STATISTICAL MODEL OF ROUGHNESS INDUCED CRACK CLOSURE

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ABSTRACT

A new model is proposed enabling to assess the influence of structure coarseness on roughness induced crack closure in the fatigue threshold region. It takes the plastic zone size effect into account in terms of the grain size statistical distribution. In this approach, only grains larger then the cyclic plastic zone size are assumed to contribute to the roughness induced crack closure. The validity of this model was verified by a very good reproduction of experimental dependencies of the threshold stress intensity factor ΔK_{th} on the mean grain size obtained on the ARMCO iron specimens. In this analysis, two fitting parameters with clear physical meaning get expected and reasonable values.

INTRODUCTION

Since 1971 [1], the phenomenon of crack closure became very important in elucidating the threshold behaviour and transient effects in fatigue crack growth. Basically, three different mechanisms of crack closure can be distinguished - plasticity, roughness and bridging [2]. The roughness induced crack closure (RICC) is determined by the existence of asperities on fracture surfaces produced by irregular crack path. It was experimentally verified that the contribution of the RICC increases with coarsening of materials structure.

Suresh and Ritchie [3] proposed a simple two-dimensional model based on linear elastic fracture mechanics enabling a semi-quantitative understanding of the mechanism of this phenomenon. Irreversible working of the local mode II at the front of the tortuous crack causes the mutual horizontal shifting of the fracture surfaces and their premature contact during unloading. According to this two-parametrical model, the ratio of the stress intensity factors at the moments of surface contact and peak tensile load, respectively, can be expressed as

$$\frac{K_{cl}}{K_{max}} = \sqrt{\frac{\chi \cot \vartheta}{1 + \chi \cot \vartheta}}.$$

The empirical parameter χ expresses the average ratio of shear to normal displacements in the crack wake. However, it has no clear direct relation to the level of structural coarseness. It holds also for the average tilt angle ϑ between crack branches and the average direction of crack propagation. A numerical analysis made by Llorca [4] assessed the χ value to be in the range of $\langle 0.1, 0.3 \rangle$ in 2124 Al alloy for regularly tortuous crack paths. Thus, in fact, the model is not very useful for quantitative assessment of the role of structure in RICC. E.g., it can not be applied to the quantitative interpretation of the

d_m particularly ascribed to the RICC effect.

More recent model introduced by Wang and Miller [5] deals with an irreversible dislocation pile-up adjacent to the crack tip in order to assess the mode II displacement in dependence on the mean grain size. This one parametrical approach requests a complicated analysis of the fracture surface roughness to be made and, moreover, the assumption of planar slip mode is necessary for its validity. Again, the model fails to predict the RICC effect directly from the materials structure and can be applied under very special conditions only.

The aim of this article is to introduce a new model of the RICC enabling the direct quantitative assessment of its level according to a simple structural analysis. It develops the Suresh and Ritchie model to a more detailed form and, particularly, statistically introduces the plastic zone size effect into the consideration. The experimental verification is based on measurements performed by Pippan et al. [6, 7] on ΔK_{th} vs. d_m dependence for ARMCO iron. The experiments were completed by the statistical evaluation of the grain size distribution.

THE BASIC IDEA OF THE STATISTICAL MODEL

It is commonly accepted (e.g., [8, 9]) that the microstructure has negligible influence in the stage II of fatigue crack propagation where the cyclic plastic zone size r_{pc} embraces several and more grains. Clearly, large-scale crack tip plasticity in comparison with the characteristic structure size means the

