Propagation of Surface Cracks Under Cyclic Loading

T. BOUDAOUD, N. OUALI, A. AHMED-BENYAHIA and T. BOUKHAROUBA.

Laboratory of Mechanical Reliability and Materials (LF2M), IGM–USTHB, BP 32
El-Alia, 16111 Bab-Ezzouar, Algiers / Algeria. Fax : (213 2) 24 71 82.

Abstract: In this work we propose to predict the evolution of the semi-elliptical surface crack geometry in fatigue, contained in thick plates and in thick tubes under internal pressure. The use of the global approach, based on the minimization of the potential energy, permits us to evaluate the energy release rate, the global stress intensity factor and to follow the evolution of the two half-axes of the crack during its propagation, this without the recourse to use a propagation law. A count code based on a three dimensional finite element method has been made by us. The energy release rate so calculated from experimental data permits us to predict with a very high precision the evolution of the crack geometry during its evolution under two types of loading.

1. Introduction

The life duration of high security structures, obliges us to predict the evolution of crack shape which are contained in the structures or appeared in service. It can be made by theoretical prediction, by numeric simulation or by the mechanical tests. One of the most present cracks shapes in the mechanical structures, are the semi-elliptical shape, this type of defects presents a particular complexity by the fact of its geometry and by the difficulty of choice of a three dimensional propagation law in fatigue, binding all mechanical and geometrical parameters. Several authors tried to predict the shape evolution of semi-elliptical surface cracks under cyclic loading, contained in thick plates and in thick pipes. Three methods were used :

1. the first, uses the local or average stress intensity factor and the law of propagation proposed by Paris, called local and average approach respectively,
2. the second based on the determination of the effective stress intensity factor and the utilization of a propagation law in order to define the relation ships propagation by parameters C and m at the deepest point and at the surface,
3. the third uses the different empirical, giving the evolution of the two half-axes of the semi-elliptical crack during the propagation of this one.

In this work, we propose to use a fourth method, the global approach. This latter is based on the minimization of the potential energy. The minimal potential energy available permits to determine the energy-release rate as well as the global stress intensity factor.

A three dimensional finite-element method with experimental results has allowed us to conclude that the energy-release rate so calculated, permits to describe with a good accuracy the evolution of the two half-axes ratio of semi-elliptical surface crack, in thick bending plate and in thick pipe submitted to internal pressure.

We suppose that the material is elastic and that the crack remains semi-elliptic during the propagation, which is verified experimentally. From the initial values of the two half-axis a_0 and c_0, we supposes several fictional crack fronts with the same incremental change in crack area ΔA and we calculate for each shape the energy-release rate.

We notice that values of the potential energy available $\Delta\Pi$. For different fictional crack fronts, relatively to the parameter (c) show a parabolic curve with a minimum. We considers that the crack propagation yields maximum the energy-release rate G, which corresponds to the real crack front a_1 and c_1. Using the same procedure we can follow the progression of a crack until $G = G_c$.

2. Fatigue propagation law

In fatigue, the propagation law expresses the growth rate by cycle of the crack surface $\frac{\delta A}{\delta N}$ according to the stress intensity factor K_i, the integral of Rice J or the energy-release rate G [1, 2] :

$$\frac{\delta A}{\delta N} = f\left(K_i, \text{J or G}\right) \qquad (1)$$

Generally, we use the Paris propagation law that is simple and easy in its application, for which formulation is :

$$\frac{\delta A_i}{\delta N} = C_i \left(\Delta K_i\right)^{m_i}, \qquad (2)$$

With C_i, m_i, are the characteristic coefficients of the material, to determine by basis experiences on normalized specimens.

In 2D case, the problem of the choice of the stress intensity factor and the coefficients of Paris law is not ask, because the propagation of the crack depends only on one parameter, that is the growth rate of the crack length da. In the case of surface cracks, several problems can be posed :

- what propagation law to use ?
- if we use Paris law, what variable A_i, is it necessary to take ?
- what stress intensity factor is necessary to take in consideration in the calculus local, average, effective or global ?
- is it necessary to use energy-release rate G, instead of the stress intensity factor for the determination of the life duration and the evolution of the crack geometry during its propagation?
- what are the values of coefficients C_i and m_i of the propagation law is it necessary to take in consideration ?

