PREDICTION OF CRACK GROWTH UNDER MODE (I+II) CYCLIC LOADING

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ABSTRACT

In the paper the approach to determine the crack growth direction under mixed-mode loading conditions has been done. Fatigue crack propagation behaviour has been studied mainly based on the stress distribution at the crack tip in terms of the stress intensity factor K_I and K_{II}. The plastic zone shape around the crack tip is applied for evaluating angle of crack propagation. Under mixed mode conditions to calculate angle crack propagation assuming that crack growth initiates in a direction along factor W possesses a relative minimum. The prediction of the proposed criterion is compared with the experimental data and other models. The agreement is good.

INTRODUCTION

The stress and strain range near a crack-tip generally controls the phenomenon of fatigue crack growth. A characteristic of mixed mode fatigue cracks is that they usually propagate in a non-self similar manner. Cracks will change their growth direction when subjected to mixed mode loading. Therefore, under mixed mode loading conditions, not only the fatigue crack growth rate is of importance but also the crack growth direction.

From 1963, when Erdogan [1] brought forward the criterion of maximum tangential stress, several kinds of fracture criteria have been developed [2], among which the most widely used: the criteria of minimum strain energy density factor, Det-criterion [4], J-integral and some empirical criteria obtained from experimental results.

As different hypotheses are used in the formulation of each of these criteria, the results for a given problem can be somewhat different depending on the criterion used. This leads to limitations on their applicability.

ANALYSIS OF DET. CRITERION

In linear elastic fracture mechanics, the stress field near the crack tip is given in terms of the stress function derived by Wiliams [7]:

$$\phi = r^{\frac{3}{2}} f_1(\theta) + r^2 f_2(\theta) + r^{\frac{5}{2}} f_3(\theta) + \mathrm{K} \qquad (1)$$

Where r is the radius and $f_1(\theta),\ f_2(\theta),\ f_3(\theta)\mathrm{K}$ are functions of θ.

The elastic strain-energy dW stored in a parallel-piped of volume dV dominating at the strained plate is expressed by:

$$\frac{dW}{dV} = \frac{1+\nu}{4E}\left[\kappa_{1,2}\left(\sigma_x+\sigma_y\right)^2+\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}^2\right] \tag{2}$$

Where:

E - modulus of elasticity of the material,

ν - Poisson ratio,

$$\kappa_1 = \frac{1-\nu}{1+\nu}\text{, for plane stress,} \tag{3}$$

$$\kappa_2 = 1-2\nu\text{, for plane strain.} \tag{4}$$

The total elastic strain density is divided into two components:

- the dilatational strain-energy density T_V,
- the distortional strain-energy density T_D.

For plane stress they have the form:

$$T_V = \frac{1-2\nu}{6E}\left(\sigma_x+\sigma_y\right)^2, \tag{5}$$

$$T_D = \frac{1+\nu}{3E}\left[\left(\sigma_x+\sigma_y\right)^2-3\left(\sigma_x\sigma_y-\tau_{xy}^2\right)\right]. \tag{6}$$

For plane strain T_V and T_D have the form:

$$T_V = \frac{(1-2\nu)\ (1+\nu)^2}{6E}\left(\sigma_x+\sigma_y\right)^2, \tag{7}$$

$$T_D = \frac{1+\nu}{3E}\left[\left(\nu^2-\nu+1\right)\left(\sigma_x+\sigma_y\right)^2-3\left(\sigma_x\sigma_y-\tau_{xy}^2\right)\right]. \tag{8}$$

Substituting relations (5) and (7) into relations (6) and (8), respectively, we obtain:

$$T_D = \frac{2(1+\nu)}{1-2\nu}T_V - \frac{1+\nu}{E}\left(\sigma_x\sigma_y-\tau_{xy}^2\right)\text{, for plane stress,} \tag{9}$$

$$T_D = \frac{2\left(\nu^2-\nu+1\right)}{(1-2\nu)(1+\nu)}T_V - \frac{1+\nu}{E}\left(\sigma_x\sigma_y-\tau_{xy}^2\right)\text{, for plane strain,} \tag{10}$$

$$T_V = \frac{1-2\nu}{2(1+\nu)}T_D + \frac{1-2\nu}{2E}\ \mathrm{Det.}\left(\sigma_{ij}\right)\text{, for plane stress,} \tag{11}$$

$$T_V = \frac{(1-2\nu)(1+\nu)}{2\left(\nu^2-\nu+1\right)}T_D + \frac{(1-2\nu)(1+\nu)^2}{2\left(\nu^2-\nu+1\right)E}\ \mathrm{Det.}\left(\sigma_{ij}\right)\text{, for plane strain.} \tag{12}$$

