentally obtained ones, is constructed. Having found the unknown parameters we determine the strain field and calculate components of the residual stress tensor, among them those which cannot be obtained experimentally.

BASIC EQUATIONS AND RELATIONS

We consider a thin piecewise-homogeneous cylindrical shell of thickness *2h*. Let *R* be the radius of its median surface. We introduce a triorthogonal coordinate system (α, β, γ) and direct its α-axis ($\alpha = x/R$) along the generatrix and its γ-axis along the outer normal to the median surface. The shell consists of two different joined semi-infinite closed cylindrical shells and is weakened by a longitudinal crack of length $2l_0$. We place the origin of coordinates in the median surface of the shell at the point lying in the middle of the crack line (i.e., $|\alpha| \le \alpha_0$, $\beta = 0$ $(\alpha_0 = l_0 / R)$). The parts of this piecewise-homogeneous structure are welded together by a circular weld and suffer the action of residual stresses induced by welding. The axis of the weld $\alpha = \alpha_1$ $(\alpha_1 = l_1 / R)$ passes through the interface of the shell, and we assume that the conditions of ideal mechanical contact are satisfied at this interface.

To describe the entire piecewise-homogeneous shell and physicomechanical processes in it, we use the following representation:

$$p(\alpha) = p_1(\alpha) + [p_2(\alpha) - p_1(\alpha)]S_-(\alpha - \alpha_1) \quad (1)$$

where $p(\alpha)$ and $p_k(\alpha)$ $(k = 1,2)$ are unknown or given functions defined in regions occupied by the entire shell and its kth part, respectively, and $S_-(\alpha)$ is the asymmetric unit function [1,2].

To determine inherent stresses and their redistribution we use the following representation of the components of the total strain tensor e_{ij} [3-6]:

$$e_{ij} = e_{ij}^s + e_{ij}^0, \quad (2)$$

where e_{ij}^0 are components of the stress-free strain tensor and e_{ij}^e are components of the elastic strain tensor expressed via the inherent stresses according to Hooke's law.

For a piecewise-homogeneous cylindrical shell a system of partly-degenerated differential equations of elastic equilibrium in displacements obtained using the method of generalized coupling problems has the form:

$$L_{i1}(\alpha)u + L_{i2}(\alpha)v + L_{i3}(\alpha)w = g_i'\left(\alpha, \beta, \varepsilon_{kl}^0, \kappa_{kl}^0\right) + g_i''(\alpha, \beta), \quad i = \overline{1,3}, \quad (3)$$

where $L_{ij}(\alpha)$ and $g_i'\left(\alpha, \beta, \varepsilon_{kl}^0, \kappa_{kl}^0\right)$, $i, j = \overline{1,3}; k, l = 1,2$, are the same differential operators and functions as in the case of a homogeneous shell [6] but with discontinuous coefficients due to the representation of Poisson's ratio $\upsilon(\alpha)$ in the form of Eq. 1; u, v and w are components of the displacement vector;

$$g_1''(\alpha, \beta) = \left\{[\partial_1 u]_1 - R\left(\left[\varepsilon_{11}^0\right]_1 + \upsilon_k\left[\varepsilon_{22}^0\right]_1\right)\right\}\delta_-(\alpha - \alpha_1),$$

$$g_2''(\alpha, \beta) = \frac{1}{2}\left\{\left(1 + 4c_1^2\right)(1 - \upsilon_1)[\partial_1 v]_1 - R(1 - \upsilon_1)\left(\left[\varepsilon_{12}^0\right]_1 + 2Rc_1^2\left[\kappa_{12}^0\right]_1\right)\right\}\delta_-(\alpha - \alpha_1),$$

$$g_3''(\alpha, \beta) = c_1^2\left\{\left[\partial_1^3 w\right]_1 - (2 - \upsilon_1)\partial_2[\partial_1 v]_1 + R^2\left(\left[\partial_1 \kappa_{11}^0\right]_1 + \upsilon_1\left[\partial_1 \kappa_{22}^0\right]_1\right)\right\}\delta_-'(\alpha - \alpha_1) +$$
$$+\left\{\left[\partial_1^2 w\right]_1 + R^2\left(\left[\kappa_{11}^0\right]_1 + \upsilon_1\left[\kappa_{22}^0\right]_1 + 2(1 - \upsilon_1)\partial_2\left[\kappa_{12}^0\right]_1\right)\right\}\delta_-'(\alpha - \alpha_1),$$