3. The utilization of the stress intensity factor

The stress intensity factor is the first parameter used in approaches to predict the evolution of surface crack shape. Three approaches are used, to know: the local approach, mean, and effective.

3.1. Local Approach

The local approach uses the local stress intensity factor as a propagation criteria, as the growth rate of each point on the crack front depends on the stress intensity factor amplitude which is calculated in this point :

$$\left(\frac{\delta R_i}{\delta N} = C_i \left(\Delta K_i\right)^{m_i}\right), \qquad (3)$$

Generally, for the symmetrical pieces in geometry and loading axis containing a semi-elliptical crack, we only consider the two half-axes of the ellipsis (a, c):

$$da/dN = C_a \left(\Delta K_a\right)^{m_a} \quad \text{and} \quad dc/dN = C_c \left(\Delta K_c\right)^{m_c} \qquad (4)$$

Several authors [3 to 8] noticed that $C_a \neq Cc$ and that $m_a = m_c$.

For the calculus of the local stress intensity factor, different methods were used. In according to their results, we notice a great dispersion of stress intensity factor values, which reaches 80% [9]. One of the more used formulations, for the calculus of the local stress intensity factor, is that proposed by Newman Raju [6], established from a fitting of the results based on the finite element method, below :

$$K_I = \left(S_t + H\,S_b\right)\sqrt{\pi\frac{a}{Q}}\,F\left(\frac{a}{t}, \frac{a}{c}, \frac{c}{b}, \phi\right) \qquad (5)$$

H, F and Q represent the geometric correction functions and ϕ represents the parametric angle of the ellipsis. For the case of a tubes under internal pressure, similar formulation has been proposed [10] :

$$K_I = \sqrt{\pi\frac{a}{Q}}\,F_i\left(\frac{a}{t}, \frac{a}{c}, \frac{t}{R}, \phi\right) \qquad (6)$$

By using the formulations (5 and 6), the prediction of the crack shape evolution remains bound to the choice and the use of a propagation law and its coefficients. The difference between the value of coefficients (C_a and C_c) makes Newman and Raju [6] to propose the following relationship :

$$C_c = 0.9^m \, C_a \tag{7}$$

then :

$$\Delta a = \frac{C_a}{C_c}\left(\frac{\Delta Ka}{\Delta Kc}\right)^m \Delta c = \left(\frac{\Delta K_a}{0.9\,\Delta K_c}\right)^m \Delta c \tag{8}$$

Mahmoud and Hosseini [7] noticed that the correction of Newman and Raju gives a better prediction of the geometry. On the other hand, for $C_c = C_a$ the evaluation of the life duration is better. These same authors, proposed a propagation law for bending plates, by combining the formulation of the local stress intensity factor established by Newman and Raju and the Paris law, given by :

$$C_c = 0.9^m \, C_a \; : \; \Delta a / \Delta c = \left[1.1 (a/c)^{0.5} \left(1.1 + 0.35 (a/t)^2\right) H_1 / H_2\right]^{-m} \tag{9}$$

$$C_c = C_a : \; \Delta a / \Delta c = \left[(a/c)^{0.5} \left(1.1 + 0.35 (a/t)^2\right) H_1 / H_2\right]^{-m} \tag{10}$$

3.2. Average approach

The average approach use the average stress intensity factor range $\Delta \overline{K}$ defined by Cruse and Besuner [11]; calculated from the values of the deepest point (a) and at surface (c), (figure 1).

$$\left(\overline{\Delta K_a}\right) = \frac{1}{\Delta S_a} \int_{\Delta S_a} \Delta K_i^2(\phi)\, dS_a$$

(11)

$$\left(\overline{\Delta K_c}\right) = \frac{1}{\Delta S_c} \int_{\Delta S_c} \Delta K_i^2(\phi)\, dS_c$$

Using a propagation law in fatigue, like that of Paris, the values of the growth velocity of the length of the two half-axis length can be determined by :

$$da/dN = C_a \left(\overline{\Delta K_a}\right)^{m_a} \quad \text{and} \quad dc/dN = C_c \left(\overline{\Delta K_c}\right)^{m_c} \tag{12}$$

