PREDICTION OF NOTCH-EMANATING CRACK INITIATION LIFE IN FATIGUE: EFFECT OF INPUT PARAMETERS VARIATION

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ABSTRACT

Dimensioning in fatigue of structures requires knowledge of physical and mechanical phenomena involved in sites such as stress raisers. Two different approaches are studied in this paper for evaluating the notch-emanating crack initiation life in fatigue: the local strain approach, and the short crack approach. The latter is based on the assumption that the so-called 'initiation life' is in fact spent in propagating a short crack from a crack-like defect up to a detectable size. Predictions based on both approaches are compared, and it is shown that allowing variation of input parameters within their confidence range can lead to the evaluation of the scatter observed in experimental tests.

INTRODUCTION

The effects of notches in the endurance limit range are taken into account by the introduction of the fatigue notch factor, K_f, which represents the ratio between the endurance limits of a smooth specimen and a notched one. In the finite life range, local strain approaches developed by Neuber and Glinka are widely used to determine the fatigue crack initiation life at notch (Molski & Glinka, [1]). The main drawback of this approach is that one does not have any idea of the crack size at initiation. Recent studies (Blöm [2], Newman [3]) have successfully shown that fracture mechanics based predictions can be used to evaluate the propagation of short cracks at the notch-tip. The aim of this paper is to compare the predicted notch emanating crack lives to the experimentally obtained ones based on the conventional local strain approach and a short crack model developed recently. A particular attention is given to the effects of variation of input parameters representing material properties, on the predicted lives as compared to experimental scatter when available.

LOCAL STRAIN VERSUS SHORT CRACK APPROACH

To predict fatigue crack initiation in notched components, earlier approaches use the local strain parameters (Molski & Glinka, [1]) as presented below.

Local strain approach

In this method, the notch root strain and stress amplitude for a remote loading defined by the stress

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + (\frac{\Delta\sigma}{2K'})^{\frac{1}{n'}} \tag{1}$$

$$\frac{(K_T\Delta S)^2}{E}(1-\nu^2) = \frac{\Delta\sigma^2}{4E} + \frac{\Delta\sigma}{1+n'}(\frac{\Delta\sigma}{2K'})^{\frac{1}{n'}} \tag{2}$$

A fictive smooth specimen is assumed to exist at the notch tip and damage associated to the local stress amplitude $\Delta\sigma$ is determined by Miner's law after suitable correction for the local mean stress level. Available corrections are the Morrow (eq. 3), Manson-Halford (eq. 4) and Smith-Watson-Topper (eq. 5) expressions, added to the simple Manson-Coffin (eq. 6) expression, that does not provide any correction for the mean stress level.

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f' - \sigma_{moy}}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{3}$$

$$\frac{\Delta\epsilon}{2} = \frac{(\sigma_f - \sigma_{moy})(2N_f)^b}{E} + \epsilon_f'(2N_f)^c(\frac{\sigma_f' - \sigma_{moy}}{\sigma_f'})^{\frac{c}{b}} \tag{4}$$

$$\sigma_{max}\frac{\Delta\epsilon}{2} = \frac{{\sigma_f'}^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c} \tag{5}$$

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{6}$$

All these corrections are incorporated in a computer program, initially developed by G. Glinka, and extended later to include the short crack model by N. Ranganathan (salient details are given below) .

Short crack approach

This approach is based essentially on the following assumption : it is believed that from the first fatigue cycle, a crack initiates at a crack like defect (whose size is near comparable to a microstructural feature size, such as an inclusion, or a dispersoïd particle). The so-called initiation life is thus spent in propagating this crack up to the defined initiation size. The equivalence of both approaches have been discussed before (Dowling [4]). As previously, the notch root strain and stress histories are evaluated using the same method (equations (1) and (2)), and the local stress amplitude is incorporated in Kujawski's [5] expression (eq. 7) for a short notch-emanating crack intensity factor. The main advantage of this expression is the fact that it takes into account accurately of the evolution of the local stress at the notch root, as comparisons with finite element method (FEM) shows it. A power law is then used, to relate crack propagation rate and stress intensity factor range.

$$\Delta K_I = Qf\frac{K_T\Delta S}{2}((1+\frac{a}{\rho})^{-\frac{1}{2}} + (1+\frac{a}{\rho})^{-\frac{3}{2}})\sqrt{\pi a} \tag{7}$$

where Q is a factor depending of the crack geometry (semi-circular shape or through crack), f is a correction factor depending upon the crack length (expressed as a in the above expression) and ρ is the notch-tip radius, ΔS is the remote stress range, and K_T the notch stress concentration factor.

