Stress Intensity Factor for Small-to-Medium Edge Cracks

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ABSTRACT: *This paper presents a new and relatively simple engineering method for calculating the stress intensity factors for small-to-medium cracks emanating from a notch under arbitrary loading. The formulation can be used in calculating the fatigue life of notched components as well as in the shape optimisation problems with durability constraints. Several examples are considered to demonstrate the advantages of the present method in comparison with both existing approximate approaches and finite element techniques.*

INTRODUCTION

Fatigue failures invariably initiate at some form of a geometrical discontinuity. It is therefore not surprising to find that a great deal of fatigue related research has been devoted to studying the behaviour of notched specimens. The fatigue life prediction of notched components can be approached using fatigue crack growth analysis. To characterize the fatigue crack growth rate we require a knowledge of the stress intensity factors for a crack emanating from the notch. The stress intensity factor for edge cracks can be expressed as:

$$K = F\sigma\sqrt{\pi l} \quad , \tag{1}$$

where l is the crack length, σ is the applied stress and F is the geometry factor.

In the absence of an analytical solution for F, its determination requires extensive numerical computations for each particular notch profile, crack length and loading condition. Such a computational approach is not very efficient for problems in which large numbers of crack configurations and/or loading conditions need to be considered.

The aim of this paper is to develop a new and relatively simple method for calculating the stress intensity factors for small-to-medium cracks at notches under arbitrary loading including the shear loading. The present paper is based on the assumption that cracks occurring in similar stress fields with similar local geometries should produce similar stress intensity factors. This assumption has been widely exploited in the past to build up approximate solutions for the stress intensity factor for various geometries and loading conditions. Some examples are given in [1] and [2].

The proposed method is simple and transparent and can be used for numerous practical applications, viz: fatigue life calculations, and shape optimisation problems with durability constraints. For the last class of problems we require a knowledge of the stress intensity factors associated with cracks, of variable lengths, located at all points along the boundary being optimised.

PREVIOUS APPROACHES

Broek [3] was amongst the first to suggest a simple engineering solution for estimating the stress intensity factors for cracks emanating from a notch [3]. His idea was to consider the crack length as including the notch depth. Smith and Miller [4] proposed a simple formula for the stress intensity factor of small cracks at the root of a notch of finite depth. Another approximation was suggested by Lukas [5]. Due to the origin of this approach it is expected to provide good estimates for the cases in which $K_t < 3$ (where K_t is the stress concentration factor at the notch) i.e. blunt notches.

Karlsson and Backlund [6] used an analytical method to estimate K values for small cracks. The method was based on the work of Benthem and Koiter [7] who gave a solution for an edge crack in a semi-infinite sheet with a linear distribution of the tensile stress on the crack edges. A comparison of this method with numerical results shows that there is a systematic difference between equation given in [6] and the results of Newman [8]. This is not surprising because the actual stress distribution was not a linear function and the notch edge was not a straight line.

Schijve [9] developed another method in which the stress intensity factor was written in the form:

$$K = F\sigma_{\max}\sqrt{\pi l} \tag{2}$$

where $\sigma_{\max}$ is the peak stress.

The basic arguments for this method are as follows: Approximately similar stress distributions in the uncracked condition are obtained if the same values of σ_{max} and ρ apply (where ρ is the notch radius). Kujawski [10] proposed to replace the term σ_{max} by the local stress distribution $\sigma_y(l)$ at the distance l (crack length) to reach a better correlation with the numerical results of Newman [8] and Nisitani [11]. Kujawski estimated that in these cases the formulae differed from the numerical predictions by less than 5% [10].

Another general method for the calculation of the stress intensity factors is the weight function method, which is based on a number of fundamental LEFM relationships and solutions for references problems with a use of additional hypotheses [12]. The weight functions have been obtained for a wide range of geometries and loading conditions, in particular for a crack emanating from a notch when applied stress is normal to the crack faces. However, the case of the shear loading has not been considered.