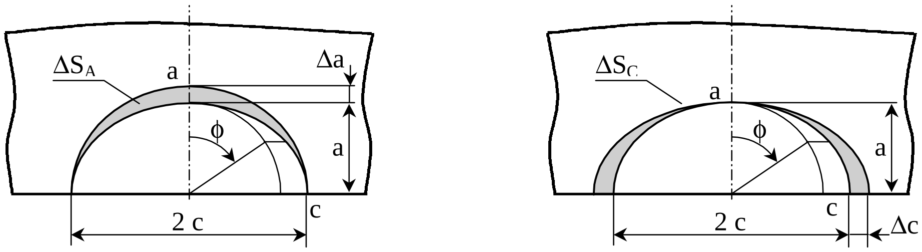

Figure.1 : Definition of the surface crack growth : ΔS_A et ΔS_C given by Cruse and Besuner

3.3 Effective Approach

Of the fact that the values of C_a and C_c are different on the crack front, some authors [12] proposed corrections on the stress intensity factor range ΔK_i (equation 2), and not on the coefficients of the Paris law C_i, by defining an effective stress intensity factor range:

$$\Delta K_{e,i} = \gamma_i \, \Delta K_i \; = C\left(\gamma_a \; \Delta K_a\right)^{m_a} \tag{13}$$

i: represent a point of the crack front and γ_i the correction factor, which is determined experimentally the use of the correction factor is justified by taking into account some complex phenomena like the plastification of the crack front. To predict the evolution of the crack shape, we use the Paris law, applied to the deepest point (a) and in the surface point (c):

$$da/dN = C_a \left(\Delta K_a\right)^{m_a} = C\left(\gamma_a \, \Delta K_a\right)^{m_a}$$

(14)

$$dc/dN = C_c \left(\Delta K_c\right)^{m_c} = C\left(\gamma_c \, \Delta K_c\right)^{m_c}$$

with : $\eta = \gamma_a / \gamma_c = \left(C_a / C_c \right)^{\frac{1}{m}}$ (15)

Authors [12] adopt the value of 1.1 for η. we notices that this approach coincides with the one proposed by Newman and Raju $C_c = 0.9^m \; C_a$.

4. Empirical Relations

Because of the insufficiencies and the contradictions of previous methods, several authors [13 to 18], proposed an empirical relation ships giving the evolution of the semi-elliptic crack geometry during its propagation for the case of bending and traction. For the case of tubes submitted to internal pressure, rare are the experimental and empirical results. Generally we supposes that the cracks contained in tubes propagate like the cracks contained in plates under a traction loading. The different formulations giving the evolution of (a/c) = f(a/t) are regrouped in the table (1).

Table.1: Different empirical formulations giving the evolution of (a/c) in function of (a/t).