APPLICATIONS

Al 2024-T3

Constant amplitude loading

The material properties are shown in table 1. Notched specimens of Al 2024-T3 have been at first submitted to constant amplitude loading, with a stress ratio of 0,1 (Bleuzen, [6]). Using the four different corrections, the local strain approach was applied. For the short crack model, the following assumptions are made:

- The short crack is assumed to propagate without any crack closure, and the crack growth law consists of three ranges with appropriate constants C and n as given in table 1.

Predictions made with mean material properties can be seen in the table 2.

TABLE 1

MATERIALS PROPERTIES OF AL 2024-T3

Material	Al 2024-T351
Young's Modulus	74000 MPa
Yield strength	365 MPa
Cyclic - strength hardening coefficient K'	618 MPa
Cyclic - strength hardening exponent n'	0,051
Fatigue strength coefficient $\sigma_f^{'}$	842 MPa
Basquin's exponent b	-0.102
Fatigue - ductility coefficient $\epsilon_f^{'}$	12,12 %
Fatigue - ductility exponent c	-0,564
Poisson's coefficient	0.33
Propagation law $\frac{da}{dN} = C_i \Delta K^{n_i}$	Coefficients
$\Delta K < 2 : C_i$	1.3333e-11
$\Delta K < 2 : n_i$	7.229
$2 < \Delta K < 4 : C_i$	3.3e-10
$2 < \Delta K < 4 : n_i$	2
$\Delta K > 4 : C_i$	1.5e-11
$\Delta K > 4 : n_i$	4.1

TABLE 2

COMPARISON OF PREDICTED AND MEASURED CRACK INITIATION LIFE, IN NUMBER OF CYCLES, CAL, R=0,1

$S_{max}(MPa)$	M.C.	Mo	M.H.	S.T.W.	Exp.	Short crack
180	374491	36127	20427	15715	67959 to 578000	63100
200	148700	19595	9945	9284	44330 to 55530	44400

It can be seen here that the better estimations are made with the short crack approach, slightly conservative, and the Morrow correction. Smith-Topper-Watson and Manson-Halford corrections are over conservative, whereas Manson-Coffin shows a highly non conservative trend.

Effect of input parameters variations

In a previous study, it has been shown that variations in parameters representing mechanical properties of the material within their confidence range can lead to significant variations in predicted lives. In the present study similar concepts are applied with more emphasis on parameters representing the short crack model. Variations concern two types of input parameters : intrinsic, representing material properties such as strain-life curve coefficients or cyclic strain-stress curve. The extrinsic parameters considered here are geometric properties of the component (by evaluating the stress concentration factor at the notch corresponding to machining tolerance). A crack deflection of $\pm 13,5^o$ is assumed for the short crack model, as observed by Hillberry [9] in this material. The equivalent ΔK_{eq} is determined by :

$$\Delta K_{Ieq} = sin^2(\frac{\pi}{2} - \alpha).\Delta K_I \quad (8)$$

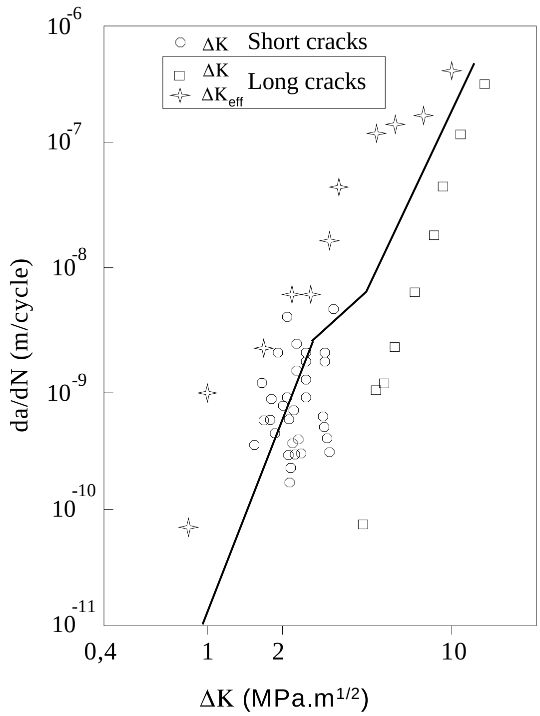

Figure 1: Experimental data and mean crack propagation law (Al 2024-T3, Desforges [8]).

where α is the crack deflection. It is assumed that in spite of this deflection, a mode II crack propagation is not considered. This assumption may need further study for larger deflection. The crack propagation law was modified to roughly cover the scatter band observed in experimental measures (see figure 1). Finally, the initial crack size was modified, to take in account the scatter in the size of the crack-like defect where the crack initiates . Results of such variations are shown for a typical stress level in table 3.

