A Proposal for Parameters Identification of the GTN Model – Application to CT Specimen

J. Chambert and Ph. Bressolette

LERMES (Laboratoire d'Etudes et de Recherches en Mécanique des Structures), Université Blaise Pascal – Clermont-Ferrand II, 63177 Aubière, France – e-mail: chambert@lermes.univ-bpclermont.fr

ABSTRACT: *This paper deals with a new ductile fracture model based on incorporating coalescence criteria into the Gurson-Tvergaard-Needleman (GTN) model in order to determine the critical porosity of coalescence f_C and the final void volume fraction f_F (GTN2P model). Because this model is only valid for axisymmetric specimens, we propose a practical approach based on a two-step procedure. In the first step, using the GTN2P model on an axisymmetric notched tensile specimen, the parameters are identified. In the second step, the GTN model, with parameters values previously defined, is applied to structural components or fracture specimens to predict the ductile crack growth. We have performed a numerical analysis of a CT25 specimen made of 16MND5 in order to validate the proposed methodology.*

INTRODUCTION

Ductile fracture of metals is classically described by three idealized stages: nucleation, growth and coalescence of micro-voids. Since classical fracture mechanics has not accurately predicted the geometric and loading effects, local approaches have been increasingly used to simulate ductile fracture. In this context, Gurson [1] has proposed a micro-mechanical model for porous plastic solids containing spherical voids. In order to better account for the void coalescence process, Tvergaard and Needleman [2] have introduced into Gurson's original model two coalescence parameters: the critical void volume fraction f_C and the final void volume fraction f_F (GTN model).

CONSTITUTIVE EQUATIONS

In this context, the overall ductile porous material is modelled by a periodic array of axisymmetric unit cells (see Figure 1a). Each cylindrical cell of current *2H*-height and *L*-radius contains one spherical microvoid of current *R*-radius, which is embedded in Von Mises matrix materials. Subsequently, the microscopic quantities concern the local state in the unit cell, they are represented by small letters ($\sigma_{ij}, \varepsilon_{ij}$). Capital letters ($\Sigma_{ij}, E_{ij}$) are used for the 'mesoscopic' conditions applied to the cell. Mesoscopic quantities are defined by averaging the microscopic variables over the cell volume.

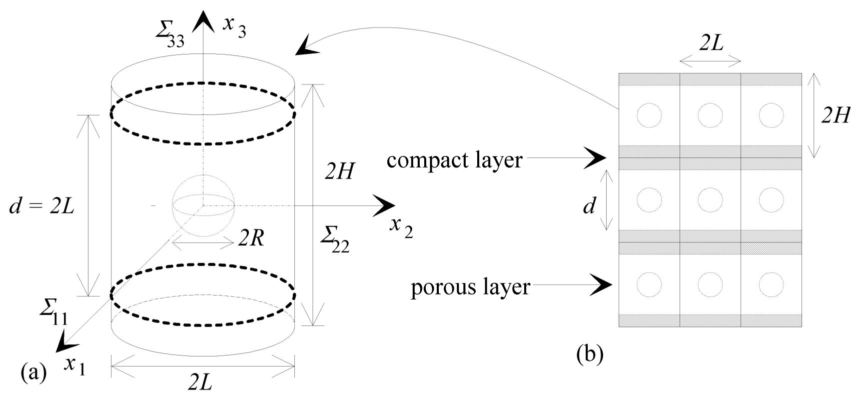

Figure 1: (a) Axisymmetric unit cell – (b) Porous/compact layers.

The GTN Model

The yield condition of the GTN model [2] reads:

$$\Phi\left(\Sigma_{ij}, \bar{\sigma}, f\right) = \frac{\Sigma_{eq}^2}{\bar{\sigma}^2} + 2q_1 f^* \cosh\left(\frac{3}{2} q_2 \frac{\Sigma_m}{\bar{\sigma}}\right) - 1 - \left(q_1 f^*\right)^2 = 0 \qquad (1)$$

where Σ_{ij} is the Cauchy stress tensor, $\Sigma_{eq} = \sqrt{3\Sigma_{ij}^{'} \Sigma_{ij}^{'} / 2}$ denotes the equivalent stress with $\Sigma_{ij}^{'}$ being the stress deviator, $\Sigma_m = \Sigma_{kk} / 3$ is the hydrostatic stress, $\bar{\sigma}$ is the flow stress of the matrix material. For the 'constitutive' parameters q_1 and q_2, we use the common values $q_1 = 1.25$ and $q_2 = 1.$ (cf. Chambert [3]). In order to account for void coalescence, Tvergaard and Needleman [2] have replaced the void volume fraction by the bilinear function $f^*(f)$:

$$f^*(f) = \begin{cases} f & \text{for } f \le f_C \\ f_C + \dfrac{1/q_1 - f_C}{f_F - f_C}(f - f_C) & \text{for } f > f_C \end{cases} \qquad (2)$$