Plates in bending	Tubes submitted to internal pressure
Portch [13] $(a/c)_i = [1.05-(a/t)_i] - [1-(a/t)_i]^4 (1.05-\lambda_P)$ for $(a/t)_i \geq (a/t)_b$ $(a/c)_i = I(a/t)_i$ for $(a/t)_i < (a/t)_b$ $(a/t)_b = \frac{3.2 - I}{3(I+1.0)}$	**T. Boukharouba [18]** $\left(\frac{a}{c}\right)_i = \left[A - 0.75\left(\frac{a}{t}\right)_i\right] - 0.000727\left[\left(\frac{a}{t}\right)_i - \lambda_B\right]^{-2.6}$ A constant and λ_B depend of the initial values of a and c.
	Plates in tension
Kawahara [14] $a_i = 1.05 \Big/ \left\{ \left(c_i^2 - \lambda_K^2\right)^{-0.5} + (1.0/t) \right\}$ λ_K is calculated from the initial values a_0 and c_0	**Portch [13]** $(a/c)_i = [1.1 - 0.35(a/t)_i] - [1-(a/t)_i]^4 (1.1-\lambda_P)$ for $(a/t)_i \geq (a/t)_b$ $(a/c)_i = I(a/t)_i$ for $(a/t)_i < (a/t)_b$ and $(a/t)_b = \frac{4.05 - I}{3(I+0.35)}$
Gorner and al. [15] $\left(\frac{a}{c}\right)_i = \left[1.05\left(\frac{a}{t}\right)_i - \left(\frac{a}{t}\right)_i^2\right] \left\{ \left(\frac{a}{t}\right)_i^2 + \mu\left[1.05 - \left(\frac{a}{t}\right)_i\right]^2 \right\}^{-0.}$ μ is calculated from a_0 and c_0	**Gorner and al. [15]** $\left(\frac{a}{c}\right)_i = \left[0.98\left(\frac{a}{t}\right)_i - 0.06\left(\frac{a}{t}\right)_i^2\right]$ $\left\{ \left(\frac{a}{t}\right)_i^2 + \mu\left[1.05 - 0.06\left(\frac{a}{t}\right)_i\right]^2 \right\}^{-0.5}$ μ is calculated from a_0 and c_0
Iida [16] $\left(\frac{a}{c}\right)_i = \left[0.85 - 0.75\left(\frac{a}{t}\right)_i\right] \pm 0.0063\left[\left(\frac{a}{t}\right)_i - \lambda_I\right]^{-3.8}$ λ_I is a constant calculated from the initial values of a and c, the sign (+) is for the case a/c > 0.85-0.75a/t, and the sign (–) is for a/c < 0.85-0.75a/t.	**Iida [16]** $\left(\frac{a}{c}\right)_i = \left[0.78 - 0.07\left(\frac{a}{t}\right)_i\right] \pm 0.00834\left[\left(\frac{a}{t}\right)_i - \lambda_I\right]^{-2.7}$ λ_I is a constant calculated from the initial values of a and c, the sign (+) is for the case a/c > 0.85-0.75a/t, and the sign (–) is for a/c < 0.85-0.75a/t.
T. Boukharouba [17] $\left(\frac{a}{c}\right)_i = \left[A - \left(\frac{a}{t}\right)_i\right] - 0.0063\left[\left(\frac{a}{t}\right)_i - \lambda_B\right]^{-5.65}$ A is calculated from the initial values of a & c and λ_B is a constant depending on the final shape of the defect $(a/c)_{fin}$ et $(a/t)_{fin}$.	

5. Global approach

We noticed the physical justification deficiency and the insufficiency in the quoted approaches (points 3 and 4) to predict the evolution of the surface crack geometry. Besides we noticed also the dispersion of results and sometimes their contradiction. We proposes the global approach as a criteria of crack propagation taking into account the minimization of the potential energy Π of the structure, which corresponds to a maximization of the energy release-rate G.

Thus, an initial shape crack of dimension (a_1, c_1) propagate with an increment dA, toward a final shape of dimension (a_2, c_2), among all possible shapes. This last shape minimizes the potential energy of the cracked structure. Applying this Principle, we can construct a procedure to follow-the crack shape evolution during its propagation, without having recourse to a propagation law, that should be used for the calculus of the fatigue life of the structure life duration. For our case, we worked out a code of calculus based on a three-dimensional finite-element method, with a procedure of minimization of the potential energy. In the case of the symmetry of geometry and loading, we formulate the hypothesis that the crack remains semi-elliptic during its propagation, who is verified experimentally [3, 7] and that the material is elastic linear. Then, calculus have been effectuated to follow the crack evolution in the case of a thick plate in bending and in a thick tube submitted to internal pressure.

5.1. Computation Method

From an initial defect definite by its half-axes (a_0, c_0), we calculated the potential energy for every studied structure :

$$\Pi = U - W \quad (16)$$

Π : Potential energy, W: Work of external loads and U: elastic strain energy.
For our calculus code we use the formulation :

$$W = \sum F_i \, q_i = \{q\}^t [K]\{q\} \quad (17)$$

F : external loads, q : displacements, and K: rigidity matrix.
we suppose several possible fictional crack fronts (a_i, c_i) with the same surface increment ΔA (figure 2).
for every fictional front, we calculate the potential energy variation ΔP or the energy release rate G using the perturbation method :

$$\Delta W \approx -\{q\}^T [\Delta K]\{q\} = -\sum F_i \cdot q_i \quad (18)$$

$$G = -\frac{d\Pi}{dA} \approx -\frac{\Delta \Pi}{\Delta A} = \frac{1}{2} \cdot \frac{\Delta W}{\Delta A} \quad (19)$$

We notice that the energy variation for the two studied load cases for the different fictional front, represents a parabolic curve, with a minimum, which has been predicted. The values corresponding to this minimum indicate the new real crack front a_1 and c_1, with the same way, we can follow the progression of a crack until reaching a critical crack size (figure 2) using the criterion ($G < G_c$ or $K < K_{glo}$).

