ONE APPROACH TO FATIGUE ASSESSMENT OF THE COMPONENTS SUBJECTED TO COMBINED HCF/LCF LOADING

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ABSTRACT: *The way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated. This new derived formula suggests an additional damage increase when crossing from one stress block to another. Herein, the reshaped crack growth rate formula is applied for the fatigue design of aircraft components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. For the stress history simplified in the way that it consists of one LCF stress block at load ratio r = 0, followed by one HCF stress at load ratio r > 0, the closed form expression is derived for estimating the crack propagation life at combined HCF/LCF loading. Smith and Haigh diagrams as design tools for estimating the fatigue strengths for designed fatigue life, known load ratio and various number of HCF cycles per one combined stress block, are obtained and presented for the parts made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading.*

INTRODUCTION

The parts of high-speed engines, especially the turbine and compressor discs and blades, are subjected to the combined low cycle fatigue (LCF) and high cycle fatigue (HCF) loading. LCF stresses are actually the "steady" stresses, which result in one cycle for every start-up and shutdown operation [1], and HCF stresses are caused by in-service vibrations. Thus, the stress history consists of N_B stress blocks (one for each operation) with n_{HCF} HCF cycles and one LCF cycle (Fig. 1). Actually, such type of stress history is usual for all machine parts subjected to substantial load due to start-stop operations. The integrity of these parts is particularly critical, because the usually extremely high cyclic frequencies of in-service loading spectra, causes that the fatigue life of e.g. 10^7 cycles can be reached in few hours. It was one of the reasons that a number of fatigue failures has been detected e.g. in US fighter engines [1]. It is important therefore, to keep looking for the simple procedure enabling designer the reliable estimation of both crack initiation and crack propagation life for a given applied load, or to obtain the (boundary) load (or strain), at which the component would not experience the unpermissible damage during the designed life. This procedure is proposed in this paper.

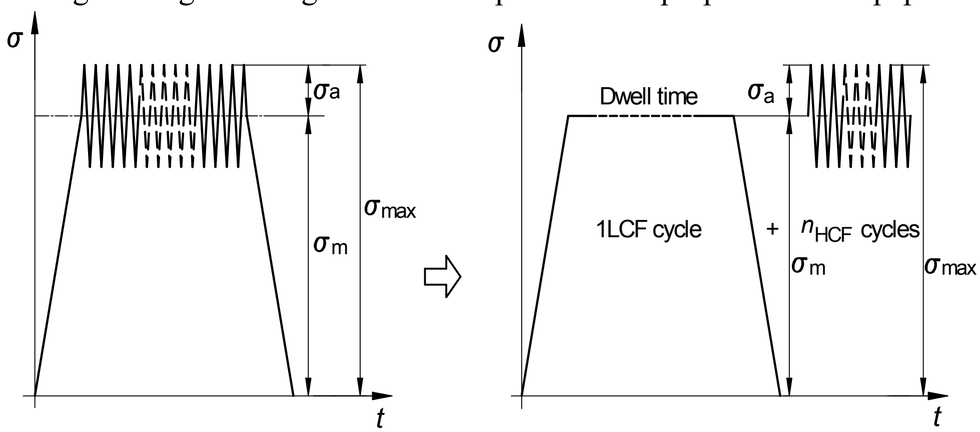

Fig. 1: Common stress history of one combined stress block and its separation in one LCF stress cycle and one HCF stress block.

CRACK INITIATION ASSESSMENT

The *S*-*N* curve for crack initiation (CI) at constant amplitude loading is described by the Wöhler type equation [1,2,3]

$$N_i \sigma^{m_i} = C_i \,, \tag{1}$$

where N_i is the crack initiation life for a certain stress level σ, and m_i and C_i are the material constants.

At steady loading (N = 1/4), the CI curve equals ultimate strength σ_U, and for the fatigue life N_{gr} at the knee of the *S*-*N* curve, it equals the endurance limit σ_0, which mean the entire fatigue life at the endurance limit level consists of the crack initiation life. On the basis of assumption that there is a unique CI curve between these two points, its slope was approximated [3] as

$$m_i = \frac{\log(4N_{gr})}{\log(\sigma_U / \sigma_0)} \tag{2}$$

where N_i is the crack initiation life for a certain stress level σ, and m_i and C_i are the material constants.

This expresion was found to be in good correllation with experimentaly obtained values. For example, the fatigue strength exponent b of steel 42 Cr Mo 4V (after DIN) for initiation life at r = -1 loading, was found to equal

0,0692 [4], thus $m_i = 1/b = 14{,}5$. Exactly the same value was obtained after Eq. 2 for $N_{gr} = 3\cdot 10^7$. It is also in line with novel investigations of Singh [2].