According to Det.-criterion, the angle of crack extension on mixed mode loading is determined by the condition that the determinant of the stress tensor must take maximum value. This condition is expressed mathematically by the relations:

$$\left.\frac{\partial \, Det.(\sigma_{ij})}{\partial \theta}\right|_{\theta=\theta^*} = 0, \tag{13}$$

$$\left.\frac{\partial^2 \, Det.(\sigma_{ij})}{\partial \theta^2}\right|_{\theta=\theta^*} < 0. \tag{14}$$

The critical stress of crack initiation is calculated by:

$$Det.(\sigma_{ij}) = Det.(\sigma_{ij})_{cr}. \tag{15}$$

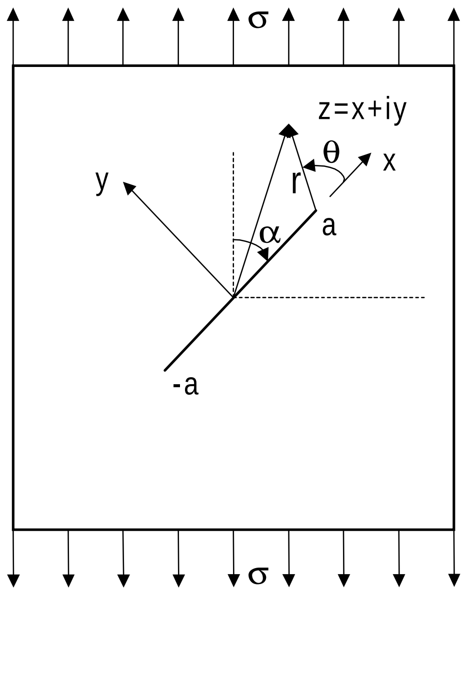
Where $Det.(\sigma_{ij})_{cr}$ is a constant of the material and

$$Det.(\sigma_{ij}) = \begin{vmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{xy} & \sigma_y \end{vmatrix} \tag{16}$$

is the determinant of the tensor.

ANALYSIS OF THE STRESS FIELDS NEAR CRACK TIP

In the present analyses, all stress components are calculated based on the exact elastic stress solutions of an infinitive plane under loading configuration shown in Figure 1.



The stress and displacement field in the vicinity of crack tip for Mode I and Mode II can be written by superimposing the stress and displacement components:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\left[1 + \sin\frac{\theta}{2}\ \sin\frac{3\theta}{2}\right] + \frac{K_{II}}{\sqrt{2\pi r}}\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}, \tag{17}$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] - \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left[2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right], \tag{18}$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right], \quad (19)$$

$$u_x = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}} \cos\frac{\theta}{2}\left[\frac{1}{2}(\kappa - 1) + \sin^2\frac{\theta}{2}\right] + \frac{K_{II}}{G}\sqrt{\frac{r}{2\pi}} \sin\frac{\theta}{2}\left[\frac{1}{2}(\kappa + 1) + \cos^2\frac{\theta}{2}\right], \quad (20)$$

$$u_y = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}} \sin\frac{\theta}{2}\left[\frac{1}{2}(\kappa + 1) - \cos^2\frac{\theta}{2}\right] + \frac{K_{II}}{G}\sqrt{\frac{r}{2\pi}} \cos\frac{\theta}{2}\left[\frac{1}{2}(\kappa - 1) + \sin^2\frac{\theta}{2}\right]. \quad (21)$$

Stress intensity factors K_I and K_{II} have form:

$$K_I = \sigma\sqrt{\pi a} \sin^2\alpha, \quad (22)$$

$$K_{II} = \sigma\sqrt{\pi a} \sin\alpha \cos\alpha. \quad (23)$$

One of the problems to be resolved in a mixed mode situation is the prediction of the direction of crack growth. Some difficulties arise concerning the nature of the area around the crack tip.

