STIFFNESS CHANGES IN FIBER-REINFORCED POLYMERIC MATRIX LAMINATES CAUSED BY INTRALAMINAR DAMAGE

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ABSTRACT: *In the present paper the damage of fiber-reinforced polymeric matrix laminates is considered with the aim to examine their mechanical properties.*
The damage mode in the form of intralaminar cracks ("primary matrix cracking") is described in terms of second order, symmetrical damage tensor by Vakulenko and Kachanov. It allows for considering the orientation of the defects, which is determined by the orientation of the constituent layers. The crack discontinuity parameter is estimated in frame of the linear elastic fracture mechanics.
To describe the changes of the mechanical properties of a body, namely longitudinal Young modulus and Poisson ratio, caused by the damage - the constitutive relations by Adkins, based on the theory of irreducible integrity basis and polynomial functions are employed.
Theoretical results obtained due to above procedure are compared with the experimental results for carbon/epoxy laminates of cross and angle-ply stacking sequences.

INTRODUCTION

The mechanisms of the deterioration of unidirectional fiber-reinforced polymeric matrix laminates are quite good clarified by e.g. Reifsnider et al. [1]. For the laminate under fatigue load they specified intralaminar matrix cracking, cracks coupling and interfacial debonding, delamination, large scale fiber breaking and finally formation of a failure path leading to the total material deterioration.

In a case of monotonically increasing tensile load and laminates with *off-axis* plies being separated by *on-axis* plies, the author [2] observed in experiments carried out on symmetrical cross-ply and angle-ply specimens, that the dominant failure mechanism was connected with intralaminar transverse matrix cracking, while the delamination and fibers breaking occurred nearly simultaneously with specimen deterioration. Thus, in such a case the „life period" is determined mainly by matrix cracking.

The present paper deals with this specific damage mechanism and its influence on material elastic characteristics.

INTRALAMINAR MATRIX CRACKING

Matrix cracking along fibers in *off-axis* layers takes place at relatively low level of the tensile load and results in an array of cracks with nearly parallel midplanes, evenly distributed, laying within an *off-axis* lamina - see Figure 1. The important factor determining cracking process and crack density is ply orientation angle θ_m, i.e. the angle between the material axis (*1, 2*) and any arbitrary reference axis (*x, y*).

Almost regular pattern of a number of cracks within an individual ply, possibility of initiation of similar pattern within the other *off-axis* plies, as well as very small crack size - make approach based on Continuous Damage Mechanics (CDM), dealing with continuous description of the discreet crack field, an effective tool in the analysis of composite's intralaminar damaging.

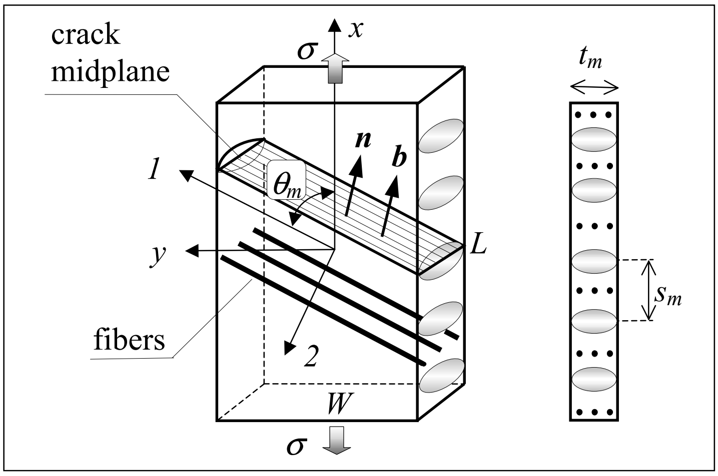

Figure 1: Orientation of intralaminar matrix cracks within *off-axis* ply.

CDM APPROACH FOR COMPOSITE BODY

Let us define a single matrix crack as an elementary damage entity, the collection of damage entities of similar geometrical features as damage mode - depending on the layers sequence we can distinguish a several damage modes in a laminate. The set of damage modes is called "damage".

Following paper by Kachanov [3] one can construct at any point on surface S of a single crack, second order damage tensor $\boldsymbol{D}'$ in the form of dyadic product of the two vectors, namely: $\boldsymbol{b}$ - the displacement jump vector across a surface S, and $\boldsymbol{n}$ - a unit outward normal to the surface S. Thus, we have:

$$\boldsymbol{D}' = \sum_k \boldsymbol{b}_k \otimes \boldsymbol{n}_k \, d\, S_k \tag{1}$$

Transition from discreet model to the continuum one is made by averaging the damage field (1) over a volume V, containing representative sample of „k" damage modes. Confining analysis to the normal discontinuities (cracks in mode I), the damage tensor takes the form:

$$\left\langle \boldsymbol{D}' \right\rangle_n = \boldsymbol{D} = \frac{1}{V} \sum_k \int_{S_k} \beta_k \, \boldsymbol{n}_k \otimes \boldsymbol{n}_k \, d\, S_k \tag{2}$$

where symbol "< >" means an average over volume V, S means the projection of crack surface on a crack "midplane", and multiplier β is a measure of average crack opening displacement. It is calculated in frame of LEFM with use of averaging procedure. Possibility and admissibility of such an approach is discussed by Varna et al. [4]. In order to estimate β_k, some additional assumptions and approximations, as well as proposed by the author the concept of „imaginary strip" [5,6], constructed from the considered damaged ply and the parts of adjacent layers, are also employed.

