A Unified Description of Nucleation, Propagation, and Proliferation of Cracks under Creep Conditions

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ABSTRACT*: In the paper the results of the extension of previous analyses of cracking of metallic structures in creep conditions has been presented. A comparison between Continuum Damage Mechanics (CDM) and Fracture Mechanics (FM) was aimed for. It proofed capabilities of CDM to detect localisation of first macrocrack and its further development until final collapse of a structure occurred. Form practical point of view it has been shown that the analysis within limited time scale is sufficient for evaluation of safety factor defined as a ratio of time of crack proliferation to time of its initiation. Thus, the fields of CDM and FM applications obtained again their firm definition of practical usefulness.*

INTRODUCTION

Process of failure of structures caused by crack development is very complex one. It consists of three characteristic stages: nucleation, propagation and proliferation of the cracks throughout the whole structure. The possibility of distinction of these stages depends on different factors like stress inhomogeneity, time-dependent properties of a material or loading regimes (e.g. cycling loading). For example, in the case of uniaxial tension of a bar made of elastic material submitted to constant load, all three above stages practically coincide. In engineering, complex structures, however, the distinction between consecutive stages allows for a quantification of failure process, and, therefore, on determination of a safety margins. From this point of view it bears an important practical implications.

In the present paper, the plates under constant external pressure (resulting in three-dimensional state of stress) made of a material that exhibits distinctive rheological properties (steel under high temperature) will be considered. In this case the characteristic stages of failure process can be identified by three characteristic time intervals.

- nucleation period ($0,\ t_1$), where t_1 is the time of first macrocrack appearance,
- propagation period ($t_1,\ t_2$), where t_2 is the time of first crack to span a characteristic length (plate's thickness),
- proliferation period ($t_2,\ t_3$), where t_3 is the time of final collapse of a structure caused by forming a critical network of the cracks.

It is generally accepted that the nucleation period can be described by means of Continuum Damage Mechanics (CDM), which can answer questions on time and space location of the first macrocrack. The series of works by the authors of present paper has demonstrated that CDM can be applied to the analysis of the second stage of failure process [1,2,3]. Here, even further extension of damage mechanics is made, to describe the last stage of failure process, and – in such a way – to build a unified description of the whole process.

GOVERNING EQUATIONS

Material properties of the considered structures are assumed to obey theory steady-state non-stationary creep (elastic stress is taken into account) coupled with classical Kachanov-Rabotnov damage theory [4,5]. The damage evolution law will make use of the concept equivalent stress [6], which allows simple generalisation of Kachanov-Rabotnov theory for multiaxial states.

Consequently, the total strain tensor ε_{ij} is decomposed into elastic ε_{ij}^{e} and creep ε_{ij}^{c} components:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{c} \tag{1}$$

and the constitutive equations for strain tensors components and damage variable are:

$$\varepsilon_{ij}^{e} = D_{ijkl}^{-1}\,\sigma_{kl} \tag{2}$$

$$\frac{\partial \varepsilon_{ij}^{c}}{\partial t} = \gamma \left(\frac{\sigma_{eff}}{1-\omega} \right)^{n} \frac{\partial \sigma_{eff}}{\partial \sigma_{ij}} \tag{3}$$

$$\frac{\partial\omega}{\partial t}=A\left(\frac{\sigma_{eq}}{1-\omega}\right)^{m} \tag{4}$$

where: σ_{ij} is the stress tensor, D_{ijkl} the elastic constants tensor, ω the scalar damage parameter, γ, n, A, m creep and damage material constants, t time.

The equivalent stress σ_{eq} in Eq. (4) is given by:

$$\sigma_{eq}=\alpha\,\sigma_{max}+(1-\alpha)\sigma_{eff} \tag{5}$$

where: σ_{max} is the maximal principal tensile stress, and σ_{eff}, the Huber – Mises effective stress, α parameter $(0 \le \alpha \le 1)$ which characterises local failure mechanism mode.

The case of $\alpha=0$ corresponds to ductile (transgranular) fracture controlled by the effective whereas for $\alpha=1$ the brittle (intergranular) fracture governed by the maximal principal tensile stress occurs. The intermediate values of α correspond to mixed modes of failure.

One can observe that governing equations used consists of evolution laws for deformation and deterioration variables are as simple as possible but reflecting main characteristic features of processes i.e. its time dependence, and coupling between deformation and deterioration. The latter allows for the description of stress redistribution due to damage growth from initial elastic state to zero (loss of a bearing capacity) at instant of time when damage reaches its critical value in a given point. In such a way fully damaged material point is excluded from further considerations.