NEW CONCEPT FOR CALCULATING CRACK GROWTH DIRECTION - W-CRITERION

Fatigue crack propagation behaviour under mixed mode loading has been studied mainly based on the stress distribution at the crack tip in terms of the stress intensity factor K_I and K_{II}.
Under mixed mode conditions we propose to calculate angle crack propagation assuming that crack growth initiates in a direction along factor W possesses a relative minimum. Factor W was derived based on the analysis of the fracture process zone ahead of crack tip:

$$W = \bar{\sigma}\left(f_1 + f_2 - f_3 + f_4 - f_5 - f_6 + f_7 + f_8\right). \quad (24)$$

Where:

$$f_1 = \frac{83}{32}\cos^2\alpha \sin^2\alpha - \frac{163}{64}\cos^2\alpha \cos\theta \sin^2\alpha, \quad (25)$$

$$f_2 = \frac{63}{32}\cos^2\alpha \cos 2\theta \sin^2\alpha - \frac{21}{32}\cos^2\alpha \cos 3\theta \sin^2\alpha, \quad (26)$$

$$f_3 = \frac{3}{64}\cos^2\alpha \cos 5\theta \sin 2\alpha + \frac{3}{16}\cos 2\theta \sin^4\alpha + \frac{3}{4}\cos\alpha \sin^3\alpha \sin\frac{\theta}{2}, \quad (27)$$

$$f_4 = \frac{3}{16}\cos^2\alpha \cos 4\theta \sin 2\alpha + \frac{7}{16}\sin 4\alpha + \frac{1}{4}\cos\theta \sin 4\alpha, \quad (28)$$

$$f_5 = \frac{7}{32}\cos\alpha \sin^3\alpha \sin\theta + \frac{3}{16}\cos\alpha \sin^3\alpha \sin 3\theta, \quad (29)$$

$$f_6 = \frac{3}{16}\cos\alpha \sin^3\alpha \sin\frac{3\theta}{2} + \frac{3}{32}\cos\alpha \sin^3\alpha \sin 5\theta, \quad (30)$$

$$f_7 = \frac{3}{16}\cos\alpha \sin^3\alpha \sin\frac{3\theta}{2} + \frac{3}{8}\cos\alpha \sin^3\alpha \sin^2\theta, \quad (31)$$

$$f_8 = \frac{9}{16}\cos\alpha\ \sin^3\alpha\ \sin\frac{5\theta}{2} + \frac{3}{16}\cos\alpha\ \sin^3\alpha\ \sin\frac{9\theta}{2}\ . \qquad (32)$$

Mathematical formulation of new criterion:

$$\left.\frac{\partial W}{\partial\theta}\right|_{\theta=\theta^*} = 0, \quad \left.\frac{\partial^2 W}{\partial\theta^2}\right|_{\theta=\theta^*} > 0 \qquad (33)$$

for $\theta = \theta^*$, where $-\pi < \theta < \pi$.

DISCUSSION OF THE RESULTS FOR UNIAXIAL TENSION LOADING PROBLEM

The aim of this part is to examine the application of the W-theory to the study of the angled crack problem. The crack is situated in the centre of the plate. Crack angle α is an independent variable in the study. This problem is a combination of the opening mode I and the edge-sliding mode II. The applied load has constant amplitude of value. The direction of the initial crack extension is a subject of interest. The configuration of the computed crack problem is shown in Figure 2.

Values of angle crack growth under uniaxial loading computed from criterion W are compared to experimental data for LY12-CZ (similar to American 2024-T3) material published by Yang-Xian Qi and Wei-Xun Fan [6] and to Det-criterion [4].