Using geometrical relations arising from the Figure 1, we get finally for m-th intralaminar damage mode within the layer in its *on-axis* configuration the following form of only non-zero component of damage tensor:

$$D_{22}^m = D_2^m = \left(\pi t_m / 4 E_2\right) \rho_m \, v_m \, f\left(t_m / c_m\right) \sigma^\infty \tag{3}$$

E_2 denotes the transverse Young modulus of a ply, c_m is an imaginary strip width, $f\,(t_m / c_m)$ denotes the finite width correction factor for the stress intensity factor K_I, $\rho_m = 1/s_m$ denotes average crack density within m-th damaged ply, s_m is cracks spacing and v_m stands for the ply volume fraction.

STIFFNESS MATRIX FOR THE LAMINATE WITH DAMAGE

Stiffness matrix for the single damaged ply in the material axis

To construct the relationship between stress σ_{ij}, strain ε_{ij} and damage D_{ij} tensors, an approach by Adkins [7] is applied. For the orthotropic body and two symmetrical II order kinematic matrices e_{ij}, a_{ij} he derived the general relation:

$$\sigma_{ij} = A_{ijtt}\Theta_{tt} + \varepsilon_{rst}\varepsilon_{rst}A_{ijrs}\left[P^{(1)}_{rs;t} + P^{(2)}_{rs;t} + Q^{(1)}_{rs;t} + Q^{(2)}_{rs;t} + R_{rs;t}\right] \quad (4)$$

A_{ijrs} is equal to 1 for $i=r$ and $j=s$, otherwise - A_{ijrs} is equal to zero, ε_{rst} denotes the Ricci's symbol. The terms $\Theta_{tt}, P^{(1)}_{rs;t}, P^{(2)}_{rs;t}, Q^{(1)}_{rs;t}, Q^{(2)}_{rs;t}$ and $R_{rs;t}$ are quite complex combinations of matrices e_{ij} and a_{ij} (identified in further analysis as, respectively, a strain tensor ε_{ij} and damage tensor D_{ij}), as well as invariant polynomial functions - constructed as linear combinations of the elements of irreducible integrity basis [7] (the basis for the orthotropy and the two symmetrical II order tensors e_{ij} and a_{ij} was employed).

It is assumed here, that stresses are linearly dependent on kinematic matrices, what in fact is equivalent to the assumption of strains and damage being small quantities.

From the general form of constitutive equation:

$$\boldsymbol{\sigma} = \boldsymbol{C}\,\boldsymbol{\varepsilon} \quad (5)$$

we get, after tedious transformations, the stiffness matrix $\boldsymbol{C}$ for *on-axis* ply configuration, decomposed into two parts, namely matrix $\boldsymbol{C^o}$ and $\boldsymbol{C^d}$. The first one relates to undamaged ply and can be taken from e.g. [8]. The second matrix characterises the change of m-th ply stiffness caused by the damage. Matrix $\boldsymbol{C^d}$ for the m-th ply takes the following form:

$$\boldsymbol{C}^{d\,m} = \begin{bmatrix} A_6 & A_2 & 0 \\ A_2 & A_{10} & 0 \\ 0 & 0 & A_8 \end{bmatrix} D_2^m \quad (6)$$

The procedure being used to determine the values of unknown „new" material parameters A_2, A_6, A_8, A_{10} will be given in later part of a paper.

Stiffness matrix for the damaged laminate

The matrices $\boldsymbol{C^o}$ and $\boldsymbol{C^d}$ are the basis for calculating stiffness matrix for damaged laminate. Taking into account that laminate consists of many layers with different orientation in relation to the reference co-ordinate system (x, y), these matrices have to be transformed from material axis (1, 2) to reference axis (x, y). After transformation - with use of Tsai-Pagano procedure described in e.g. [8] - we get transformed reduced stiffness matrix for any constituent ply. Now, one can employ the classical lamination theory and determine the global extensional stiffness matrix $\boldsymbol{A}$, which can be decomposed into the matrix $\boldsymbol{A^o}$ for „virgin" material and matrix $\boldsymbol{A^d}$ describing an influence of the damage state developing within some layers on the stiffness of laminate. The matrix $\boldsymbol{A^o}$ depends on volume fraction and orientation angle θ_m of each laminate's constituent layer, as well as on "standard" material characteristics i.e. four independent engineering constants for *on-axis* ply: E_1, E_2, G_{12} and ν_{12}. The matrix $\boldsymbol{A^d}$ depends on volume fraction, orientation angle θ_m and damage state within damaged layers only. It depends also on "new" material parameters A_2, A_6, A_8 and A_{10}.