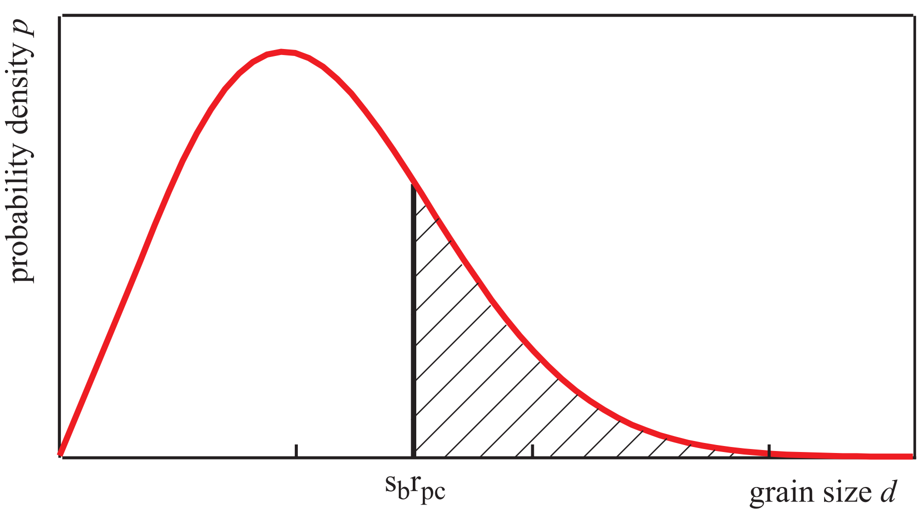

Figure 1: The dashed area under the probability density function corresponds to the relative number of grains involved in the roughness induced crack closure process

easily overcoming of structural barriers by multiple slip and the homogeneous deformation does not allow the formation of large pile-ups. As a result, a very limited and almost reversible mode II slip is to be expected within the relatively small grains inside the large plastic zone. On the other hand, the crack path is very sensitive to the microstructural features in both the stage I and the transient I $\rightarrow$ II growth regime. The crack path controlled by crystalography, secondary phases and, particularly, the grain boundaries dominates here. The r_{pc} is less (or at the most comparable) with that of the mean grain. Obviously, such small-scale plasticity must produce a single (planar) slip followed by crystalographical crack growth accompanied by long dislocation pile-ups initiating either the local plastic strain in the adjacent grains or the local intergranular cracking. Both processes cause a high degree of mode II displacement irreversibility and the mixed trans-intergranular fracture mode is often observed [6, 10, 11]. Therefore, a high level of RICC is present. Thus, the influence of the materials structure can be associated with the range and homogeneity of plastic deformation ahead of the crack tip in relation as the size effect [12, 13]. Further we denote the important structure sensitivity parameter - the ratio of the grain size d to the cyclic plastic zone size r_{pc} - as the size ratio s. The cyclic plastic zone size is preferred before the static one since the reversal dislocation motion is crucial in the mechanism of RICC and in the fatigue damage process as a whole. According to Rice [14] it holds

$$s = \frac{d}{r_{pc}} = \pi d \left(\frac{2\sigma_y}{\Delta K_{eff}} \right)^2 . \tag{1}$$

Here σ_y is the yield strength and both the small-scale yielding and plane stress conditions are to be fulfilled. It should be noted that owing to the butterfly shape (not circular) of the real plastic zone, this relation holds for conditions in between the plain stress and strain. The effective value $\Delta K_{eff} = K_{max} - K_{cl}$ must be used instead of the applied ΔK value to take the crack closure into account.

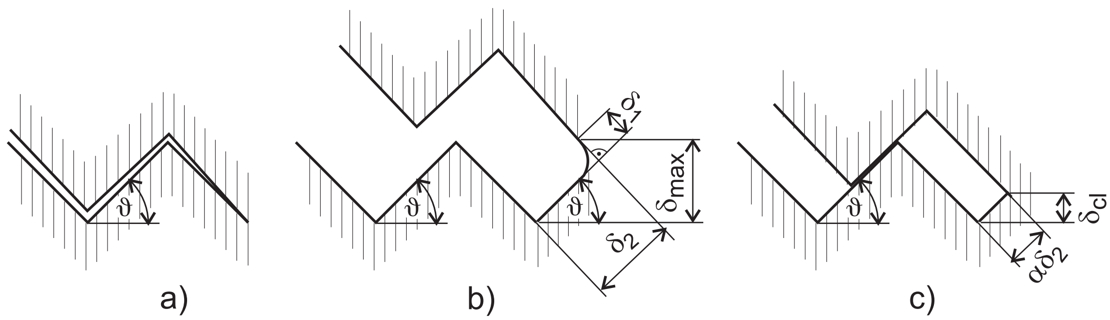

Figure 2: Scheme of the roughness induced crack closure mechanism

In spite of the fact that the size effect is used to be connected with the mean size d_m, it must work also locally. Owing to the enormous grain size scatter in polycrystalline materials, different local deformation modes operate in individual grains with different local values s. Grains much smaller than r_{pc} do not contribute to the RICC mechanism and, on the other hand, grains larger than r_{pc} contribute to the RICC mechanism substantially. The basic idea of the statistical model lies in the assumption that the RICC level is determined exactly by the latter part of the grain size probability density function only. In this model, the grains are simply divided into two main categories - with low and high s values. In