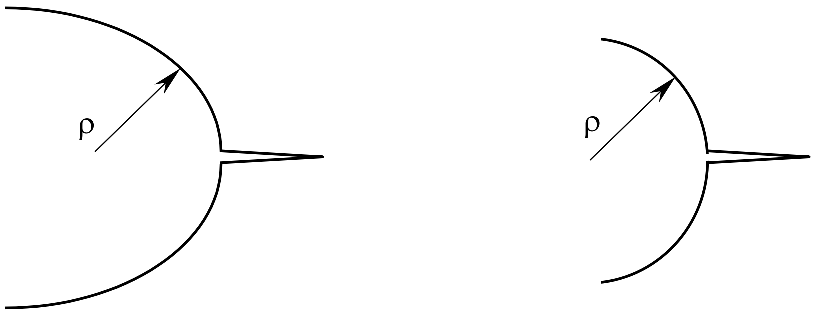
In general, the approximate approaches considered above work well if applied appropriately. However, the application of these approaches to different loading conditions or notch geometries can lead to significant errors, as demonstrated in the present paper. This finding is not surprising since approximate solutions only partially account for the local stress distribution in the vicinity of the notch and the local geometry (i.e. that the notch edge) is not a straight line.

PRESENT APPROACH

In this section we will develop an approximate method for calculating the stress intensity factors for an edge crack, with a length up to the order of the characteristic dimension of a local structural detail. A crack is defined to be small when $l \ll \rho$, medium when $l \sim \rho$ and long $l \gg \rho$, where ρ is the local radius at the point of interests. In the current investigation only small and medium length cracks are considered

Whilst for many structural components the fatigue life is dominated by the time taken in growing from a "small"-to-a "medium" length crack, it should be noted that there are instances when this is not true. This tends to occur in lightly loaded structures with large initial defects.

In the method presented by Broek [3] to solve a problem of the type shown in Figure 1a) we consider the ancillary problem of a crack emanating from a hole having the same radius as the notch-tip, see Figure 1b). We also need to determine the stress field associated with the (uncracked) notch as shown in Figure 1a).



(a) (b)

Figure1. Initial and ancillary geometries

The stress distribution for several types of holes in an infinite body can be obtained analytically based on the complex variable method or numerically. After replacing the notch geometry and determining the stress field the stress intensity factor for the edge crack can be related to the solution of the following Fredholm equation [13] and [14]

$$q_i(t) - \int_1^b \frac{q_i(\eta) M_i(t,\eta)}{\sqrt{(\eta-1)(b-\eta)}} d\eta = s_i(t) \quad , \tag{3}$$

where

$$s_i(t) = \frac{t-1}{\pi t} \int_1^b \sqrt{\frac{b-\xi}{\xi-1}} \frac{\xi t_i(\xi)}{\xi - t} d\xi \tag{4}$$

Here $b = l/\rho$, the index $i = I$ corresponds to the normal loading, $i = II$ corresponds to the shear loading, and the functions t_i represent the stress distributions on the crack, q_i is an unknown function. With this formulation the stress intensity factor can be expressed as:

$$K_i = -\frac{\sqrt{2\rho}}{\sqrt{b-1}} q_i(b) \quad . \tag{5}$$

Using Gauss-Chebyshev quadrature we can replace the integral equation (3) by a system of linear equations, viz:

$$q_i(t_k) - \frac{\pi}{n}\sum_{m=1}^{n} q_i(t_k) M_i(t_k, t_m) = s_i(t_k) \quad . \tag{6}$$

where M_i are given by Tweed and Rooke (1973) for mode I and in Kotousov and Jones (2001), for mode II.

$$t_k = \frac{b+1}{2} + \frac{b-1}{2}\cos\left[\frac{(2k-1)\pi}{2n}\right] \qquad k = 1,2 \ldots n \tag{7}$$

With this formulation the stress intensity factor can be rewritten as

$$K_i = -\frac{\sqrt{2\rho}}{\sqrt{b-1}} \left\{ s_i(b) + \frac{\pi}{n}\sum_{m=1}^{n} q_i(t_k) M_i(b, t_m) \right\} \tag{8}$$

Thus, the problem of determining the stress intensity factor can be reduced to the solution of a system of linear algebraic equations. It is a straightforward task to programme the system of equations and to use a computer library routine to invert the resulting $N \times N$ matrix. N may typically be chosen to be around 5-30, although more integration points will be needed when high accuracy is required and the complexity of the right hand side of equation (3) (i.e. the stress gradient) is expected.