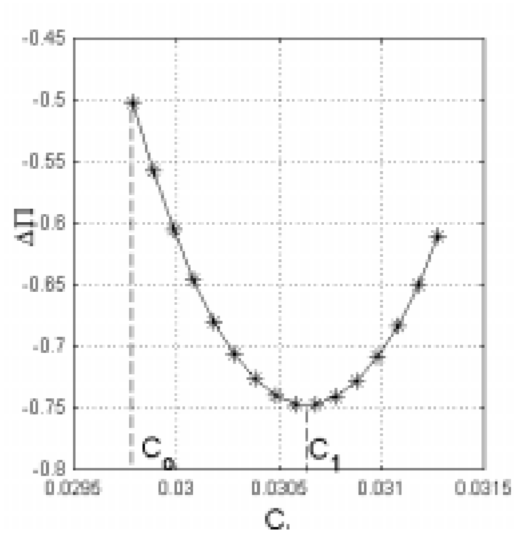

Figure 2 : Variation of the potential energy for the different fictional fronts

6. Results

6.2. Bending plate

the plate used in the present study is submitted to three points bending, with a maximal load of 90 KN and ratio load R of 0.1, 0.3, and 0.5. Because of symmetry, the modelling has been done only for the quarter of the piece, the used element is a 20 nodes isoparametric solid. the central nodes of the adjacent elements, at the crack front, are moved back of a quarter of the elements dimension towards the singular point [1, 6] the total number of elements is 285 with a total number of 1543 nodes to assure a good convergence this sufficient number, is inferior to the number of elements used for the computation of the local stress intensity factor. The stability of this procedure is based on the choice of the ratio value of the element height (h) by the default depth as well as the increment ΔA. in our case we adopt the following values : h varies from a/10 to a/20, and ΔA varies from A/20 to A/40; The evolution results of the two ratios (a/c) and (a/t) of the crack are represented on the figure (3).

6.3. Tube submitted to internal pressure

we have considered a thick tube, submitted to a maximal internal pressure of 420 bars with a ratio of R = 0.3. containing an identical default as the plate, we have considered the quart of the cylinder. The modelling has been done with the same type of elements as in the plate. The total number of elements is 360 with 1973 nodes. The height of element on the crack front and the surface increment for the perturbation method are respectively h = (1/15)a and ΔA = (1/30)A. The figure(4) shows the computation results compared to the experimental curve. We notice that the crack shape evolution converges towards a quasi-horizontal asymptote which is less inclined than the bending plates. Figure (4) evolution Prediction of the ratio (a/c) according to (a/t) in the case of Tube submitted to internal pressure.

