Circumferentially Notched Pipe with an External Surface Crack under Tension and Bending

Andrea Carpinteri, Roberto Brighenti and Sabrina Vantadori

Department of Civil Engineering, University of Parma, Parco Area delle Scienze 181/A, 43100 Parma, Italy; E-mail: andrea.carpinteri@unipr.it

ABSTRACT: *The fracture behaviour of a round pipe with a circular-arc circumferential notch is analysed in the case of tension and bending loading , by assuming that an elliptical-arc external surface crack exists at the notch root. The stress concentration factor (SCF) is determined through theoretical expressions and a finite element analysis. Then, the stress-intensity factor (SIF) along the surface crack front is computed for four values of the dimensionless notch radius. The effect of stress concentration is discussed for both thick- and thin-walled pipes.*

INTRODUCTION

Several Authors have examined smooth structural components with surface cracks [1-6], whereas only a few papers analyse round bars [7,8] and pipes [9,10] with hoop grooves. As is well-known, the SIF distribution for a surface crack embedded in a stress field due to a notch can be quite different from that evaluated for the same crack in a homogeneous stress field.

In the present paper, a pipe with a circular-arc circumferential notch is considered. An elliptical-arc external surface crack is assumed to exist in the notched (reduced) cross section of the pipe (Fig. 1). The dimensionless stress-intensity factors (SIFs) for tension (F) and bending (M) are determined through a finite element analysis, by varying the stress concentration factor (SCF), K_t, from 1.0 (smooth pipe) to about 3.0 (pipe with a small notch radius).

GEOMETRICAL PROPERTIES

The circumferential notch is characterized by a depth c and a constant curvature radius r (Fig. 1), and a transversal surface crack is assumed to exist at the notch root. The relative notch radius and the relative notch depth