The above constitutive equations completed together with equilibrium and compatibility equations form the set of problem governing equations which for given initial and boundary conditions make possible to describe stresses, strains, displacements and damage development history. It means that they allow for effective solving of a problem of description the whole process of cracking structures.

NUMERICAL PROCEDURES

The set of problem governing equation has to be solved using discretisation technique. The Finite Element Method for structure discretisation and Euler's procedure for time integration was used. In the computer code the layered isoparametric eight-node Serendipity shell elements with reduced integration were employed. Ten layers and two-point Gaussian quadrature for volume integration were adopted. The time t_1 is identified with $\omega = 1$ condition fulfilled in any layer and Gaussian point (that is numerical integration point). For time $t > t_1$ calculation those numerical integration points were excluded from further integration. When critical condition for damage parameter is reached in all ten layers of a Gaussian points the time is referred to as t_2. Time of structure collapse t_3 were identified with critical values of damage parameter in a whole finite element, which in turn lead to instability in numerical calculations.

Details of the algorithm and conditions for numerical stability can be find elsewhere [1].

RESULTS OF NUMERICAL CALCULATIONS

As an example rectangular plates under constant pressure uniformly distributed over its upper surface were analysed. Two types of plate support were considered, namely simply supported along all edges, and those with all edges clamped. The plates have thickness 0.10 m and sides length equal to 1.0 and 2.0 m. The material of the plates was *Ti-6Al-2Cr-2Mo* alloy whose material constants at a temperature of *675 K* are given in paper [7]: $E = 0.102*10^6$ *MPa*, , $n = 6.8$, $m = 5.79$, $A = 1.08*10^{-20}$ $(MPa)^{-m}$ h^{-1}, $\gamma=1.38*10^{-24}$ $(MPa)^{-n}$ h^{-1}. For this material $\alpha = 0.5$, however in the paper it was assumed to take also values of *0* and *1*, since it seemed to be interesting to reflect the influence of local failure mechanism modes upon development of cracking in the structures.

To make possible a comparison of results and to draw some conclusions the loading for analysed plates was chosen in such way that in all cases maximum equivalent stress at time zero had the same value.

Some numerical results of analysis are summarised in the Tables 1 and 2.

TABLE 1. Results of analysis for simply supported plates

Load p (MPa)	α	t_1 (10^5 hrs)	t_2 (10^5 hrs)	t_3 (10^5 hrs)	t_2/t_1	t_3/t_1
10.323	0	0.421188751	0.457839897	0.457844314	1.087018	1.087029
9.583	0.5	0.533371647	0.664524815	0.664524826	1.245894	1.245894
8.942	1	0.430631366	0.560250140	0.560250337	1.300997	1.300998

TABLE 2. Results of analysis for clamped plates

Load p (MPa)	α	t_1 (10^5 hrs)	t_2 (10^5 hrs)	t_3 (10^5 hrs)	t_2/t_1	t_3/t_1
13.249	0	1.056595640	1.264425990	1.266108610	1.196698	1.198291
12.510	0.5	0.870886386	1.972037870	1.972039220	2.264403	2.264405
11.850	1	0.521956624	1.359254660	1.359255460	2.604152	2.604154

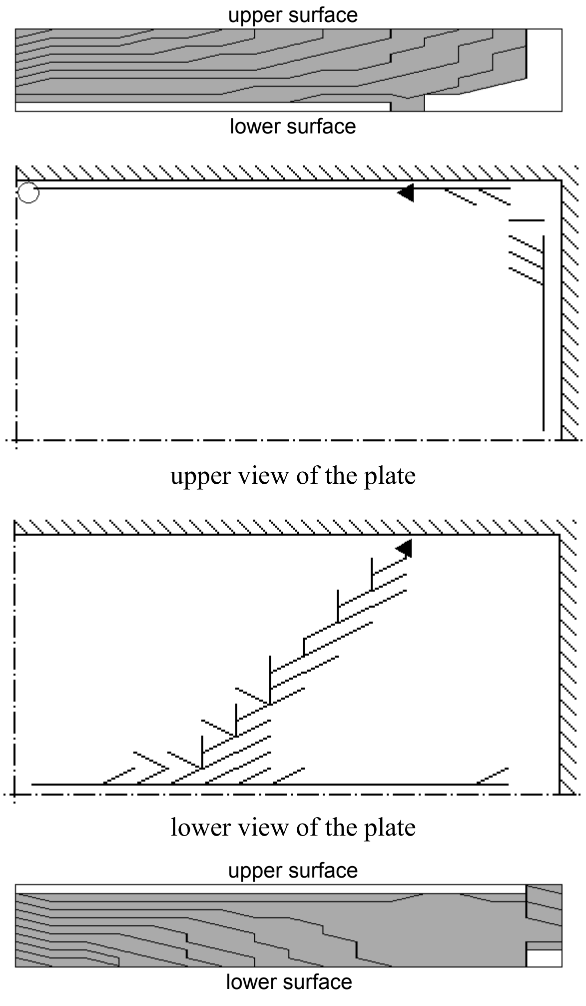

Figure 1: Surface cracking at time t_2.