$$\varepsilon_{mn}^0 = \frac{1}{2h}\int_{-h}^{h} e_{mn}^0(\alpha, \beta, \gamma)d\gamma, \quad \kappa_{mn}^0 = \frac{2}{3h^2}\int_{-h}^{h} e_{mn}^0(\alpha, \beta, \gamma)\gamma d\gamma, \quad m, n = 1,2$$

$$[p(\beta)]_1 = p_2(\alpha_1 + 0, \beta) - p_1(\alpha_1 - 0, \beta), \quad \delta_-^\lambda(\alpha) = \frac{d^\lambda S_-(\alpha)}{d\alpha^\lambda}, \quad c_1^2 = \frac{h^2}{3R^2}.$$

In deducing the system of Eqs. 3, we have taken into account only the fact that the components u, v and w and the rotation angle $\theta_1 = R^{-1}\partial_1 w$ are continuous. The jumps of the derivatives of displacements with respect to α at the interface $\alpha = \alpha_1$ are determined from other conditions of ideal mechanical contact:

$$N_1^{(2)} = N_1^{(1)}, \quad S^{*(2)} = S^{*(1)}, \quad M_1^{(2)} = M_1^{(1)}, \quad Q_1^{*(2)} = Q_1^{*(1)} \text{ for } \alpha = \alpha_1, \quad (4)$$

where $N_1^{(k)}$, $S^{*(k)}$, $M_1^{(k)}$ and $Q_1^{*(k)}$ (k=1,2) are generalized forces and moments in the kth part of the shell [6].

The Procedure for Obtaining Singular Integral Equations

Let the considered shell be subject to the action of forces symmetric about the axis of the crack. The crack faces are free of stresses. To analyze of the limit equilibrium of the shell we use the method of distortions [6]. According to this method, the cracked shell is associated with a similar continuous shell in which sources of inherent stresses with unknown densities are located along the line of the crack. Further, we use the theory of generalized functions and relate the sources of internal stresses ($\varepsilon_{kl}^0(\alpha,\beta)$, and $\kappa_{kl}^0(\alpha,\beta)$) to the jumps of generalized displacements (displacements and angles of rotation) and their derivatives passing the crack line. The expressions for these sources are substituted in the key system of partly-degenerated differential Eqs. 3. As a result, the right-hand sides of these equations contain the following relations for $\varepsilon_{kl}^0(\alpha,\beta)$ and $\kappa_{kl}^0(\alpha,\beta)$:

$$\varepsilon_{11}^0 = \varepsilon_{12}^0 = 0, \; \varepsilon_{22}^0 = R^{-1}[v(\alpha)]_c \chi(\alpha)\delta(\beta),$$

$$\kappa_{11}^0 = \kappa_{12}^0 = 0, \; \kappa_{22}^0 = -R^{-1}[\theta_2(\alpha)]_c \chi(\alpha)\delta(\beta), \quad (5)$$

where $\delta(\beta)$ is the Dirac delta-function; $\theta_2(\alpha) = R^{-1}(\partial_2 w - v)$, $\chi(\alpha) = S_+(\alpha + \alpha_0) - S_-(\alpha - \alpha_0)$, $[p(\alpha)]_c = p(\alpha,+0) - p(\alpha,-0)$.