TABLE 3

EFFECT OF INPUT PARAMETERS VARIATION, CAL, R=0,1

Parameter	Effect on prediction local strain (Morrow)	Effect on prediction short crack
K', n'	+10%	0,1 %
$\sigma_f^{'}$, b	-20% to +1%	X
$\epsilon_f^{'}$,c	-48%	X
K_T	-16% to +27%	±10 %
Applied stress	-11% to +16%	± 7%
Crack deflection	X	+22 %
Propagation law	X	±60%
Initial size	X	-23 %

Assuming that all variations are cumulative, the variation due to the propagation law leads to an over prediction of the scatter band. Nevertheless, excluding this effect, the predicted minimum and maximum initiation lives are about ± 40% from values predicted with mean material properties. These variations are shown in figure 2. The ratio between maximum and minimum predictions is close to 2. The it is quite longer for 200 MPa. It is believed that the smaller number of experimental measurements for the last stress level does not lead to an accurate measure of scatter.

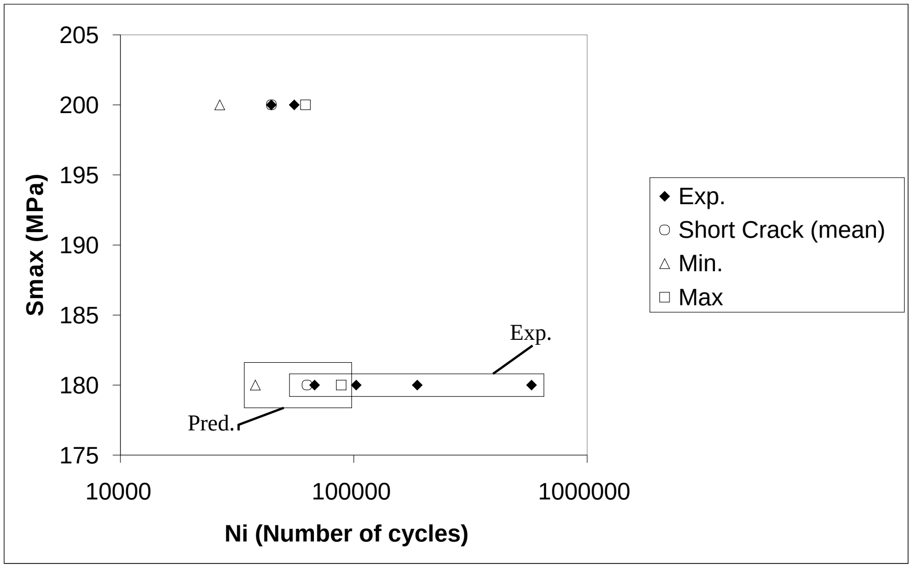

Figure 2: Predicted scatter for the CA conditions, $R = 0.1$, Short Crack approach.