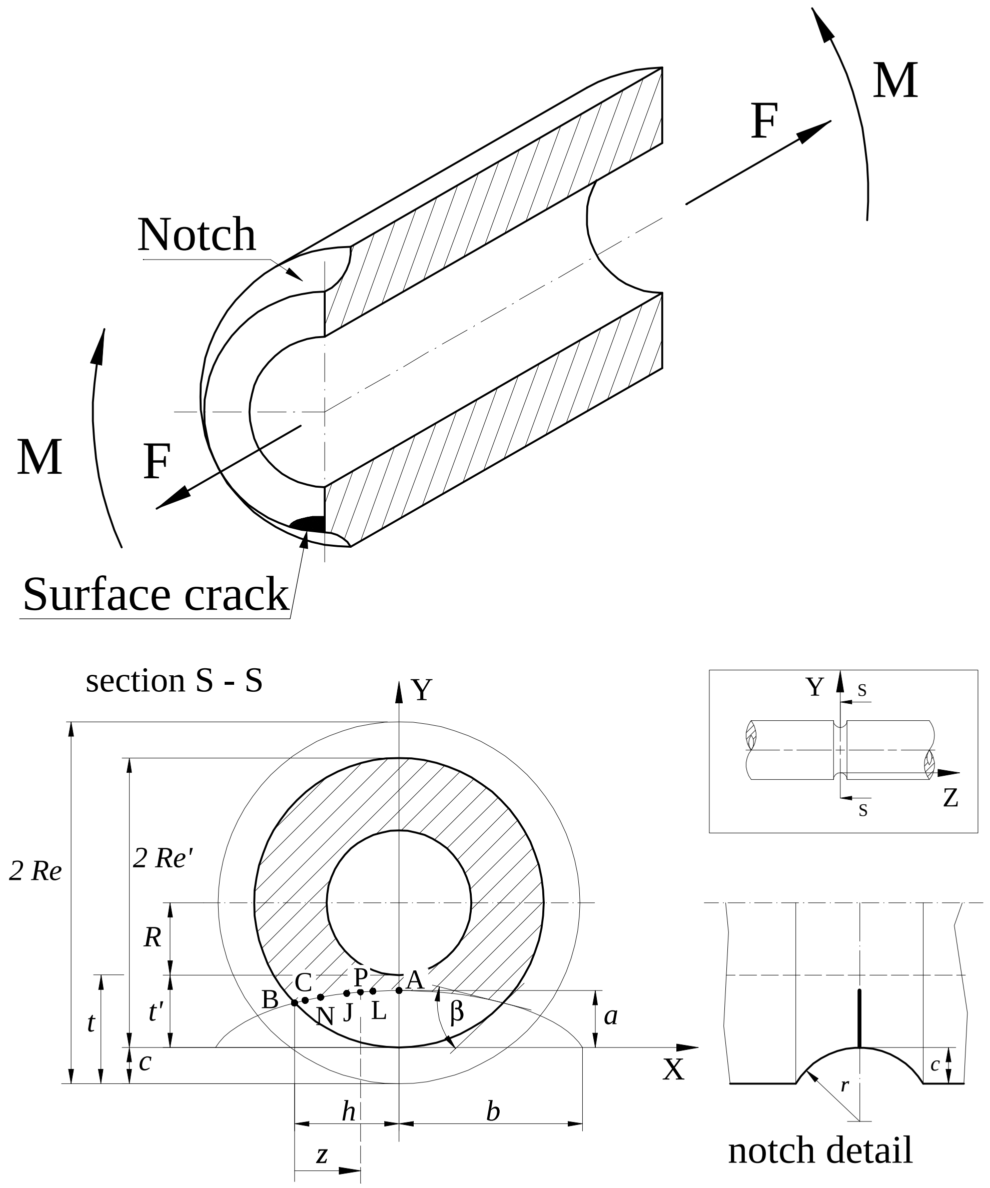

Figure 1: External surface flaw in a circumferentially notched pipe.

are defined as $r^* = r/t$ and $d = c/t$, respectively, where t is the pipe wall-thickness in an unnotched cross section. In the following, d is assumed to be equal to 0.1 for all the notched configurations considered, whereas r^*

is equal to 0.1 (sharp notch), 0.2, 0.7 (blunt notch) and ∞ (unnotched pipe). For a given value of r^*, a stress concentration factor can be computed for each loading condition analysed.

An external surface crack is assumed to exist in the notched cross section of the structural component. Such a flaw presents an elliptical-arc shape (Fig.1). The crack configuration being examined is described by the relative crack depth $x = a/t'$ of the deepest point A on the defect front (with $0.1 \leq x \leq 0.8$), and the flaw aspect ratio $a = a/b$ (with $0.0 \leq a \leq 1.2$), whereas the generic point P along the crack front is identified by the dimensionless coordinate $z^* = z/h$.

STRESS CONCENTRATION FACTOR (SCF) EVALUATION

Making use of the spherical section assumption [11], Zhang et al. [9] have determined SCF theoretical expressions for notched pipes under tension. Two ranges, based on the geometrical parameters, have been distinguished : (i) $R < R_e^{'} - a_0$, and (ii) $R > R_e^{'} - a_0$, where R is the inner radius of the pipe, $R_e^{'}$ is the outer radius of the notched section and $a_0 = 1.3\sqrt{r\,c}$.

In the case of $R < R_e^{'} - a_0$, the SCF value can be obtained from the following expression :

$$K_{t,F} = \frac{R_e^{'2} - R^2}{1.3r\,R_e^{'}\left[\dfrac{(R_e^{'} - a_0)^2 - R^2}{1.3r\,R_e^{'} + 2R_e^{'}a_0 - a_0^2} - \ln\left(\dfrac{1.3r\,R_e^{'}}{1.3r\,R_e^{'} + 2R_e^{'}a_0 - a_0^2}\right)\right]} \qquad (1)$$

In the case of $R > R_e^{'} - a_0$, Zhang et al. [9] have proposed :

$$K_{t,F} = \frac{R_e^{'2} - R^2}{1.3\,r\,R_e^{'}\ln\left(\dfrac{1.3\,r\,R_e^{'} + R_e^{'2} - R^2}{1.3\,r\,R_e^{'}}\right)} \qquad (2)$$

Since the longitudinal stress in a pipe under bending linearly varies along the thickness, the SCF, $K_{t,M}$, for this kind of loading can be approximately computed by combining those for a plate under tension ($K_{t,F}^{plate}$) and bending ($K_{t,M}^{plate}$), respectively [12,13] :

$$K_{t,M} = C_F \ K_{t,F}^{plate} + C_M \ K_{t,M}^{plate} \qquad (3)$$
$$\text{where :} \quad C_F = \frac{R_e + R}{2R_e} \ ; \quad C_M = 1 - C_F$$

Figure 2 shows SCF values for both tension and bending loading against the relative notch radius $r^* = r/t$, in the case of $R^* = R/t = 1$ (a) and $R^* = R/t = 10$ (b), respectively. Results determined by the present authors through a 2D finite element (FE) analysis are also displayed. A good agreement between numerical and theoretical values can be observed. Furthermore, it can be remarked that the results for tension are about coincident with those for bending.