TABLE 1

MECHANICAL PROPERTIES AND LOADING PARAMETERS OF ARMCO IRON [6]

Sample No.	σ_y MPa	R	ΔK_{th} MPa m$^{1/2}$	K_{max} MPa m$^{1/2}$
1	96	0,1	10,3	11,7
2	108	0,1	8,7	9,4
3	150	0,1	6,8	7,6
4	240	0,1	5,3	5,9
5	530	0,1	4,5	5,0
6	96	0,7	3,6	13,3
7	108	0,7	3,2	10,3
8	150	0,7	2,8	9,3
9	240	0,7	2,75	9,2
10	530	0,7	2,75	9,2

grains of the first type the local contribution to RICC can be neglected whereas in grains of the latter type an extended RICC process takes place. This is clearly seen in Fig. 1 where the area under the part (with high s and d) contribute to the RICC. The s value corresponding to the sharp boundary between both types of grains is denoted by s_b (the related grain size $d_b = s_b r_{pc}$). This value should lie somewhere within the transient range $s_b \in \langle 0.5, 2.0 \rangle$ where the cyclic plastic zone embraces the space not much different from one grain.

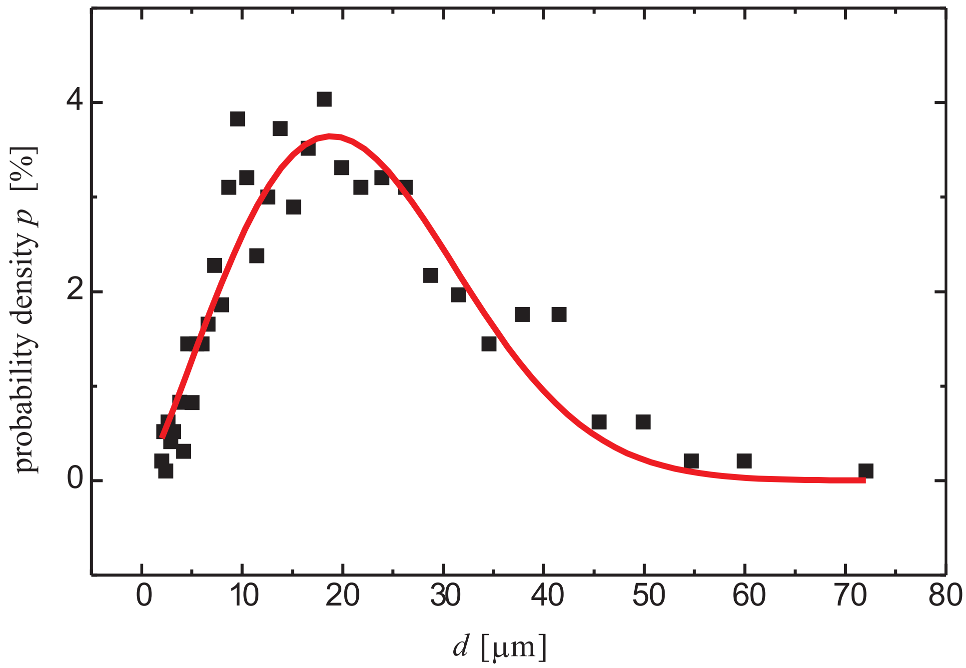

Figure 3: Weibull fit of experimental grain size data for ARMCO iron ($d_m = 20\,\mu$m)

TABLE 2
STATISTICAL CHARACTERISTICS OF THE GRAIN SETS (WEIBULL PARAMETERS μ,ξ, MEAN GRAIN SIZE d_m, STANDARD DEVIATION SD)

Sample No.	(d_m) μm	μ μm	ξ	d_m μm	SD μm
1, 6	(3000)	4520	2.17	3550	1700
2, 7	(500)	1850	2.17	410	200
3, 8	(70)	1170	2.17	90	44
4, 9	(10)	23	2.17	20	10
5, 10	(3)	5	2.17	2	1.0