The material looses its stress carrying capacity at Gauss points from which the porosity reaches the final void volume fraction f_F. The porosity rate comes partly from the growth of existing voids and partly from the nucleation of new voids, which are controlled by the plastic strain:

$$\dot{f} = (1-f)\dot{E}^{p}_{kk} + A\dot{\bar{\sigma}} \quad ; \qquad A = \frac{1}{h_M}\frac{f_N}{s_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\bar{\varepsilon}^{p}-\varepsilon_N}{s_N}\right)^2\right] \tag{3}$$

Here, $\dot{E}^{p}_{ij}$ means the mesoscopic plastic strain rate tensor, f_N is the volume fraction of void nucleating particles, ε_N is the mean strain for nucleation and s_N is the corresponding standard deviation. The tangent modulus of the matrix material is defined by $h_M = \mathrm{d}\bar{\sigma}/\mathrm{d}\bar{\varepsilon}^{p}$ where $\bar{\varepsilon}^{p}$ is the local equivalent plastic strain.

The GTNP Model

According to Perrin [4], void coalescence comes from the plastic flow localization in the ligament between the neighboring cavities. He has assumed that coalescence results from progressive concentration of cavities in some horizontal *porous layers* bounded by *rigid zones* (see Figure 1b).

Subsequently, (*p*)-exponent denotes quantities inside the porous layers. The unit cell is subjected to mesoscopic axisymmetric loading ($\Sigma_{33} > \Sigma_{11} = \Sigma_{22}$) with a constant stress triaxiality ratio. The porosity *f* and mesoscopic stresses are calculated by applying the GTN model to the overall unit cell. Let ($\Sigma^{(p)}_{11} = \Sigma^{(p)}_{22}, \Sigma^{(p)}_{33}$) and $f^{(p)}$ be the mesoscopic principal stresses and porosity into the highly porous layer.

In order to calculate $f^{(p)}$, Perrin has supposed that the virtual material, composed by stacking the *d*-height porous layers, is always defined by isotropic void distribution, this entails that $f^{(p)} = f\ H/L$ and $d = 2L$ (see Figure 1). These porous layers should adhere to the GTN yield condition:

$$\left(\frac{\Sigma^{(p)}_{eq}}{\bar{\sigma}}\right)^2 + 2q_1 f^{(p)}\cosh\left(\frac{3}{2}q_2\frac{\Sigma^{(p)}_{m}}{\bar{\sigma}}\right) - 1 - \left(q_1 f^{(p)}\right)^2 = 0 \tag{4}$$

A standard numerical scheme enables us to calculate $\Sigma^{(p)}_{11}$, the unknown quantity of Eq. (4). After some algebraic manipulations, Chambert [3] has found the critical condition describing the onset of void coalescence. When the criterion is reached, the porosity is then equal to the critical value f_C. In the case of rigid-plastic matrix material and for a material with single void population, the condition is restricted to the criterion of incipient void coalescence, proposed originally by Perrin [4]. The GTNP model consists of combining incipient coalescence criterion with the GTN model.

The GTN2P Model

By evaluating the slope of the equivalent stress-strain curve just after the onset of void coalescence, Perrin [4] has also proposed an analytical approximate formula, which determines the final void volume fraction f_F. Chambert [3] has generalized Perrin's post-coalescence analysis to the case of strain hardening material and nucleation of secondary voids in metals.

We assume that the overall material is subjected to the GTN model and to axisymmetric proportional loading. Subsequently, the exponent or index *C* (respectively, *C*+) means that the value is taken at (respectively, just after) the onset of coalescence. Let introduce the following notation:

$$S_{C+} = \frac{1}{\Sigma_{eq}^{C}} \left. \frac{\mathrm{d}\Sigma_{eq}}{\mathrm{d}E_{eq}} \right|_{C+}$$

Here, E_{eq} is the equivalent strain. The expression S_{C+} is calculated by two different *approaches. In the first one*, the slope S_{C+} is obtained by an approximate analysis without taking account of the GTN yield function. At incipient coalescence, it is assumed that the void remains spherical and that the deformation of the unit cell becomes uniaxial (*i.e.* $\dot{L} \approx 0$). After much calculation, Perrin [4] has found:

$$S_{C+} \cong -\frac{3}{2}\left(\frac{3H_0}{2L_0}\right)^{2/3} f_C^{-1/3} \exp\left(E_{eq}^{C}\right) \frac{1-\frac{2}{3}\left(\frac{3H_0}{2L_0} f_C\right)^{2/3} \exp\left(E_{eq}^{C}\right)}{1-\left(\frac{3H_0}{2L_0} f_C\right)^{2/3} \exp\left(E_{eq}^{C}\right)} \tag{5}$$