Whereas at the endurance limit stress level the initiation life practically equals the total fatigue life, the constant C_i can be assessed as $C_\mathrm{i}=N_\mathrm{gr}\sigma_0^{m_i}$.

For the purpose of this paper, the CI curve at $r = 0$ is used, which enables determining the level of the pulsating stress at the CI boundary for certain N_i, by knowing the crack initiation life N_{gr} at the endurance limit level:

$$\sigma_{0N,i} = \sigma_0\left(N_{gr} / N_i\right)^{1/m_i} \tag{3}$$

For the stress history described in Fig. 1., the crack initiation life is derived [3] on the basis of Palmgren - Miner hypothesis of linear damage accumulation

$$N_i = N_{B,i} \cdot n_{HCF} = \frac{1}{\dfrac{1}{N_{HCF,i}} + \dfrac{1}{N_{LCF,i} \cdot n_{HCF}}} \tag{4}$$

where $N_{B,i}$ is a number of combined stress blocks till the crack initiation, and n_{HCF} is the number of HCF stress cycles per one combined stress block.
The initiation life $N_{LCF,i}$ is obtained after the CI curve (3) at $r = 0$:

$$N_{LCF} = N_{gr}\left(\sigma_0/\sigma_m\right)^{m_i} \tag{5}$$

Since the Palmgren-Miner hypothesis is valid for various stress blocks at the same stress ratio, this equation is also used for the calculation of the HCF initiation life, but by substituting in it an equivalent stress range obtained by reducing a HCF stress range (with stress ratio $r_{HCF} > 0$) to an equivalent stress range at $r = 0$ [1, 3]

$$\Delta\sigma_{eq} = 2\sigma_{a,eq} = \frac{2\sigma_a\sigma_U}{\sigma_U + \sigma_a - \sigma_m} \tag{6}$$

where σ_U is an ultimate strength and σ_a is HCF amplitude stress. Thus, by substituting Eq. 5 in Eq. 4 twice (for a LCF stress σ_m , and for a reduced HCF stress after Eq. 6, the explicit formula is obtained for determining the crack initiation life at combined HCF/LCF loading:

$$N_i = \frac{N_{gr}\sigma_0^{m_i}}{\left(\dfrac{2\sigma_U\sigma_a}{\sigma_U + \sigma_a - \sigma_m}\right)^{m_i} + \dfrac{\sigma_m^{m_i}}{n_{HCF}}} \tag{7}$$

CRACK PROPAGATION ASSESSMENT FOR COMBINED HCF/LCF LOADING

As most appropriate for the purpose of this paper, because obtained for the titanium alloy Ti-6Al-4V, used hereafter in calculation example, the Ritchie fatigue crack growth rate formula [5]

$$\frac{da}{dN} = C\Delta K^m K_{\max}^n \tag{8}$$

is applied for determining the damage size. In this formula $\Delta K = \Delta\sigma Y\sqrt{\pi a}$ is the stress intensity range, $K_{\max} = \sigma_{\max} Y\sqrt{\pi a}$ is the upper value of the stress intensity factor, m and n are material constants, $\Delta\sigma = 2\sigma_a$ is a stress range, $\sigma_{\max}$ is a maximum stress, Y is a crack form factor, and a is a crack size. For titanium alloy Ti-6Al-4V, the following values of material constants were obtained: $C = 5{,}2\cdot 10^{-12}$, $m = 2{,}5$ and $n = 0{,}67$.

By introducing into the Eq. 8 the damage ratio $D = a/a_c$, where a_c is a critical crack size and fracture toughness $K_c = \sigma_{\max} Y\sqrt{\pi a_c}$, it can be reshaped in the form

$$\frac{dD}{dN} = \frac{B}{a_c}(1-r)^m D^{\frac{m+n}{2}} \tag{9}$$

where $B = 2^m C K_c^{m+n}$ is a material constant. By integrating this equation, it is easy to determine the damage ratio after N propagating cycles:

$$D = \frac{D_0}{\left[1 - D_0^{\frac{m+n}{2}-1}\dfrac{B}{2a_c}(1-r)^m(m+n-2)N\right]^{\frac{2}{m+n-2}}}, \tag{10}$$

where $D_0 = \pi a_0 (Y\sigma_m/K_c)^2$ is initial damage ratio, a_0 is an initial crack size, $r = \sigma_{\min}/\sigma_{\max} = K_{\min}/K_{\max}$ is a load (stress intensity) ratio, and the form factor is approximated after Raju and Newman [6] as $Y = 0{,}78(1 + a/d)$, where d is a bar diameter.