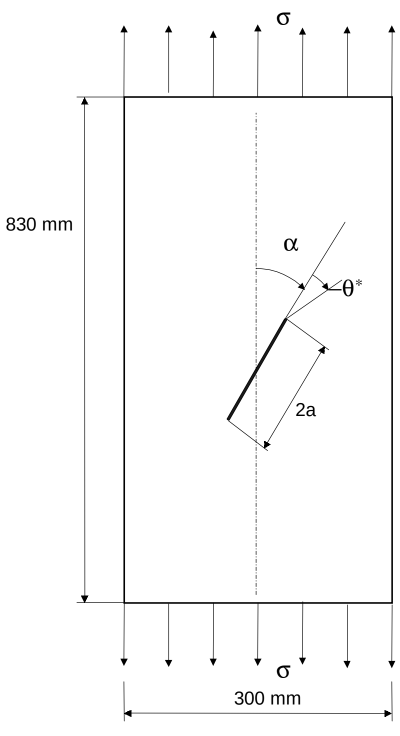


Figure 2: Tested crack with geometry

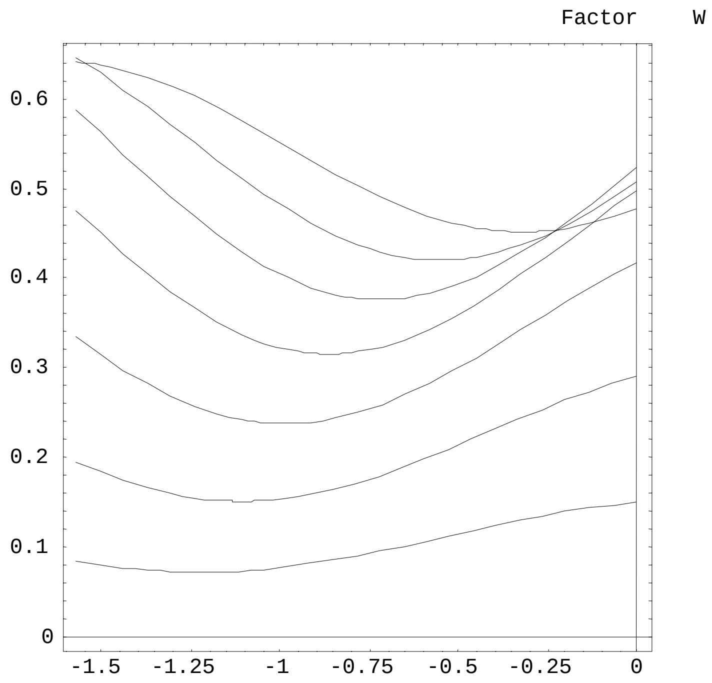


Figure 3: Stationary values of the W factor

The length of the crack was taken as a constant, whereas the angle α of slantness to the loading axis was changing from 15^o to 90^o at 15^o interval.

Figure 3 presents the variation of the W factor vs θ for parametric values of angle crack initiation α.

The mechanical properties of the LY12-CZ material are:

$\sigma_u = 456\ MPa, \sigma_{02} = 332\ MPa, E = 70\,600\ MPa, \nu = 0.33, \delta = 11.7\ \%$.

There is a good agreement between results computed based on with proposed criterion and experimental data as shown in Figure 4.

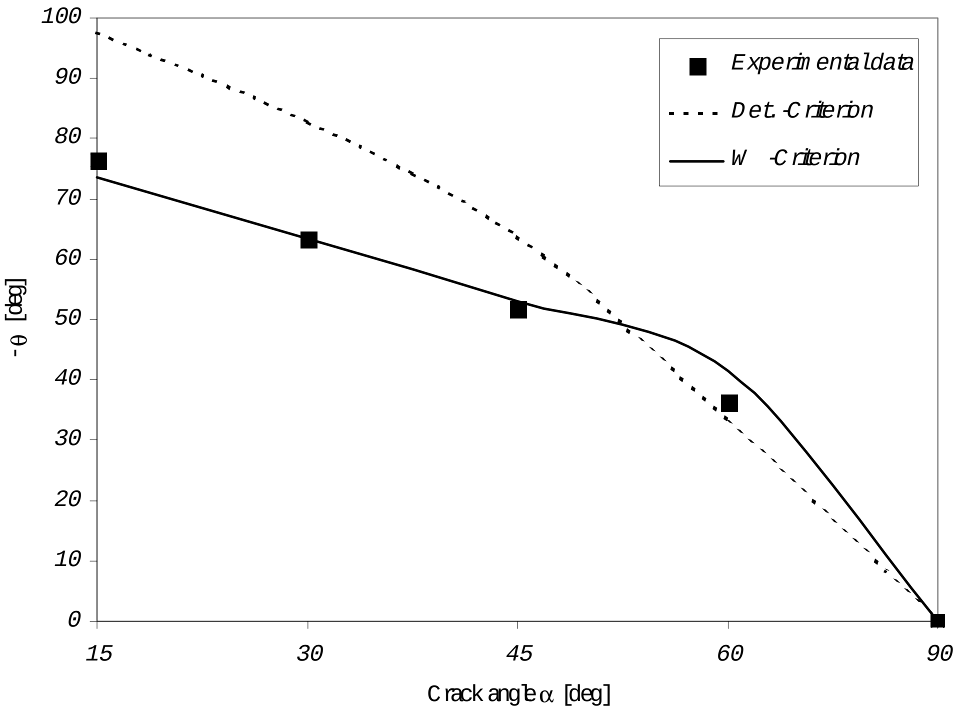


Figure 4: Comparison between the W-criterion and Det. criterion and experimental data for LY12-CZ

CONCLUSIONS

An approach to determine the crack growth direction under mixed-mode loading conditions is presented. The plastic zone shape around the crack tip is applied for evaluating angle of crack propagation. It is proposed that a mixed-mode crack will extend along the plastic zone radius with a minimum value. The prediction of the proposed criterion is compared with the experimental data and other models. The agreement is good.

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