where H_0 and L_0 are the initial height and width/radius of the unit cell. In this study, we consider that the initial distribution of voids is homogeneous, *i.e.* $H_0 = L_0$. *In the second one*, Chambert [3] has evaluated the slope S_{C+} by using the GTN yield function Φ and the consistency condition $\dot{\Phi} = 0$. By equalling both relations S_{C+} and using Eq. 2, Chambert [3] has obtained the final porosity f_F:

$$f_F = f_C + \frac{2(1-q_1 f_C)(\mathrm{ch}_C - q_1 f_C^*)\left[3q_1q_2 f_C^*(1-f_C)\mathrm{sh}_C + \frac{A h_M^{C+}\,\omega_C}{1-f_C}\right]}{\omega_C\left[\frac{h_M^{C+}\,\omega_C}{(1-f_C)\bar{\sigma}_C} - 2S_{C+}\frac{\Sigma_{eq}^C}{\bar{\sigma}_C}\right]} \qquad (6)$$

with
$$\omega_C = 2\left(\frac{\Sigma_{eq}^C}{\bar{\sigma}_C}\right)^2 + 3q_1q_2 f_C^* \frac{\Sigma_m^C}{\bar{\sigma}_C}\mathrm{sh}_C$$

sh_C and ch_C denote the hyperbolic sine and cosine of $(3q_2\Sigma_m^C)/(2\bar{\sigma}_C)$. The GTN2P model consists of combining Eq. 6 with GTNP.

THE TWO-STEP PROCEDURE

Presentation

Because the GTN2P model is only valid for axisymmetric specimens, we propose a practical use of the GTN model based on a *two-step* procedure.

In the first step, the parameters f_C and f_F are identified by using the GTN2P model on an axisymmetric notched tensile specimen (AE4 for instance, see Figure 2). By using this model, all the parameters except the 'free' parameter f_N are fixed. The value of f_N is determined by comparing the numerical and experimental results of the specimen.

In the second step, the GTN model with parameters previously defined is applied on structural components or fracture specimens to predict the crack growth.

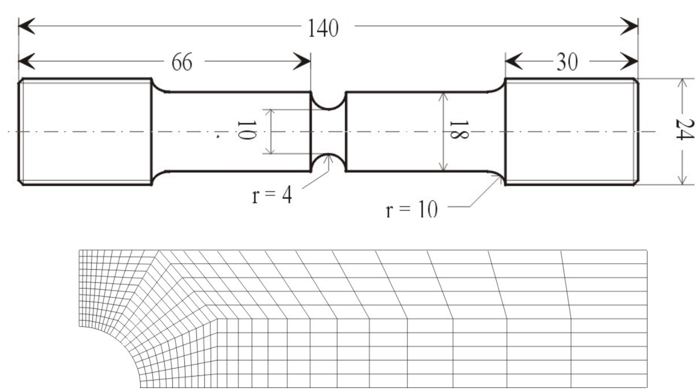

Figure 2: Axisymmetric notched specimen AE4 (dimensions in mm).

In order to validate the proposed methodology, we have performed a numerical analysis of a CT25 specimen. The material under consideration is a ferritic steel 16MND5 (French designation), which have been studied by Geney [5]. The chemical composition (in weight percent) is : C=0.16, Mn=1.35, Si=0.21, S=0.002, P=0.005, Ni=0.74, Cr=0.14, Mo=0.48 and N=0.004. At room temperature, the tensile properties are: *E*=193 GPa, R_{eL}=473 MPa, R_{eH}=475 MPa, R_m=605 MPa and A=25.5 %.

Numerical Results

The GTNP and GTN2P models have been implemented by Chambert *et al.* [3,6] into the FE code CASTEM 2000. In these implementations, the parameters f_C and f_F are no more considered as constants but as internal variable fields. The specimens have been modelled by using 8-nodes isoparametric elements with reduced integration (see Figure 2). Large displacements and strains have been assumed in this study.

First step: AE4 calculations

For both GTNP and GTN2P models, the parameters for void nucleation were set to $\varepsilon_N = 0.3$ and $s_N = 0.1$ [2]. As regards the initial void volume fraction f_0, we have chosen nearly the same value as Geney [5], *i.e.* $f_0 = 3.10^{-4}$. For the GTNP model, the final porosity was set to $f_F = 15\%$ (see Chambert [3]). By fitting the GTNP results to the experimental data, it was obtained $f_N = 3.10^{-3}$. The curves of load versus diameter reduction are shown in Figure 3. As soon as necking occurs, a loss of stiffness appears and the sustaining load is reduced significantly. Then, the initiation of a macroscopic crack is associated with a sudden drop of the load. It can be seen in Figure 3 that the GTNP model slightly overestimates the sudden drop point of experimental tests.