For the r^* values assumed in the present study (0.1, 0.2, 0.7 and ∞), the SCF under tension loading is equal to about 3.06, 2.33, 1.63, 1.0 for thick-

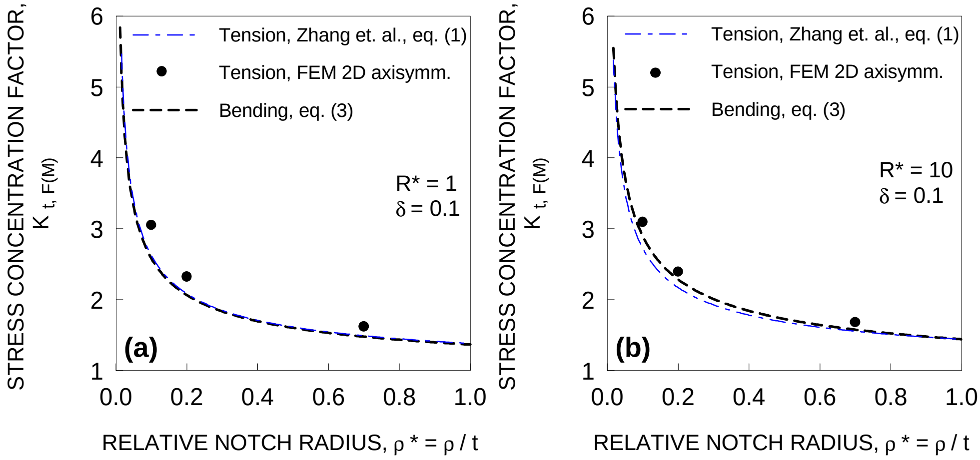

Figure 2: SCF against for (a) thick- and (b) thin-walled pipes

walled pipes ($R^* = 1$) and 3.10, 2.40, 1.68, 1.0 for thin-walled pipes ($R^* = 10$), respectively.

STRESS INTENSITY FACTOR (SIF) EVALUATION

Because of the double simmetry of the problem, only a quarter of the pipe has been modelled by 20-node isoparametric finite elements. Wedge elements in rings have been employed around the flaw front in order to improve the accuracy of the numerical solution.

The SIF values for tension (F) and bending (M) have been obtained from the quarter-point finite element displacements :

$$K_{I,F(M)} = \frac{E\ \sqrt{2\pi}}{4\ (1-\nu^2)}\ \frac{w(l/4)}{\sqrt{l/4}} \tag{4}$$

where $w(l/4)$ stands for the displacement orthogonal to the crack faces (i.e. along the Z-direction), measured in correspondence to the quarter-point position (with a distance equal to $l/4$ from the crack front, where l is the edge finite element length).

The dimensionless SIFs for tension and bending are defined as follows :

$$K^*_{I,F} = \frac{K_{I,F}}{\sigma_F\ \sqrt{\pi\, a}}\ ; \qquad K^*_{I,M} = \frac{K_{I,M}}{\sigma_M\ \sqrt{\pi\, a}} \tag{5}$$

where

$$\sigma_F = \frac{F}{\pi\ (R_e'^{\,2} - R^2)}$$

nominal tensile stress in the notched section

$$\sigma_M = \frac{M}{\left[\dfrac{\pi\ (R_e'^{\,4} - R^4)}{R_e'}\right]}$$

nominal maximum bending stress in the notched section

RESULTS AND DISCUSSION

The SCF parameter is very important especially in presence of cracks, since it heavily affects the SIF results. For example, the dimensionless SIF at point A against the relative crack depth x is displayed in Fig.3 ($R^* = 1$ in Fig. 3a and $R^* = 10$ in Fig. 3b) for different values of the relative notch depth r^* and the crack aspect ratio a, in the case of tension loading Note that the SIF distribution for a notched pipe ($r^* \neq \infty$, i.e. $K_t \neq 1$) is quite different from that for an unnotched pipe ($r^* = \infty$, i.e. $K_t = 1$) : as a matter of fact, the SIF curves for a smooth pipe monotonically increase by increasing the relative crack depth, while a minimum SIF value can be observed in correspondence to an intermediate value of x for a notched pipe, but such a