In Fig. 1 upper and lower surfaces of one of the analysed plate (clamped plate, $\alpha = 1$) are shown at time t_2. The networks of macrocracks is shown also, with indication of their onset marked by ❍, and through-body proliferation at time t_1 marked by ▲. Though these cracks are seen as surface ones, in fact they penetrate the body of a structure. Profiles of the cracks along two cross sections (along clamped edge and through a mid-span of the plate) are shown in Fig.1 as well.

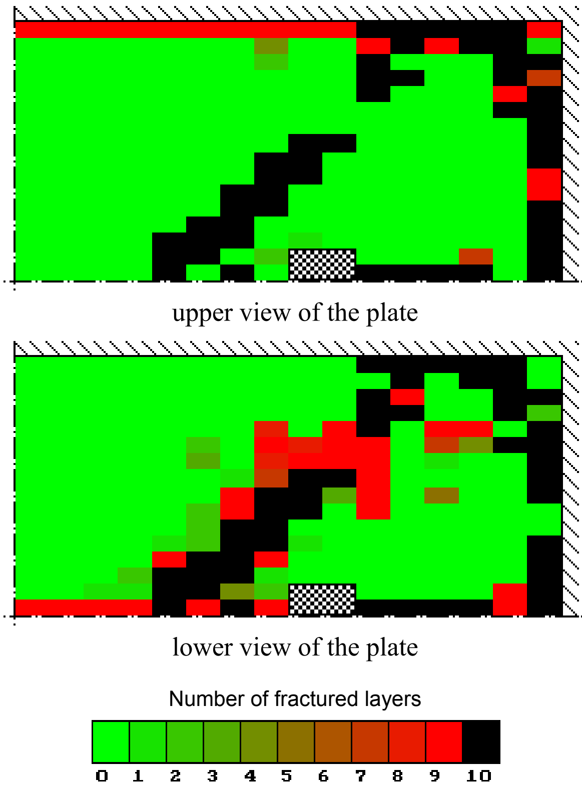

Figure 2: In-deep penetration of cracks at time t_3.

In Figure 2 depth of cracks penetration at time t_3 is indicated by different colour pattern of Gaussian points at which damage has occurred. The point determining time t_2 is shown in black, whereas the element in which all four Gaussian points suffered annihilation in time t_3 is marked by black and white chessboard pattern. A scale as indicated in Figure 2 reflects the depth of crack penetration at other points.

CONCLUSIONS

The main goal of the present paper was to extend previous analyses of plate cracking over all three periods of deterioration process marked by milestone times: t_1, t_2, and t_3. It proofed to be possible, though essential numerical problems have to be overcome. In such a way, a capability of CDM to predict not only initiation of macrocracking, but also further cracks' development has been demonstrated. It is worthwhile to mention that crack growth, usually attributed to FM, can be covered by a consistent, unified description in the frame of CDM.

A great advantage of CDM is approach, fully exploited in the present analysis, is twofold: first, the location for a point at which a macrocrack initiates comes out as a result of analysis. No assumption of this location has to be made as unavoidable assumption prior to further analysis, which is a case when FM is to be applied. Further, the direction of a macrocracks and their branching is also included into CDM analysis. Finally, the profiles of the cracks penetrating structure's body can be determined, though one has to admit that the definition of crack looses its clear meaning when a maze of macrocracks grow within material body.

From practical point of view, it seems however, that performing cumbersome and time consuming calculations to find out the values of time t_3 does not pay: the time difference between times t_2 and t_3 are marginal (cf. Tables 2 and 3). This observation conforms well with well known fact that crack propagation velocity is approaching that of sound speed for a given elastic material, and is also very high in a material which exhibit time-dependent properties. Again, the importance of analysis within time scale $(0,t_2)$ comes out: the ratio of times t_2/t_1 (proliferation/initiation) can be used as evaluation of safety factor for structures working in an environment which imposes time dependent processes: deformation and deterioration.

Finally, authors want to underline the fact that all analysis performed was done on the level of a whole structure, but not in a chosen point nor characteristic cross-section of a structure, which is a normal procedure in the case of FM analysis. Moreover, fully coupled constitutive equations allowed for the description of a whole process in a dynamic manner, with fully recorded history of its development. It is true that this requires sophisticated method of numerical analysis, but when limited to time interval $(0,t_2)$ it does not pose essential problems taking into account presently available computing power.

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