MATHEMATICAL CONSTRUCTION OF THE MODEL

The two-dimensional zig-zag crack path characterised by the average angle ϑ is assumed to produce the RICC according to the scheme in Fig. 2. During the loading part of the cycle, the crack tip is opened in the I + II mixed-mode reaching the maximum CTOD denoted by δ_{max} at the moment of the peak stress (Fig. 2b). This displacement can be considered to be composed of the reversible normal component δ_1 and the irreversible shear component δ_2. During the unloading phase, consequently, the shear displacement is not recovered so that the crack surfaces come into the contact before the applied stress becomes zero (Fig. 2c). The irreversibility level is denoted as α. For simplicity reasons, $\alpha = 1$ is assumed in grains with $s > s_b$ unlike $\alpha = 0$ in grains with $s < s_b$. The statistically averaged α_m value is considered to be the relevant irreversibility parameter.

$$\frac{\delta_{cl}}{\delta_{max}} = \frac{\alpha_m \delta_2 \sin\vartheta}{\delta_1 \cos\vartheta + \delta_2 \sin\vartheta}. \tag{2}$$

With regard to the well-known relations for the local stress intensity factors k_1 and k_2 at the tip of the small kinked crack [15], one obtains

$$\frac{\delta_1}{\delta_2} = \frac{k_1^2}{k_2^2} = \frac{K_I^2 \cos^6\frac{\vartheta}{2}}{K_I^2 \sin^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2}} = \cot^2\frac{\vartheta}{2}, \tag{3}$$

where K_I is the remote stress intensity factor. It is to stress that Eqn. 2 holds well also for the zig-zag crack path [16]. By substitution to Eqn. 2, it holds

$$\frac{\delta_{cl}}{\delta_{max}} = \frac{\alpha_m}{\cot^2\frac{\vartheta}{2}\cot\vartheta + 1}. \tag{4}$$

The parameter α_m is to be considered in terms of the grain size statistical average. Since the Weibull distribution seems to be the best fit to the real grain size scatter, the probability density is to be used in the following form:

$$p(s) = \frac{\xi s^{\xi-1}}{\mu^\xi r_{pc}} \exp\left[-\left(\frac{s}{\mu}\right)^\xi\right], \tag{5}$$

where ξ and μ are distribution parameters and the mean grain size d_m can be expressed using the Gamma-function as

$$d_m = \Gamma\left(\frac{1}{\xi} + 1\right)\mu r_{pc}. \tag{6}$$

Thus, the final relation for the RICC ratio can be written as

$$\frac{\delta_{cl}}{\delta_{max}} = \frac{K_{cl}^2}{K_{max}^2} = \frac{\int_{s_b}^{\infty} p(s)\mathrm{d}s}{\cot^2\frac{\vartheta}{2}\cot\vartheta + 1}. \tag{7}$$

TABLE 3
RESULTS OF CRACK CLOSURE CALCULATIONS

Sample No.	R	d_m/r_p	K_{cl}/K_{max}	ΔK_{th} MPa m$^{1/2}$
1	0.1	54.37	0.4553	10.0
2	0.1	7.947	0.4365	9.2
3	0.1	3.367	0.3938	7.9
4	0.1	1.916	0.2975	6.1
5	0.1	0.939	0.0621	3.9
6	0.7	54.37	0.4553	3.4
7	0.7	7.947	0.4365	3.1
8	0.7	3.367	0.3938	2.7
9	0.7	1.916	0.2975	2.0*
10	0.7	0.939	0.0621	1.3*

* virtual values

The validity of the statistical model was applied to the quantitative elucidation of the experimental ΔK_{th} vs. d dependence obtained on the ARMCO iron [6, 7]. In Tab. 1, the loading parameters and mechanical properties are summarised. The r_{pc} values calculated according to Eqn. 1 are also in Tab. 1 since the original loading conditions corresponded to the SSY and the transition between plain strain and plain stress. Values $\Delta K_{eff} = 2,75\,\mathrm{MPa\,m}^{1/2}$ were used here [6].