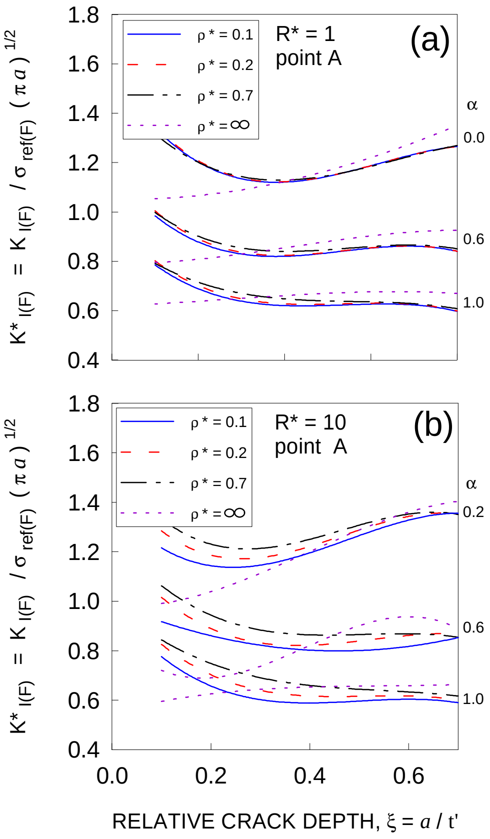

Figure 3: SIF for tension

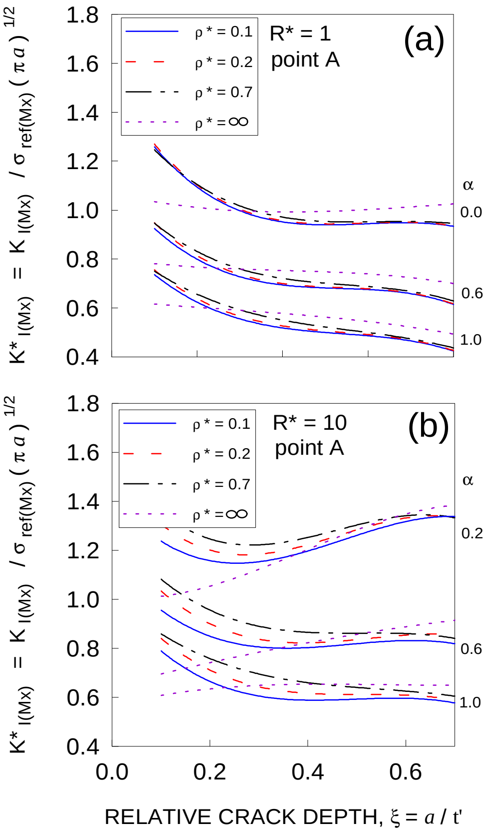

Figure 4: SIF for bending

minimum tends to disappear for high values of α .

Note that the influence of SCF on the SIF results is remarkable especially for small relative crack depths. For high values of ξ , the effect of the stress concentration on the SIFs tends to vanish, that is, the fracture behaviour of a notched pipe becomes similar to that of a smooth pipe.

Some results for a pipe under bending are reported in Fig. 4. For a thick-walled pipe (Fig. 4a), the SIF at point A is always decreasing by increasing ξ for high values of α , while it tends to present a minimum only for low values of α and low values of r^* (i.e. high values of K_t). The behaviour of a thin-walled pipe under bending (Fig. 4b) is similar to that of a tensioned pipe (Fig. 3b), and the same conclusions can be drawn.

CONCLUSIONS

The behaviour of a part-through-cracked notched pipe under tension and bending has been examined. In order to obtain the stress-intensity factor (SIF) distribution for different values of the stress intensity factor (SCF) and several crack geometries, a finite element analysis has been performed.

The notch effect on the SIF is remarkable especially for small values of the relative crack depth. The diagrams of SIF against the relative crack depth show that there exists a crack depth value, depending on the SCF and the crack aspect ratio, for which the SIF attains a minimum.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the research support for this work provided by the Italian Ministry for University and Technological and Scientific Research (MIUR).

REFERENCES

1. Raju, I.S., Newman, J.C. Jr. (1986) In: *Fracture Mechanics*: *Seventeenth Volume,* ASTM STP 905, pp.789-805, 1986.
2. Lorentzen, T., Kjaer, N.E. and Henriksen, T.K. (1986) *Engng Fract. Mech*. **23**, 1005-1014.

3. Carpinteri, A. (1994) In : *Handbook of Fatigue Crack Propagation in Metallic Structures*, pp. 653-705, Carpinteri, A. (Ed.), Elsevier Science Publishers B.V., Amsterdam.
4. Lin, X.B., Smith, R.A. (1997) *Int. J. Pressure Vessels Piping* **71**, 293-300.
5. Carpinteri, A., Brighenti, R. (1998) *Engng Fract. Mech.* **60**, 383-396.
6. Carpinteri, A., Brighenti, R. and Spagnoli, A. (2000) *Fatigue Fract. Engng Mater. Struct.* **23**, 467-476.
7. Lin, X.B., Smith, R.A. (1998) *International J. of Mechanical Sciences* **40**, 405-419.
8. Lin, X.B., Smith, R.A. (1999) *Int. J. of Fatigue* **21**, 965-973.
9. Zhang, Q.Y., Liu, Y. (1992) *Int. J. Pres. Ves. & Piping* **51**, 361-372.
10. Liu, Y., Zhou, S.S. and Cai, Sh.T. (1993) *Int. J. Pres. Ves. & Piping* **53**, 393-404.
11. Liu, H.W. (1983) *Mechanics of Materials*, Advanced Education Publishers, Beijing, pp. 20-52.
12. Cole, A.G., Brown, A.F.C. (1958) *J. Roy. Aeronaut. Soc.* **62**.
13. Leven, M.M., Frocht (1953) *Proc. Soc. Exp. Stress Anal.* **11**, No. 2.