EXAMPLES

To illustrate this approach we will first consider a crack configuration that has been widely treated in a number of previous investigations; viz: an elliptic notch with a crack in an infinite plate subjected to remote tensile loading acting normally with respect to the crack faces, see Figure 2. The solids lines represent the present solution; Kujawski's [10] solution for this problem is shown as the doted lines. The filled circles with the error range are the solution given in the Stress Intensity Factors Handbook [15]. The squares, diamonds and triangles represent the values obtained by the authors using FE method for c/a values of 0.5, 1 and 2 respectively. Here we see that all solutions are in good agreement. Note that for the case of $c/a = 1$, the solid line (present method) represents the benchmark solution because this is the exact analytical solution of this problem.

The method presented by Schijve [9] and further developed by Kujawski [10] can give significant errors if applied to a different loading configuration. To demonstrate this let us consider the notch configuration discussed above with of tensile remote load acting parallel to the crack. The results for three shapes of the ellipse, c/a equal to 0.5, 1 and 2 are shown in Figure 3. Here we see that the discrepancies between the present work and that of Kujawski become significant at relatively small ratios of the crack length to the notch radius. Again, at $c/a = 1$ (diamonds) the present values represent an exact analytical solution of the problem.

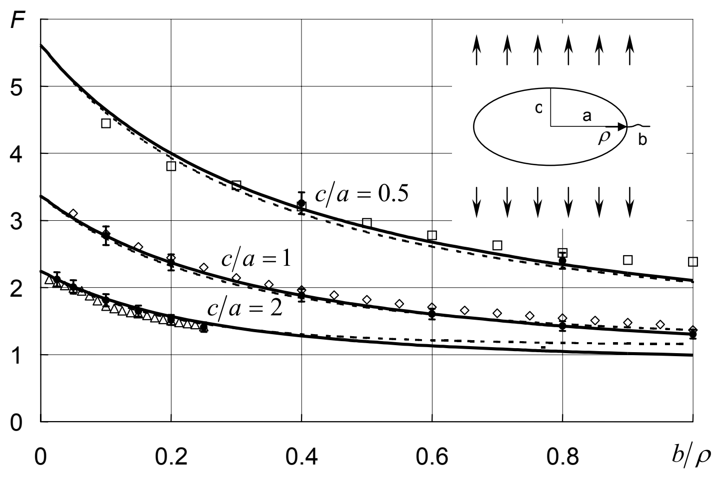


Figure 2. Comparison of approximate analytical methods: Kujawski (1991) (dot lines), present approach (solid lines) and numerical results (symbols) for various c/a ratios.

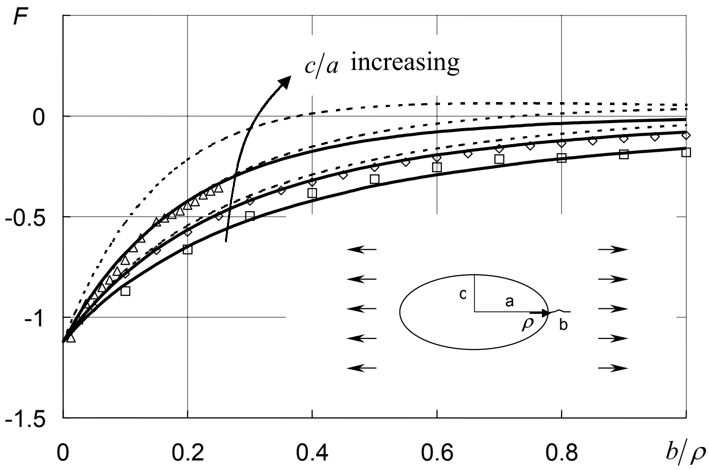


Figure 3 Comparison of approximate solutions; Kujawski (1991) (dot lines), present approach (solid lines) and numerical results (symbols) for various c/a ratios.

CONCLUSION

The paper presents a simple engineering method for calculating of the stress intensity factors for an edge crack at a notch. The results were compared with numerical calculations, which have shown a good agreement. The present method can give significant computational advantages over a direct finite element technique in fatigue life calculation and shape optimisation problems, especially where a large number of crack configurations and/or loading cases need to be considered. Our preliminary results show that the computational time for fatigue life and shape optimisation problems with durability constraints can be reduced by approximately 1000 times.

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