When taking the GTN2P model into consideration, the slope of the curve is very steep after the load-drop point (see Figure 3). For both models, the f_C-values, located next to the middle of the specimen, are nearly constant: $f_C = 2.69\%$. For the final porosity, the GTN2P model provides almost constant values ($f_F = 5.45\%$) close to the center of the specimen. This value is very similar to the f_C one. Consequently, these close values could explain the numerical problems of convergence that we have encountered with the GTN2P model.

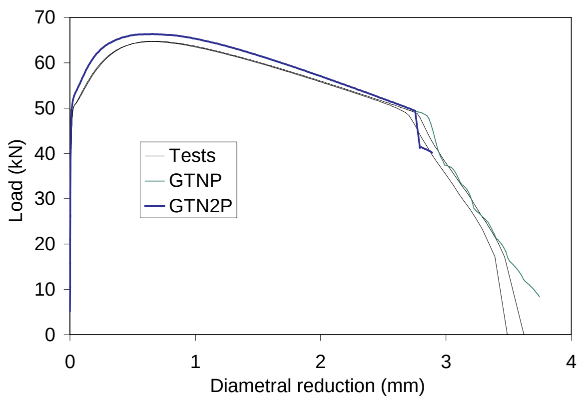

Figure 3: Load-diameter reduction curves (AE4 specimen)

Second step: CT25 calculations

Figure 4 depicts the geometry and the mesh of a half of CT precracked specimen with 25 mm of thickness. We have carried out 2D-plain strain computations. The experimental results have been performed by Geney [5].

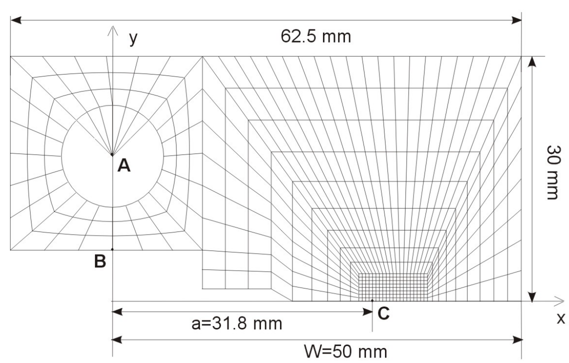

Figure 4: Compact tension specimen CT25.

The curves of load versus crack mouth opening displacement are shown in Figure 5. The curves of the GTN model have been obtained by using either the coalescence values of GTNP ($f_C = 2.69\%,\ f_F = 15\%$), or the ones of GTN2P ($f_C = 2.69\%,\ f_F = 5.45\%$). The GTN curve achieved with GTN2P coalescence values reveals sharp drops in load, which could be explained by the close values of f_C and f_F. Although the agreement between numerical and experimental curves is moderatelely good, this first result is encouranging because the cracked specimen is qualitatively modelled.

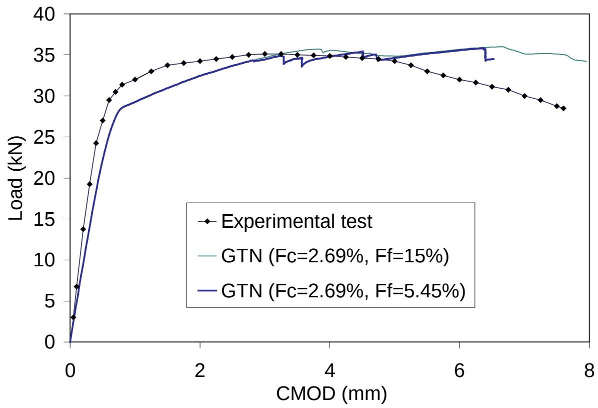

Figure 5: Load-CMOD curves (CT25 specimen)

CONCLUSION

A new ductile damage model, called GTN2P, has been formulated and implemented into the FE code CASTEM 2000. We have proposed a more practical use of the GTN model based on a two-step procedure. The results obtained by the proposed methodology are quite good. Nevertheless, it is necessary to study in more details the influence of characteristic lengths, related to the material microstructure.

ACKNOWLEDGEMENTS

The authors are grateful to Professor S. Degallaix from the "Ecole Centrale de Lille", France, for providing the experimental data of the specimens.

REFERENCES

1. Gurson, A.L. (1977) *J. Engng. Mater. Techn.* **99**, 2.
2. Tvergaard, V. and Needleman, A. (1984) *Acta Metall.* **32**, 157.
3. Chambert, J. (2001). PhD Thesis, Université Blaise Pascal, Clermont-Ferrand, France.
4. Perrin, G. (1992). PhD Thesis, Ecole Polytechnique, Paris.
5. Geney, Ch. (1998). PhD Thesis, Ecole Centrale de Lille, France.
6. Chambert, J., Bressolette, Ph. and Vergne, A. (2000). In: *ECF13 European Conference on Fracture*, 6 – 9 Sept., San Sebastian, Spain.