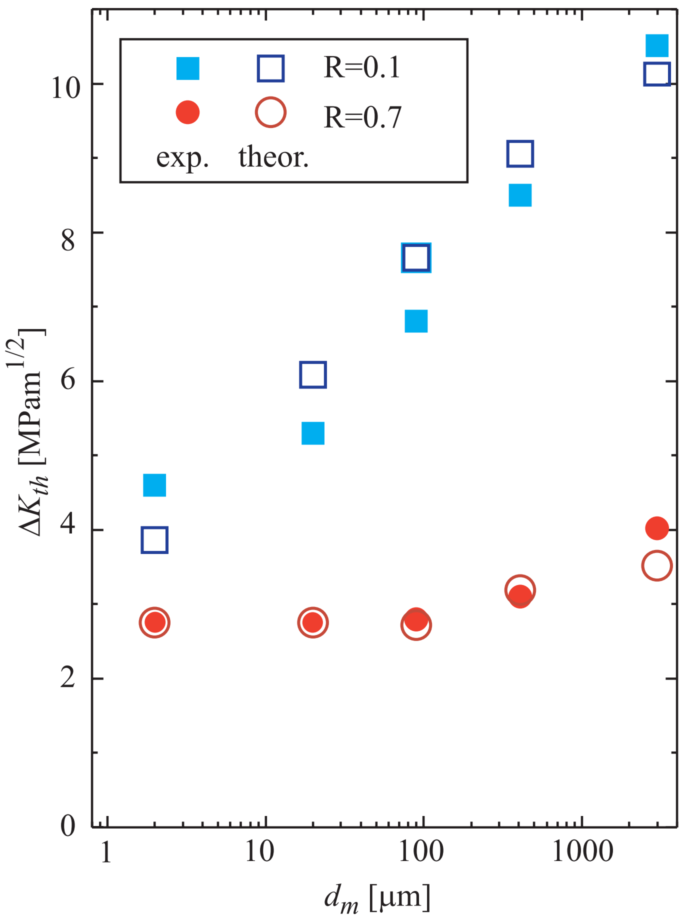

Figure 4: Comparison of experimental [6] and theoretical (Eqn. 8) data on threshold behaviour of ARMCO iron with different mean grain size

The original CT–specimens were repeatedly analysed in order to obtain the exact grain size distribution function. An example of experimental data fitted by the Weibull probability density function is shown in Fig. 3. The new, more exact, d_m values are in Tab. 2 together with the Weibull parameters ξ and μ (the originally estimated d_m values are in parenthesis). The parameter $\xi = 2.17$ was the same for all investigated grades. The K_{cl}/K_{max} values calculated by means of Eqn. 7 are shown in Tab. 3 increased by the constant value of 0.296. This value represents the contribution of another mechanism to the crack closure, most probably the plasticity induced one. It is obtained by extrapolating the theoretical data to $d_m \rightarrow 0$, where no RICC exists. The boundary value $s_b = 2$ and the averaged angle $\vartheta = 50^0$ were used giving the best fit to experimental data. Both values lie within plausible ranges of $s_b \in \langle 0.5, 2.0 \rangle$ and $\vartheta \in \langle 30^0, 60^0 \rangle$. The very good agreement between theoretical and experimental data is documented in Fig. 4.

It is to be emphasised that both the experimental data sets, for $R = 0.1$ and $R = 0.7$, were fitted by the single Eqn. 7. Values of the two fitting parameters s_b and ϑ are very reasonable and possess a clear physical meaning.

The power of presented statistical model can be clearly demonstrated by the plot K/K_{max} vs. d_m of calculated data as it is shown in Fig. 5. The arrows on the right hand side of Figs. 5a,b indicate loading ranges in both the $R = 0.1$ and $R = 0.7$ cases. During the loading cycle, the crack remains closed within the area shadowed by dashes and dots. The curve connecting K_{cl}/K_{max} points reaches the value of nearly 0.3 at the small-grain limit, where no RICC is expected to work. Thus, the hatched area corresponds to the RICC operation and the dotted area to the plasticity induced part of crack closure effect. The vertical lines show the range of opened crack corresponding to the $\Delta K_{eff}/K_{max}$ value. In case of $R = 0.1$ (Fig. 5a), a significant part of shadowed closure area (and all the K_{cl} values) lie above the K_{min} of the loading cycle. It means that extended crack closure appeared in each materials grade. In case of $R = 0.7$, however, practically all the closure area lies below the K_{min} - see Fig. 5b. Consequently, no crack closure appeared in fine grained samples No. 9 and 10 during this type of loading (see also Tab. 3) and the crack closure in coarse-grained samples is practically negligible. It exactly reproduces the experimental measurement as it is clearly visible in Fig. 4.