The stiffness matrix $\boldsymbol{A^d}$ is unknown as long as parameters A_2, A_6, A_8, A_{10} remain not determined in appropriate tests. Note, that for an orthotropic laminate without considering damage, to get full information on laminate stiffness the four *on-axis* constants are needed. When the damage is included, we also need to know four constants, which however can not be derived from tests carried out on a single ply (like in first case), but on a laminate as a whole. The first crack in a single ply means its final fracture and the damage in such a sense as in the present paper can not be defined. Therefore, the plies stacking sequence must be chosen in such a way, which makes the calculations possible, but on the other hand, as easy as possible.

MATERIAL CHARACTERISTICS FOR A DAMAGED LAMINATE

The simplest, but adequate laminate configuration is cross-ply laminate $[0/90_n]_s$, as in such a laminate the damage develops in 90° ply only.

Using the standard relations [8] between the engineering moduli and global extensional stiffness matrix $\boldsymbol{A}$, after number of transformations, along with applying assumption of damage being a small quantity, one can find formulas for material parameters A_2, A_6, A_8, A_{10}.

All needed quantities have been determined from the tensile test of laminate specimen. The constants A_2, A_6 and A_{10} have been found for specimen code $[0/90_3]_s$, manufactured from carbon/epoxy (Torayca T300/Vicotex 174) „prepreg" tape Vicotex NCHR 174B. The details of specimens' preparation, their testing and test results are given in [2]. The constant A_8 related to a shear modulus only, has not been determined. Besides, it was assumed that transverse Young modulus was constant, because transverse cracks within 90° ply do not produce the change of transverse stiffness. The following values have been found: A_2 = -192.0 GPa, A_6 = -34.7 GPa, A_{10} = -258.0 GPa. These values allow for calculating the current values of engineering moduli of any laminate.

RESULTS AND CONCLUSIONS

An example of the standard procedure for estimation of a laminate stiffness changes, based on strength analysis, is shown in the Fig. 2. Partial ply discount method (PPDM) and Azzi-Tsai-Hill criterion are used in calculations. The substantial differences - both quantitative and qualitative - between axial Young modulus (YM) and major Poisson ratio (PR), calculated on PPDM basis and those taken from tests are easily visible. It is general observation that PPDM underestimates the stiffness of a damaged laminate. The other observation is that PPDM leads to somewhat unreasonable prediction of "step" change of the engineering moduli instead of gradual, as it is observed in tests.

In order to verify the proposed theoretical model of the stiffness changes, as well as to compare it with PPDM method the two sets of specimens, namely $[0/90_n]_s$ cross-ply and $[-20/+20/-\alpha_2/-20/+20/+\alpha_2/-20/+20]_s$ angle-ply, were manufactured and tested under tensile load [2].

In the Fig. 3 dimensionless (actual to the initial value ratio) YM and PR along with crack density are plotted as functions of applied stress for $[0/90_2]_s$ and $[0/90_4]_s$ specimens. The reduction in PR is as big as 60% for $[0/90_2]_s$ specimen and 90% for $[0/90_4]_s$. Test data show that PR is reduced more when load increase is accompanied by growth of crack density. YM is much less sensitive on stress level and crack density than PR. The reduction of YM was measured as big as 13% for $[0/90_4]_s$ specimen and 7% for the specimen $[0/90_2]_s$. Reduction of engineering moduli is the biggest for the thicker specimen, while number of observed cracks in this case is the smallest. Thus, stiffness changes are not only damage, but also thickness dependent.

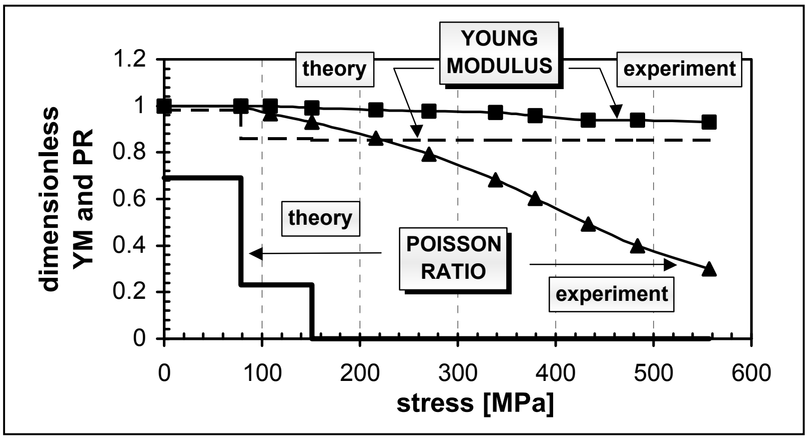

Figure 2: Dimensionless YM and PR - theoretical predictions and test data.