Using relation Eq. 5 we construct the solution of the system of Eqs. 3 on the basis of the 2π–periodic Green tensor and satisfy the boundary conditions imposed on the crack faces. These conditions reflect the fact that on the crack line the sums of forces and moments of the principal stressed state (caused by loading residual stresses in the continuous shell) and the disturbed stressed state (induced by the crack) are equal to the forces and moments acting on the crack faces, i.e.,

$$N_2^+(\alpha,0) = N_2^-(\alpha,0) = f_1(\alpha),\ M_2^+(\alpha,0) = M_2^-(\alpha,0) = f_3(\alpha).$$

Using conditions Eqs. 4 we reduce the problem under consideration to the same type system of six singular integral equations as in [5], where $f_1(\alpha) = -N_2^0(\alpha,0) - N_2^r(\alpha,0)$, $f_3(\alpha) = -M_2^0(\alpha,0) - M_2^r(\alpha,0)$, N_2^0 and M_2^0 are the normal force and bending moment on the crack line in the shell without crack, N_2^r, M_2^r are the residual forces and moment on the crack line.

This system of singular integral equations is valid for the shell with a longitudinal crack which is located in one of the parts $(\alpha_1 > \alpha_0)$, ends at the interface $(\alpha_1 = \alpha_0)$ or crosses it $(\alpha_1 < \alpha_0)$.

Determination of Residual Stresses

The appearance of technological residual stresses is explained by the incompatibility of the residual strain field e_{ij}^0 (Eq. 2). Assume that the conditions of welding and post-welding thermal treatment are such that the residual stressed-strained state of the structure is axisymmetric and can be described by a spherical tensor ($e_{ij}^0 = \delta_{ij} e^0(\alpha,\gamma)$). Then the system of Eqs. 3 implies the key equation of the fourth order for the deflection w of the median surface of the analyzed structure received in [1,2,5]. According to the chosen theoretical-experimental method for the investigation of residual stresses it is necessary to take into account the experimental data obtained e.g. using nondestructive physical methods (photoelastic, magnetic, ultrasonic, etc.). To construct a regularizing algorithm for solution of the conditionally correct axisymmetric inverse problem posed on the basis of such an equation, we take into account the available additional *a priori* information about the qualitative form of the free residual strain field e^0 caused by welding, namely, its boundedness, localization near the weld, and the possibility of description by a smooth piecewise-continuous function. This enables us to assume that this function belongs to a compact set and can be represented in the polynomial form with unknown coefficients b_λ^j $(\lambda = 1,2,\ j = \overline{0,N})$ and parameters α_λ^* (the bounds of the zone of influence of $e^0(\alpha,\gamma)$).

Thus, we arrive at the final key equation of the axisymmetric inverse problem for the piecewise homogeneous cylindrical shell with residual welding stresses. This equation contains a finite number of unknown parameters and, hence, the inverse problem of determining the free residual strain field e^0 can be formulated as a problem of their search. Further, solving such an equation we can find residual stresses at any point of the shell using the well-known formulas. To determine the unknown parameters b_λ^j and α_λ^* we use some functional [5] which minimization provides the least deviation of theoretically calculated residual stresses from the experimentally obtained ones. As a result, we obtain a system of linear algebraic equations in the unknown quantities b_λ^j for fixed values of the parameters α_λ^*. The values of these parameters are taken from a special set of probability values determined using physical arguments and taking into account the necessity of minimization of the functional. If the parameters b_λ^j and α_λ^* are determined, then we use the well-known formulas from [6] to obtain final expressions for the residual stresses at any point of the shell. After that we determine the redistribution of residual stresses in the piecewise-homogeneous shell caused by a crack using the above-mentioned system of singular integral equations.

NUMERICAL RESULTS

We carried out the numerical analysis of the stress intensity factors K_1 and K_2 for the force N_2 normal to the crack line and the bending moment M_2. Calculations were performed by the method of mechanical quadratures for a shell with crack located in one of the part $(\alpha_1 > \alpha_0)$.

The Shell with a Crack Under Normal Forces of Constant Intensity

The investigation was performed for the shell Aluminum-Epoxy structure with a radius of the median surface R = 0,15 m and a thickness $2h$ = 0,003 m without residual stresses ($N_2^r = 0$ and $M_2^r = 0$) for the case crack faces loaded by forces of constant intensity N_0 ($f_1(\alpha) = -N_0$, $f_3(\alpha) = 0$). The plots of variation of the normalized intensity factors $K_N = K_1 / \left(N_0 \sqrt{l_0}\right)$ and $K_M = K_2 / \left(N_0 c R \sqrt{l_0}\right)$ versus various values of parameter $\rho = \alpha_0 / \alpha_1$ at

$\alpha_0 = 0.1$ are presented in Fig. 1 (the index 1 indicates the crack tip close to the interface, the index 2 marks the opposite crack tip). The dashed lines show the variation of intensity factors in a piecewise-homogeneous plate made of the same materials and weakened by the similar crack as given by Cook and Erdogan [7].