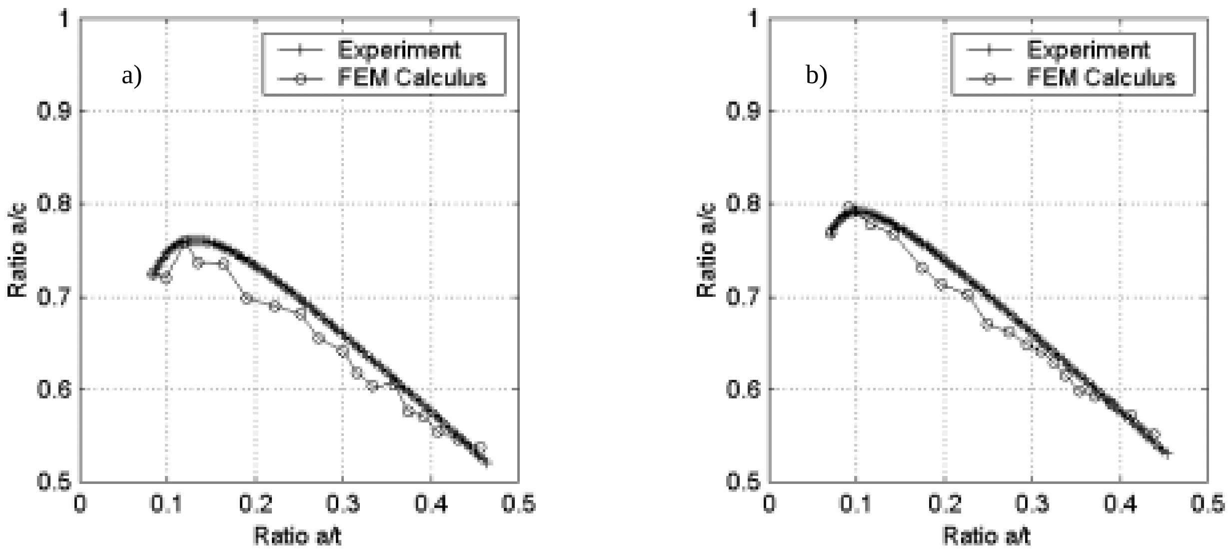

Figure 3 : Prediction of the semi-elliptic crack evolution for bending plates using the global approach. a) R = 0.1 and b) R = 0.3

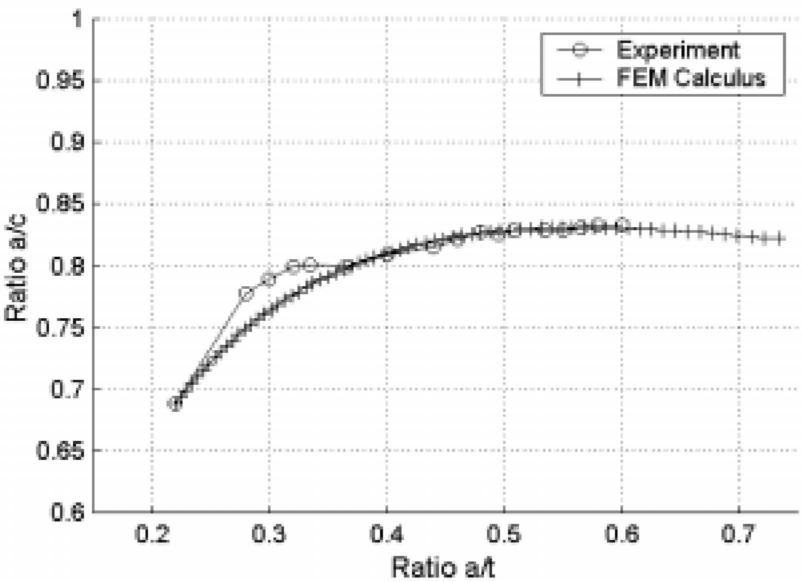

Figure 4 : Prediction of the semi-elliptic crack evolution for tube under internal pressure using the global approach. a) R = 0.1 and b) R = 0.3

7. Discussion

Figures (5 and 6) show that the global approach predicts, with a high precision, the semi-elliptical defects geometry evolution contained in the three point bending plates and in tubes submitted to internal pressure. curves (a/c) = f(a/t) converge towards an asymptote, which is in a total agreement with literature results. A comparison to empirical formulations is presented on the figure (5). The calculus results agree with the representative curve of the experimental points [3, 18] and the results of Mahmoud and Hosseini using $C_c = C_a$ [7]. The present study permits to propose an empirical model (equation 20) which describes the evolution of a semi-elliptical defect included in plates submitted to a three points bending. the model line is presented in the figure (6).

$$\left(\frac{a}{c}\right)_i = 0.915 - 0.85\left(\frac{a}{t}\right)_i + \alpha\left(\frac{a}{t}\right)_i^{-2} \qquad (20)$$

avec :

$$\alpha = \left(\frac{a_0}{c_0} + 0.85\left(\frac{a_0}{t}\right) - 0.915\right)\left(\frac{a_0}{t}\right)^2$$

