CYCLIC CRACK TIP DEFORMATION DUE TO SINGLE TRANSITION CYCLES IN BIAXIAL LOADING

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ABSTRACT: *An elastic-plastic finite element analysis is used to simulate the cyclic tip deformation of a mode I long central crack in a plate subjected to a remote in-phase biaxial cyclic stress field with the three biaxial ratios of 1, 0 and -1. One-stress amplitude at a zero stress ratio is considered. The loading and unloading modes of a cycle are simulated. Results are presented in the form of the development of both monotonic and cyclic crack tip plastically deformed zones and opening displacements as the crack tip artificially advances.*

After five increments of crack tip advance within the equi-biaxial base loading cycles, the biaxial ratio is allowed to be either 0 or -1 for a single cycle. With further crack tip advance at the original base load for both types of transient cycles, the corresponding tip deformation shows behaviour similar to the effect of a single overload injected within uniaxial loading cycles. Generally, crack tip deformation starts with a relatively high value to decrease and then increases after reaching a minimum value beyond which an increase follows until matching with the behaviour due to the original base load takes place.

INTRODUCTION

Mechanical systems are usually subjected to variable amplitude loading. In this condition, load interaction or sequence significantly affects FCG rates and consequently fatigue lives. The effect of injecting a single overload within constant amplitude uniaxial load cycles in mode I FCG tests is well-documented [1,2]. Following the overload, FCG starts with a relatively high rate and continuously decreases to a minimum value. An increase in FCG rates follows to match the FCG behaviour corresponding to the base load. Responsible reported mechanisms include the generation of residual compressive stress field ahead of the crack tip, the strain hardening of the plastically deformed material around the crack tip and the occurrence of plasticity induced crack tip closure. Those mechanical events can be cast in the behaviour of cyclic crack tip deformation [3]. Finite element analyses

were developed to simulate the cyclic deformation accommodated at the tip of physical short and long mode I crack before, during and after the application of either a single overload [3,4]. This showed that the numerically computed extents of the crack tip plastically deformed zones and opening displacements behaved similarly to the experimentally observed FCG rates.

In many structures cracks initiate and grow in biaxial or multiaxial variable stress fields. An interesting aspect of crack growth under complex loading is that the crack may grow in a mixed mode manner [5]. Compared to uniaxial loading conditions, multiaxial fatigue studies are rare. However, mode I fatigue crack growth (**FCG**) under constant amplitude biaxial loading has relatively received considerable attention. Experimentally, both stress ranges and biaxial degree, λ, substantially affect FCG rates, e.g. [6,7]. Relatively, the crack grows faster in the case of negative λ and slower for positive λ. An obvious explanation is that crack tip plasticity is of a larger extent in the former case.

In actual service biaxial fatigue loading, transient cycles can be of a quite different configuration from a sudden change in the magnitude of the maximum value of one of the involved loading cycles. There are some mechanical systems in which reversing one of the applied loads during their service is frequently possible either intentionally or out of hand. Examples include gearing systems in automobiles and handling equipment in small area yards. In this case, possible transition cycles can be formed by a change in the phase angle between the applied loads. Such a problem has not yet been addressed either experimentally or numerically. The effect on cyclic crack tip deformation of changing the biaxial ratio for a single loading cycle within a base equi-biaxial cyclic stress field remotely acting on plate having a mode I long central crack is simulated in the present work. A previously developed in-house two-dimensional elastic-plastic finite element package is used.

NUMERICALL ANALYSIS

A previously developed in-house two-dimensional elastic-plastic finite element package [8] was utilised to simulate the cyclic tip deformation of a mode I long central crack in a plate subjected to a remote in-phase biaxial