By substituting in this formula D = 1, the crack propagation life at constant amplitude loading can be determined. Eq. 9 can be used also in fatigue assessments at variable amplitude loading [7], but in such a case a_c changes, if $\sigma_{\max}$ changes, and Eq. 9 must be reshaped:

$$\frac{dD}{dN}=\frac{1}{a_c}\frac{da}{dN}-\frac{a}{a_c^2}\frac{da_c}{dN}=\frac{B}{a_c}(1-r)^m D^{\frac{m+n}{2}}+\frac{D}{D_0}\frac{dD_0}{dN} \tag{11}$$

In the case of block loading, or if the spectrum loading is approximated with block loading, the second term of this equation always equals zero, except when crossing from one stress block to another- when a_c changes, because σ_{max} changes, and consequently- the initial damage ratio changes.

During this process the first term of Eq. 11 equals zero and it becomes

$$\frac{dD}{D}=-\frac{da_c}{a_c}. \tag{12}$$

By integrating it, the increased value of damage ratio caused by the change of the critical crack size between two stress blocks, is obtained:

$$D_2=D_1\frac{a_{c1}}{a_{c2}}. \tag{13}$$

The expression in Eq. 11 is appropriate for the crack propagation assessment at any loading conditions, including non-regular one, where maximum stress, crack form factor and load ratio change.

Herein, the Eq. 11 is applied for the crack propagation life estimation in the gas turbine and compressor discs and blades made of the titanium alloy Ti-6Al-4V, at combined HCF/LCF loading. If the stress history is simplified in the way that it consists of one LCF stress block with $N_{\mathrm{LCF}} = N_{\mathrm{B}}$ cycles at maximum stress σ_m and load ratio $r = 0$, followed by one HCF stress block with $n_{\mathrm{HCF}}{\cdot}N_{\mathrm{B}}$ cycles at maximum stress $\sigma_{\max}$ and load ratio $r = (\sigma_{\max}-2\sigma_a)/\sigma_{\max}$, then the damage ratio D_{LCF} after the LCF stress block is determined after Eq. 10. According to Eq. 13, at the beginning of the HCF stress block, the damage ratio is

$$D_{0,HCF}=D_{LCF}\frac{a_{cL}}{a_{cH}} \tag{14}$$

where a_{cL} and a_{cH} are the critical values of the srack size at LCF and HCF loading, respectively. Those values can be determined by solving their equations. E.g. a_{cH} is determined from the equation

$$a_{cH}=\frac{1}{\pi}\left[\frac{K_c}{Y(a_{cH})\sigma_{\max}}\right]^2, \tag{15}$$

where K_c=50 MPa $m^{1/2}$ for Ti-6Al-4V alloy, after Ritchie [5]. The damage ratio D_{HCF} at the end of the HCF stress block, as the final damage ratio, is obtained again after Eq. 9. by substituting in it the corresponding values of initial damage ratio, stress ratio and number of cycles. The fatigue fracture occurs when this damage ratio reaches the value of one. Then, it is not difficult to solve the mentioned three equations for the N_B and consequently for the entire crack propagation life. It is obtained:

$$N_p = 2\frac{\left(a_0/a_{cH}\right)^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[\left(1-r\right)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\left(a_{cL}/a_{cH}\right)^{1-\frac{m+n}{2}}\right]} n_{HCF}. \quad (16)$$

Thus, the explicit expression is derived, enabling the estimation of the crack propagation life at combined HCF/LCF loading, for certain values of the stress levels $\sigma_{\max}$ and σ_m, which are hidden in a_{cH} and a_{cL}.

When no "block crossing" effect is applied, the expression for the crack propagation life becomes

$$N_p = 2\frac{D_0^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[\left(1-r\right)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\right]} n_{HCF}. \quad (17)$$

Assumption that stress history consists of one LCF cycle followed by one HCF stress block consisting of n_{HCF} cycles, followed by one LCF cycle etc. is much closer to real operational conditions. Thus, for more precise calculations, damage ratio is calculated after one LCF cycle, its increase according to Eq. 13., after the HCF stress block, then damage decrease according to Eq. 13., etc. The fatigue fracture occurs and calculation procedure is stopped at the moment when damage ratio reaches the value of one.