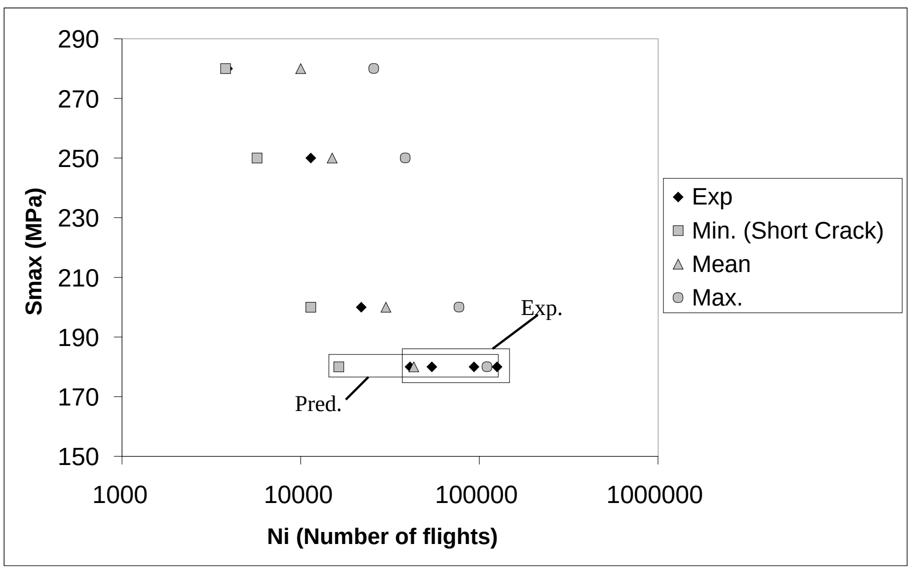

Figure 3: Predicted scatter for the 1^{st} spectrum, Short Crack approach.

Variable amplitude loading

The experimental results analyzed correspond to two kinds of aeronautical spectra, representing the wing lower surface loading for a civil aircraft, i.e. predominantly in tension. The first spectrum contains 22547 cycles, whereas the second one 266018. Each flight represents different conditions such as take off, cruise and landing phases.

1^{st} *spectrum*

The first spectrum had been studied earlier by Ranganathan [10], and the estimations closer to measured lives were obtained with the Smith-Topper-Watson correction, and the short crack approach. However, neither variation in crack propagation law nor the effect of crack deflection was studied previously.

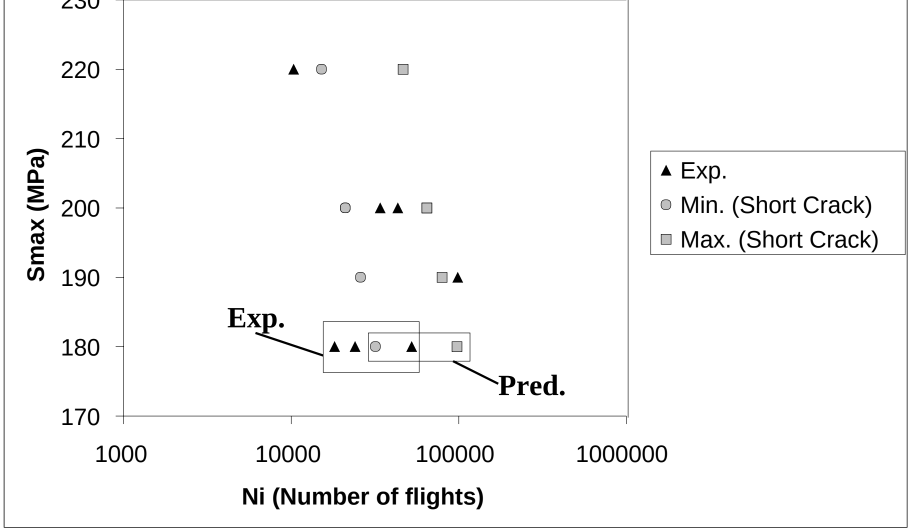

Figure 4: Predicted scatter for the 2^{nd} spectrum, Short Crack approach.

Effect of input parameters variations

The results presented here concern only the short crack approach.Results for local strain approach are found in [10]. The crack deflection led to a variation of 95%, and the crack propagation law variation can reduce the crack initiation life by 60%, or increase it by more than 200%. Figure 3 compares experimental scatter with the scatter band in the prediction results taking into account the input parameters variations, such as initial crack size and crack deflection (crack growth law variations are not considered here). It can be seen on this figure that at lowest stress level, the introduction of crack deflection leads to quite accurate estimation of the experimental scatter. Unfortunately, the number of tests involving higher stress levels are few, and no information can be extracted about experimental scatter for such levels. Nevertheless, the trend observed previously for the short crack approach is verified (Ranganathan, [10]), i.e. it seems to be slightly conservative at lower stress levels, and non conservative at higher stress levels.

2^{nd} *spectrum*

This spectrum differs from the first one, with the introduction of small cycles in the cruise condition. For this spectrum, we have studied the effect of input parameters variation for the local strain and short crack approach. The parameters were varied on the same order as that for the CA condition. It was found that the predicted scatter was on the same order as for CA conditions (see table 3). The results are given in figure 4 (mean predictions). The predicted scatter band at 200 MPa (where experimental data is available) with the short crack approach encompasses the experimental data, while at the lower stress level the prediction seems to be slightly conservative. For the Smith-Watson-Topper correction the predicted scatter band seems to be much larger than the experimental one for the two stress levels.