cyclic stress field with the three biaxial degrees 1, 0 and -1. The analysis was performed for the plane stress state. The Von-Mises yield criterion and the Prandtl Reuss flow rule were adopted. The Bauschinger effect was considered through the use of the Prager-Ziegler kinematic hardening rule. The possibility of having crack surface closure was accommodated. The mechanical properties of the plate were as follows: yield stress, $\sigma_y = 350$ MPa, modulus of elasticity, E = 206 GPa, monotonic and cyclic hardening exponent, n = 0.2, Poisson's ratio, $\nu = 0.3$. A simple power law was assumed for the stress-plastic strain behaviour of the material in the plastic regime.
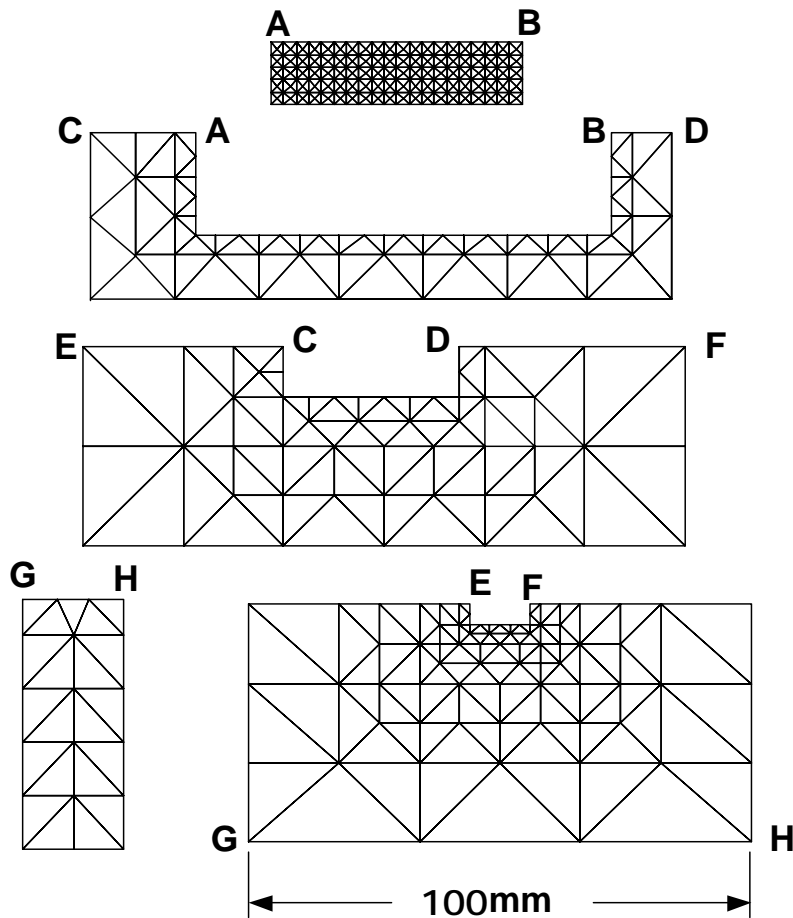

Figure1: The present finite element idealisation

Due to symmetry, one quarter of the plate was idealised as shown in Figure1. The plate was 150 mm x 100 mm with a central crack 25mm long located along the plate width. The finite element mesh was generated to consist of 693 constant strain triangular elements with 756 degrees of freedom. Four hundred elements around the crack tip with a size equal to 62.5 μm were arranged to form arrays of squares having triangles formed by their diagonals. That idealisation proved its adequacy in similar analyses on both short and long cracks.