FATIGUE STRENGTH FOR COMBINED HCF/LCF LOADING

Whereas the Smith and Haigh diagrams are the very appropriate tools for fatigue design, the computer program is made, by means of which, for certain values of fatigue lives, the fatigue strength curves are obtained indicating the stress levels causing the fatigue failure after $N_f = C_f$ cycles. The calculations are carried out for various values of C_f, and for $n_{HCF} = 10^2 \ldots 10^5$. The fatigue limit curves obtained precisely exhibit the reduction of the design area in the Smith diagram compared to HCF loading only, the more so as the share of LCF loading is greater.

As an example, the resulting N_f=10^7 curves for titanium alloy Ti-6Al-4V are exhibited in Smith diagram, Fig. 2. It is observed:

- These curves are located in Smith diagram between Goodman plot and $\sigma_{max} = \sigma_m$ straight line, the higher the n_{HCF} the higher the curve position. At the region of lower mean stresses, they make one with Goodman line, then separate from it, reach maximum, and finally fall down at the constant mean stresses. Thus, the presence of the LCF component restricts the safe design space compared to that in case of pure HCF, the more so as the share of the LCF component is greater.
- The block crossing effect does not influence significantly the curves of constant fatigue life.
- Between the curves of constant fatigue life based on initial crack sizes of 0,1 mm and 0,05 mm was not observed a significant difference.
- The curves of constant fatigue life obtained on the basis of the derived closed form fatigue life formula, and those obtained on the basis of growth increments computed for one by one combined stress block, do not differ significantly.

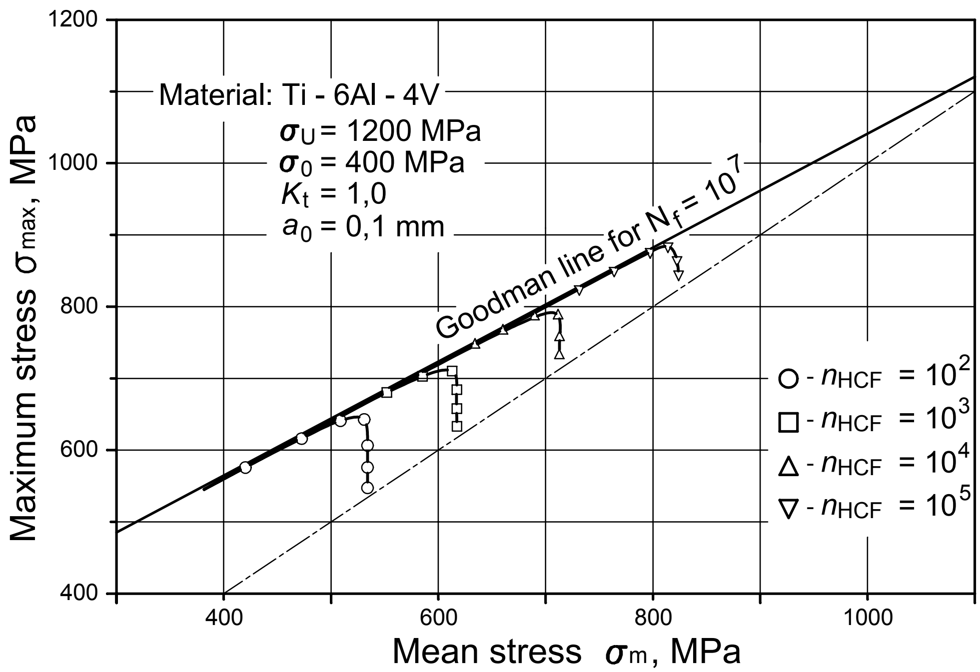

Fig. 2: Fatigue strength curves in Smith diagram for a combined HCF/LCF loading of titanium alloy Ti-6Al-4V.

SUMMARY AND CONCLUSIONS

The way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated in this paper. This new derived formula suggests an additional damage increase when crossing from one stress block to another. Unfortunately, the experimental investigations which should confirm this effect were not carried out because the technical reasons, and this will be done as soon as possible. However, the numerical results presented herein for the fatigue design of the components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading, are correct anyway, because the block crossing effect does not influence significantly the fatigue assessments at combined HCF/LCF loading .

The results obtained should be taken as a guide because

- The small crack behavior has not been taken into account,
- The presence of other damage mechanisms like creep fatigue, oxidation and other environmental effects are ignored,
- The residual stresses and the stress concentration have been ignored,
- Technology faults, material quality and operating conditions (like elevated temperature), have not been taken into account,
- Linear damage summation rule has been applied, although more precise techniques exist,
- The presence of inclusions and the service-induced damages could not be clasped in calculations,
- The reliability aspect of the design has been ignored.

At the same time, these imperfections are the signposts in the direction of building an expert system for the fatigue design of the aircraft components subjected to combined HCF/LCF loading.

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