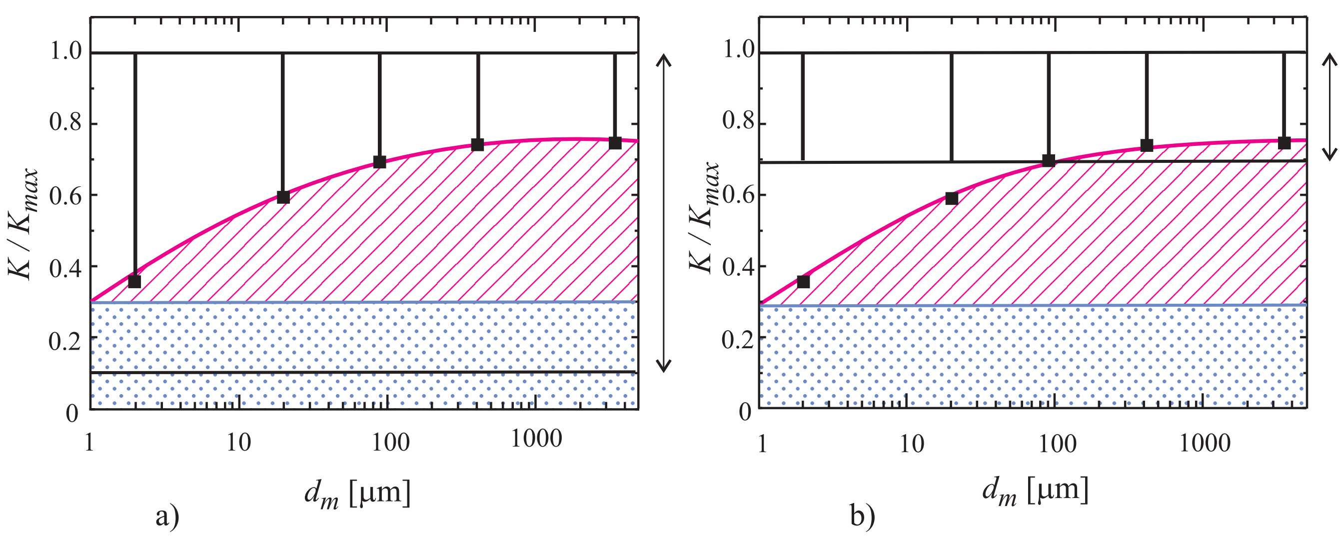

Figure 5: Scheme of the crack closure during the loading cycle with a) $R = 0.1$; b) $R = 0.7$. Within the shadowed area the crack is closed. The hatched part shows the amount of RICC contribution

By substituting $\vartheta = 50^0$, $s_b = 2$ and $\xi = 2.2$ into Eqn. 7, the crack closure ratio can be, in case of ARMCO iron, written as the following approximate formula:

$$\frac{K_{cl}}{K_{max}} = 0.45 \exp\left[-\left(\frac{1.3 r_{pc}}{d_m}\right)^{2.2}\right]. \tag{8}$$

Thus, the RICC level near the threshold can be estimated only by knowing the mean grain size and the cyclic plastic zone size. The latter value can be obtained from Eqn. 1 by knowing the yield strength and the threshold effective stress intensity range. The values of ϑ, s_b and ξ are expected to be in a close range for a wide class of steels. We believe, therefore, that the Eqn. 7 (and probably also Eqn. 8) can be used, at least, for mild steels. Anyway, the experimental verification is obviously needed.

The main results of this work can be summarised into the following points:

1. A new model was proposed enabling the quantitative assessment of roughness induced crack closure. In this approach, the plastic zone size effect is taken into account in terms of grain size statistics.
2. The validity of this model was approved by the very good reproduction of experimental ΔK_{th} vs. d_m dependencies for ARMCO iron.

It is to believe that also the threshold data for mild steels could be successfully analysed by this approach.

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