One-stress amplitude at a zero stress ratio was analysed. The loading and unloading modes of a stress cycle were incrementally applied. The crack was allowed to advance at the point of maximum load with a step corresponding to one element. Cracking was carried out by an incremental release of the existing crack tip reaction forces. After five increments of crack tip advance within the equi-biaxial base loading cycles, λ was allowed to equal either 0 or -1 for a single load cycle, see the schematic of Figure 2.
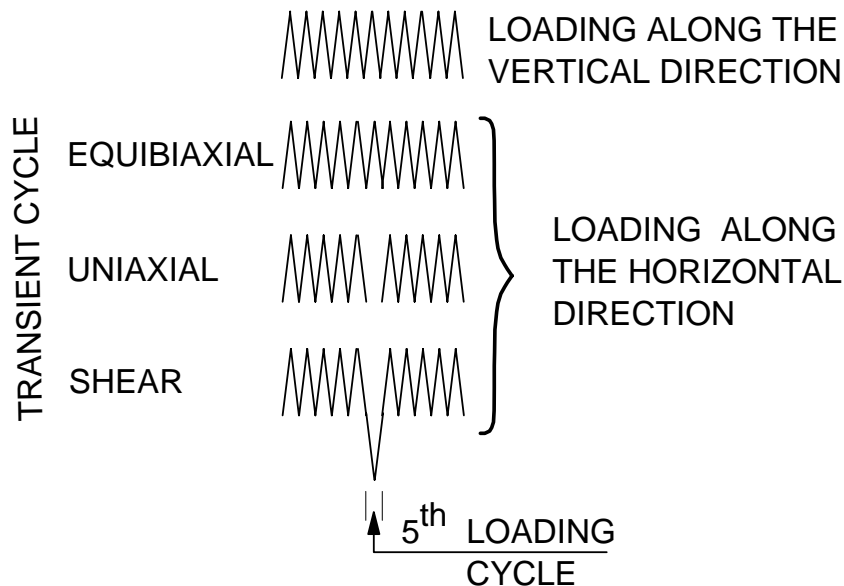
Figure 2: The present numerical program

Results were in the form of the development of both monotonic and cyclic crack tip plastically deformed zones and opening displacements as the crack tip artificially advances. In the course of each module, the extents of the crack tip plastically deformed zones and opening displacements were computed. The extent of crack tip plasticity, Δ, was considered as the diameter of a circle having the same area as the corresponding plastically deformed zone. The extent of Δ at the maximum load, P_{max}, was Δ_m. The extent of the cyclic plastic zone, Δ_c, was given by the elements plastically deformed at both P_{max} and the minimum load, P_{min}. The extent of δ, computed at the node just behind the current crack tip, were δ_{min} and δ_{max} at P_{min} and P_{max} respectively. The cyclic CTOD, $\Delta\delta$, was defined as $\delta_{max} - \delta_{min}$.
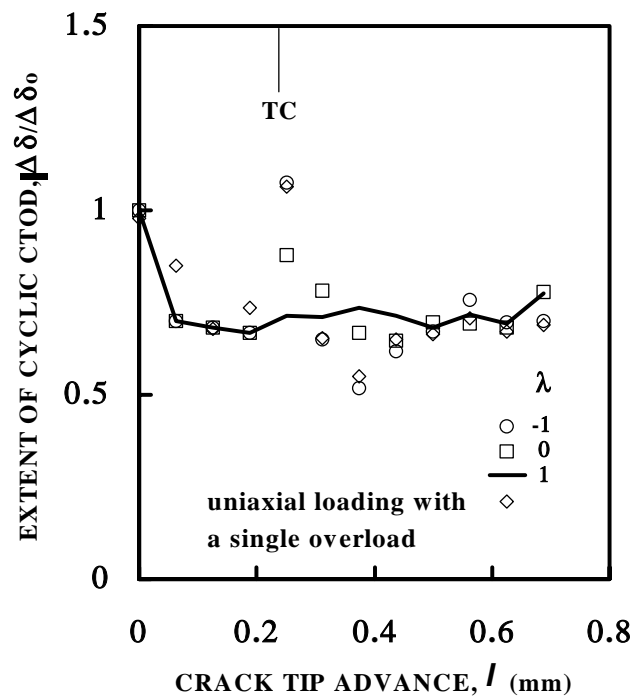
RESULTS AND DISCUSSION