Effect of input parameters variation

The effects of input parameters variation were studied as before for constant amplitude (CA) conditions. The results obtained for the short crack approach are given figure 4 and that for local strain approach using the Smith-Watson-Topper correction in figure 5. (The other corrections are not given here for the sake of clariry). It can be seen here that the predicted scatter for the short crack approach encompasses the experimental data at a maximum stress of 200 MPa while it is slightly non conservative at 180 MPa. The scatter band obtained for Smith-Watson-Topper correction is much larger than that for short crack approach, and the predictions seem to be highly non conservative at 180 MPa.

Al 7075-T6

Constant amplitude loading

The tests studied in this part of the study are from Nicolas [11], concerning notched component made of Al 7075-T6, with a K_T of 2.3. These specimens were submitted to constant amplitude loading at

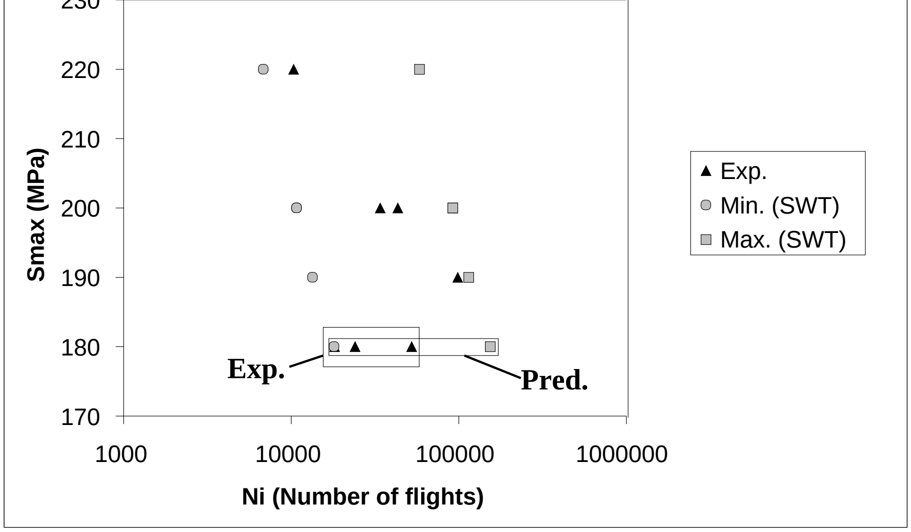

Figure 5: Predicted scatter for the 2^{nd} spectrum, Smith-Watson-Topper correction.

stress ratios $R = -1; -3; -5.3$. For each loading condition representing stress amplitude and the load ratio only one test was conducted, and hence there is no idea of experimental scatter. Previous studies using the local strain approach showed that Morrow correction gives good estimations, and the Smith-Watson-Topper correction was modified to negative R ratios (Nicolas, [11]). The emphasis made here is on the short crack model, using a crack propagation law taken from Newman [12] for this material, predictions were made with the short crack approach. The assumption of absence of closure does not seem to be valid for negative stress ratios, as it led to highly conservative predictions . The local efficiency factor U, representing the effective $\Delta\sigma_{eff}$ to total local stress amplitude ($\Delta\sigma$) at the crack tip was first estimated by adjusting the predicted results to coincide with the experimental ones. The following expression was obtained :

$$U = 0.0028\sigma_{max} - 0.2823 \tag{9}$$

This expression is quite different from the relationships proposed in the literature (Booth, [13]). The crack opening level was allowed to vary within $\pm 10\%$ from this linearized expression. The corresponding results are given figure 6. The predicted scatter is on the order of $\pm 16\%$, except for the lowest stress level at $R = -3$. These results show that, for the negative stress ratio condition, accurate determination of the local stress levels are very important for the short crack approach.