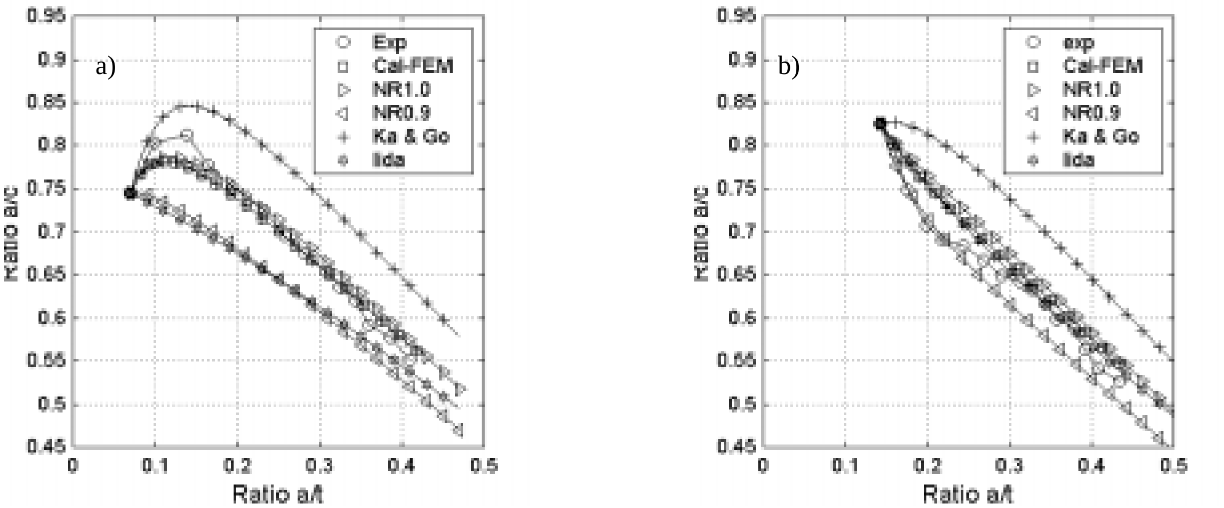

Figure 5: Comparison of the results obtained by the global approach with the other approaches for two initial defects : a) R = 0.1 and b) R = 0.3.

Cal-FE : calculation by the finite element method, exp : experimental results, NR1.0 : according to Mahmoud and Hosseini with $C_c = C_a$, NR0.9 : according to Mahmoud and Hosseini with $C_c = 0.9^m C_a$, Ka & Go : according to the empirical relation of Kawahara & Gorner and Iida: according to the empirical relation of Iida.

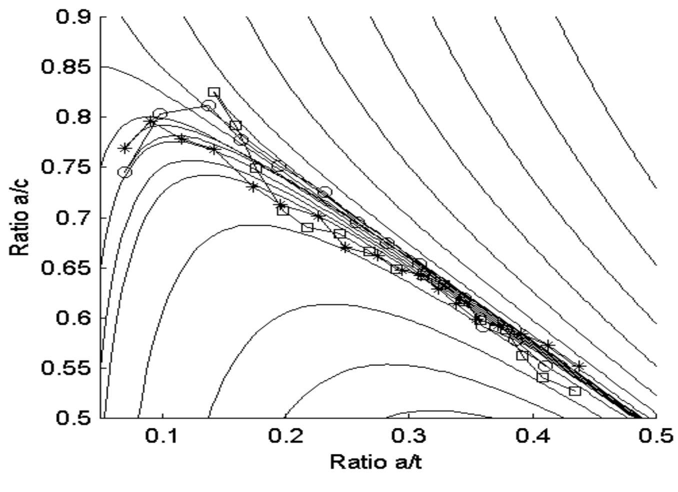

Figure 6 : Evolution of the ratio (a/c) versus (a/t), calculated by the proposed model, case of a bending plate

Conclusion

By this study, we wanted to check the reliability of the global approach in the prediction of the evolution of the of semi-elliptical surface crack, contained in bending plates and tubes submitted to internal pressure. Hence, a calculus code based on the finite element method was developed. The principle of calculation is based on the minimization of the potential energy of the cracked structure during the propagation without having recourse to the use of a propagation law.

The obtained results are in concord with the experimental results resulting from works of Boukharouba [4, 5] and those of Mahmoud and Hosseini [7] using the local approach with $C_c = C_a$.