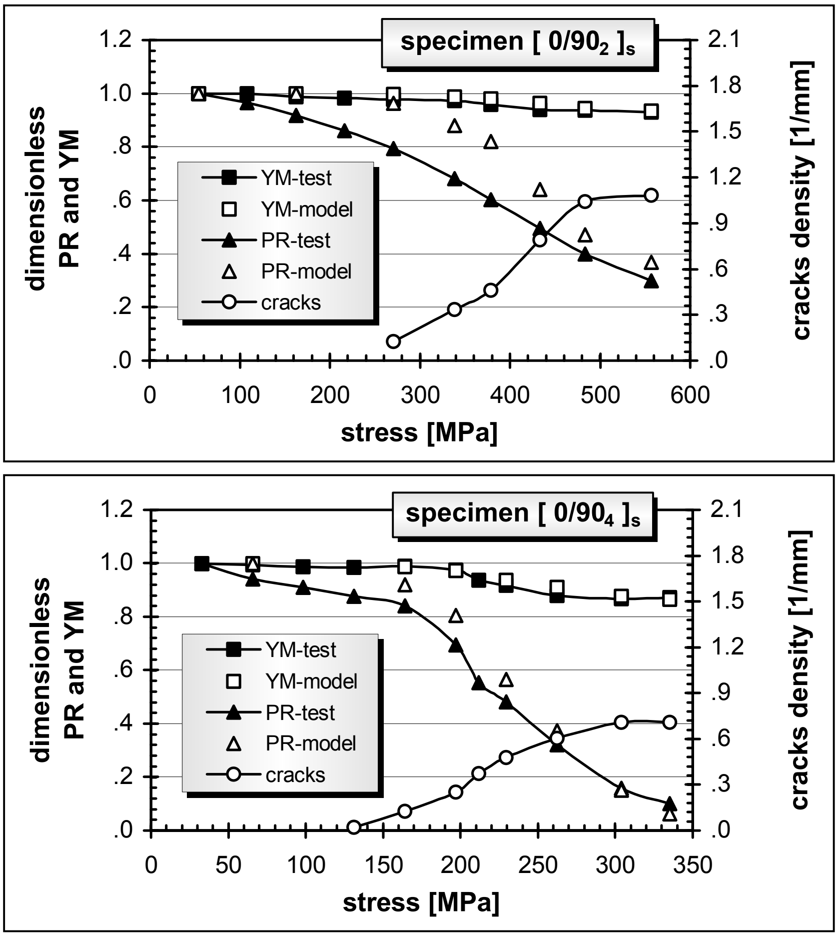

Figure 3: Dimensionless YM, PR and crack density as functions of stress.

Predictions of YM following from the proposed model match test data with very good accuracy for each cross-ply specimen and for most of angle-ply specimens. Theoretical predictions and test data do not differ more than 10%.

However, it must be pointed out that for angle-ply specimens the dependence of YM changes on the crack density was very poor, if any. In one case YM was almost constant within entire range of applied load, despite of numerous cracks and in the other one we observed reduction of YM equal to approx. 9%, whilst only single cracks have been visible. For the last case the present model was not able to predict YM change, as for a low crack density (or no crack at all) it must deliver the same result as for a nearly virgin material (or entirely virgin) and in fact it does. Therefore, the observed mismatch does not mean that the model is unreasonable.

The fitting of calculated and measured PR is in general not so good as in case of YM, but is significantly better than that given by PPDM. For cross-ply specimens the maximum difference is as big as approx. 30%, but in most cases is much less. The mechanical behaviour of angle-ply specimens is in general quite different. For most specimens the experimental results were completely out of expectations and the results obtained for cross-ply orientations. Instead of PR reduction along with load and damage growth we observed its progressive increase. The maximum increments for specimens being tested were within the range (3÷18 %). It is reasonable to assume that growth of PR is caused by laminate layout and resulting non-linear transverse deformations recorded during the tests. Intralaminar cracks reduce slightly this growth, but they are not able to cause absolute reduction of PR.

Let us finally conclude that:

* for cross-ply laminates both YM and PR are reduced due to a damage development. YM is much less sensitive on crack density than PR,
* for angle-ply laminates an influence of damage on stiffness is insignificant, if any. Despite of damage development the growth of PR was observed. Reduction of YM was noticed only for specific orientations and it was not caused by cracks,
* stiffness changes may not to be an appropriate measure of damage state.

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