DISCUSSION

local strain approach

The present study confirms previous results showing that the parameters representing the strain-life relationship ($\epsilon_f^{'}$, c, $\sigma_f^{'}$, and b) and an accurate determination of K_T are very important to have accurate predictions. For variable amplitude conditions, the accuracy of the load spectrum applied to the specimen has an important effect.

short crack model

The most important aspect is the accurate determination of the crack growth law for short cracks. Figure 1 compares the experimental short crack data (Desforges, [8]) with the mean crack growth law used in this study. It can be seen that scatter of short crack data is not taken in account. The crack growth law used in this study represents long cracks data corrected for closure. This growth law seems to represent the mean behaviour of short cracks which permitted us to assume that short crack grow

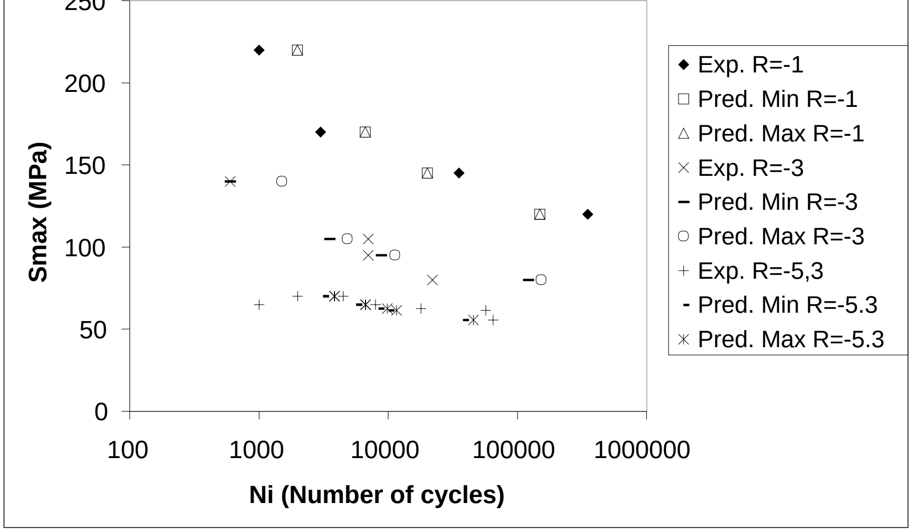

Figure 6: Effect of crack opening stress level variation (±10%) regarding to the linearized U evolution.

without any closure effect. Also in this study, it was assumed that no load interaction effect were present in the studied VA spectra. The fact that the prediction results are quite close to the measured ones seems to corroborate with this assumption, at least for the spectra studied here. For both the approaches, especially for the service loading condition, two extrinsic parameters have to be taken in account; i.e. determination of K_T and the accuracy of loads applied to the specimen.

CONCLUSION

The present study shows that accurate predictions of fatigue crack initiation lives at a notch-tip depends upon the accuracy of parameters representing material properties for the local strain approach. For the short crack approach, the most significant parameter is the crack growth law to be used to describe the crack growth behaviour. Extrinsic parameters, such as the value of K_T and loads applied to the specimen (under VAL conditions) are very important especially for service load condition.

REFERENCES

1. Molski, K., Glinka, G. (1981) *Materials Science and Engineering*, 50, pp. 93-100, .
2. Blöm A.F. (1990) *AGARD R-767*, paper 6, 6-1 6-14.
3. Newman, J.C. JR, Phillips, E.P., Swain, M.H. (1999) *International Journal of Fatigue*, 21, pp. 109-119.
4. Dowling, N.E. (1982)In *Proceedings of the SAE Fatigue Conference*, pages 161-173.
5. Kujawski, D. (1991) *Fatigue and Fracture Engineering of Materials and Structures*, 14(10), pp. 953-965
6. Bleuzen, Cl. (1992) Technical Report, CEAT.
7. Newman, J.C. JR. (1998) *Progress in Aerospace Sciences*, 34(5-6), pp. 345-388, .
8. Desforges J.R. (1996) PhD thesis, Univ. Poitiers, France.
9. Hillberry B.M. Laz P.J. (1998) *International Journal of Fatigue*, 20(4), pp. 263-270.
10. Ranganathan N., Morel F., Nadot Y. (1999) In *Engineering against fatigue*,Lindley T.C., Smith R.A., Tomkins B., Beynon J.H., Brown M.W., editors, pages 183-189.
11. Nicolas Ch. (1993) PhD thesis, Univ. Poitiers, FRANCE.
12. Newman J.C. JR, Wu X.R.,Venneri S.L., Li C.G. (1994) *NASA Reference*, A NASA/CAE Cooperative Program.
13. Booth G.S., Maddox S.J. (1988) In *Mechanics of fatigue crack closure*,Elber W., Newman J.C. JR, editors, pages 516-527.