Finally, a model of simulation of the evolution of the defects of semi-elliptical shape in the case of bending plates was proposed.

References

[1] J. Lemaître et J.L. Chaboche, « Mecanique des matériaux solides ». Ed Dunod, 1985.

[2] C. Bathias, « La fatigue des matériaux et des structures ». Maloine S.A. Ed Paris et les presses de Pirniverski de Montreal Québec. Ch. 13, pp. 412-441, 1980.

[3] T. Boukharouba, « Etude du comportement de fissures semi-elliptiques, application aux plaques en flexion et aux tubes sous pression interne », Thèse de Doctorat en Sciences de l'Ingénieur de l'Université de Metz, France, 1995.

[4] T. Boukharouba, C. Chehimi, J. Gilbert, G. Pluvinage, « Behaviour of semi-elliptical cracks in finite plates subjected to cyclic bending », Handbook of fatigue crack Propagation in Metallic Structures, Vol. 1, pp. 707-731, 1994.

[5] T. Boukharouba, J. Gilbert, G. Pluvinage, « Crack propagation of semi-elliptical surface cracks : a literature review », Proceeding of OTAN meeting on Fatigue, Varna, Bulgaria. 1996

[6] J. Newman and I. Raju, « An empirical stress intensity factor equation for the surface crack ». Engineering fracture mechanics, Vol. 15, N°. 1-2, pp. 185-192, 1981.

[7] M.A. Mahmoud and A. Hosseini, « Assessment of stress intensity factors and Aspect ratio variability of surface cracks in bending plates ». Engineering fracture mechanics, Vol. 24. N°.2. pp. 207-221, 1986.

[8] T. Boukharouba, C. Chehimi, J. Gilbert, G. Pluvinage,. « Comportement de Fissures Semi-elliptiques Contenues dans un Tube sous Pression Interne », 8ème Journées d'Etudes de AFIAP, 1995.

[9] J Newman, « A review and assessment of the stress intensity factors for surface cracks in part-through crack fatigue life ». Prediction ASTM 55P 687, pp. 16-42, 1979.

[10] I.S. Raju, J.C. Newman, Jr., « Stress intensity factors for internal and external surface cracks in cylindrical vessels ». Journal of pressure vessel technology, Vol. 104, pp. 293-298, 1982.

[11] T.A. Cruse and P.M. Besuner, « Residual life prediction for surface crack in complex structural details ». J. Aircraft 12, pp. 869-887, 1983.

[12] I. V. Varfolomeyev, V.A. Vainstok and A.Y. Krazowsky, « Prediction of part-through crack growth under cyclic loading ». Engineering fracture mechanics, Vol. 40, N°. 6, pp. 1007-1022, 1991.

[13] Q.J. Portch, « An investigation into the change of shape of fatigue cracks initiated in surface flaws ». Central electricity generation board, Repub. RD/B/N4645, Great Britain, 1979.

[14] M. Kawahara and M. Kurihara., « Fatigue crack growth from a surface plan ». in Proc. of the 4th Int. Conference on Fracture, University of Waterloo, Canada (Edited by Q. Taplin), Vol. 2, pp. 1361-1373, 1977.

[15] F. Gorner, C. Mattheck and D. Munz, « Change in geometry of surface cracks during alternating tension and bending ». Z. WOrkstoff Tech, N°.14, pp. 11-18, 1983.

[16] K. Iida, « Aspect ratio expressions for pat-through fatigue crack ». II W Doc. XIII. pp. 967-980, 1980.

[17] T. Boukharouba J. Gilgert, K. Azouaoui and G. Pluvinage, « Fatigue propagation of surface under cyclic loading », Fourth international conference in Garmisch - Partenkirchen, Berlin, sept. 1998.

[18] T. Boukharouba, G. Pluvinage, « Prediction of semi-elliptical defect form, case of a pipe subjected to internal pressure », Nuclear Engineering and Design, 188, pp. 161-171, 1999.

[19] T. Boukharouba, J. Gilbert, G. Pluvinage, « Fatigue Crack Growth in Pressurized Pipe », Mechanisms and Mechanics of Damage and Failure of Engineering - ECF11, Futuroscope de Poitiers, France, pp. 1167-1172, 1996.