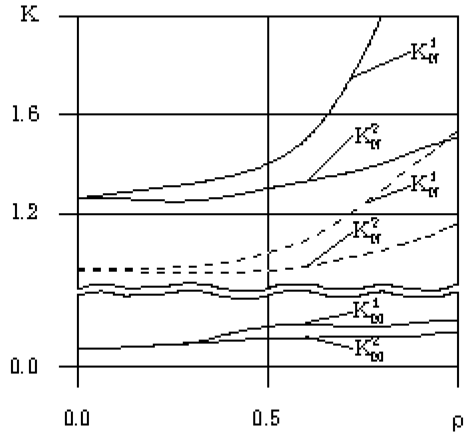

Figure 1: Variation of normalized intensity factors K_N and K_M in the shell.

We can conclude that the interface effect on the values of intensity factors begins at $\rho = 0.1$ early than in a plate. The values of normal force-intensity factors are well beyond the values of moment-intensity factors. Variation of moment-intensity factors is nonmonotonical.

Redistribution of Residual Stresses Caused by a Crack

Numerical analysis was performed for the shell with residual stresses subject to the action of internal pressure of intensity p $(N_2^0 = pR,\ M_2^0 = 0)$ with a radius of the median surface R = 0,065 m and a thickness $2h$ = 0,002 m. The distribution of technological residual stresses computed with the help of the program complex for the proposed theoretical-experimental method is depicted in [5].

In Figure 2 we have plotted the dependence of the normalized intensity factors $K_N = K_1 / \left(pR\sqrt{l_0}\right)$ and $K_M = K_2 / \left(cpR^2\sqrt{l_0}\right)$ on the relative half-length of the crack α_0 for various values of the parameter $n^0 = N_2^r(0)/(pR)$.

The influence of residual stresses on the forces and moments intensity factors increases with the parameter n^0. If the crack tip is located in the region of compression, then, for given values of the parameters, the quantity K_N is smaller than in the shell without residual stresses ($n^0 = 0$). This means that residual stresses can either promote fracture of the analyzed structure or inhibit the development of cracks in this structure (depending on their distribution and crack length).

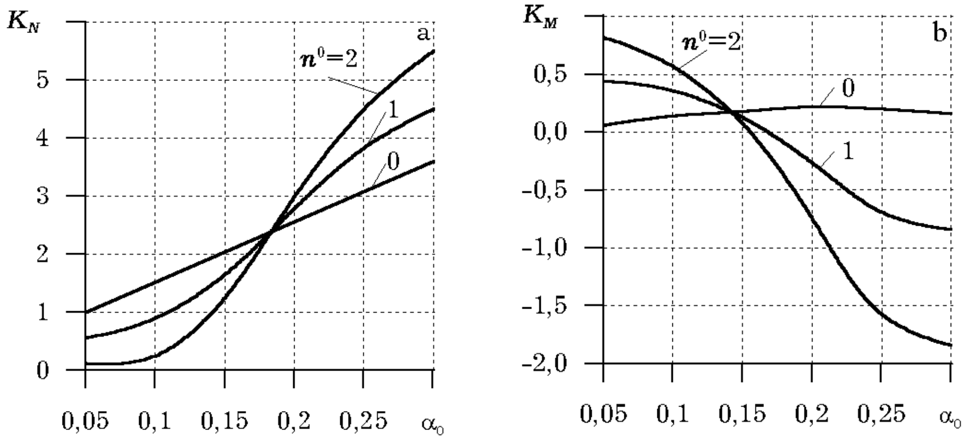

Figure2: Dependence of the normalized intensity factors K_N (à) and K_M (b) on the relative crack length α_0 and the parameter n^0.

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