Figure 3: Cyclic CTOD against crack tip advance

Figures 3-5 show the variation of the computed crack tip deformation (CTD), i.e. $\Delta\delta$, Δ_m and Δ_c respectively, as the crack tip artificially advances in the case of constant amplitude loading cycles with $\lambda = 1$, 0 and -1. In general, there is a transient behaviour, which takes place during the first three cracking increments. That behaviour is characterised by a decrease in CTD with crack tip advance (CTA) to achieve a minimum extent beyond which an increase in CTD follows to match a stabilised behaviour. For a given crack length, the shear and equi-biaxial loading cases result respectively in the largest and smallest extents of CTD.
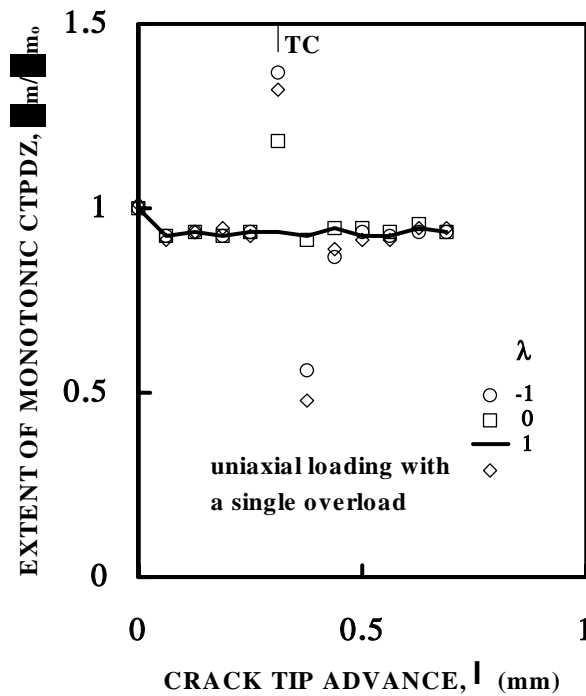

Figure 4: monotonic crack tip plasticity against crack tip advance

An artificial test was devised such that a uniaxial cyclic stress with a constant amplitude at R = 0 was applied on a cracked specimen having the same geometry. The applied stress was computed such that the extent of Δ_m after five cracking increments was the same as that generated due to the constant amplitude equi-biaxial base loading cycles. An overload cycle was, then, introduced. The behaviour resulting from two overload ratios were

analysed. An overload was estimated to generate Δ_m having the extent corresponding to the application of one of the considered transient cycles. Figures 3-5 show the behaviour of CTD with CTA for the above uniaxial case compared with the corresponding biaxial case. Such a comparison indicates that CTD behaviour after the application of a transient cycle is controlled by the extents of crack tip plasticity generated due to the base cyclic load at the instant of applying the transient cycle and that due to the application of the transient cycle.
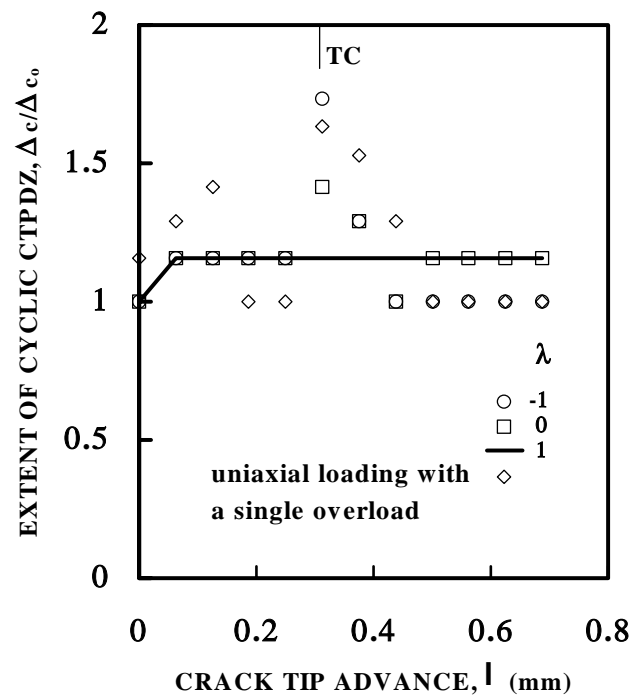

Figure 5: Cyclic crack tip plasticity against crack tip advance

CONCLUSIONS

Crack tip deformation behaviour after the application of a transient cycle within a constant amplitude cyclic base load is controlled by the extents of crack tip plasticity generated due to that load at the instant of applying the transient cycle and that due to the application of the transient cycle.

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