ON THE NEED TO CONSIDER THE VOLUMETRIC DISTRIBUTION OF STRESSES FOR HIGH CYCLE MULTIAXIAL FATIGUE STRENGTH PREDICTION

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ABSTRACT

Stress concentrations are often responsible of fatigue failure on industrial components. Fatigue strength prediction for long life is difficult on real components due to stress gradients and more generally due to complex volumetric stress distribution (gradients). This paper presents the concept of volume influencing fatigue crack initiation V* proposed by Palin-Luc et al. [1] and its use with an energy based fatigue criterion for constant amplitude fully reversed multiaxial loadings. This concept is also used with the mesoscopic approach proposed by Papadopoulos [12]. Predictions of these two criterions are in very good agreement with experimental multiaxial fatigue data showing that the volume influencing fatigue crack initiation can be used as a general concept.

INTRODUCTION

Designing metallic components against fatigue is still a difficult problem to handle for engineers since the transfer of fatigue data from specimen to component is an arduous task. Two major phenomenons are responsible for that : the stress gradient effect and the so called "size effect". In literature, many experiments show the influence of notches on the fatigue strength but the service loaded components have seldom a well defined notch factor. Consequently, the use of data from laboratory tests has to be done very carefully [4],[5]. Furthermore, it is well known, in high cycle fatigue, that the load type has a significant influence on fatigue strength (endurance limits in traction and in bending are usually different). A precise design against fatigue requires a method able to consider both the volumetric distribution of stresses and the loading kind. Several methods have been proposed in literature to take into account the stress gradient : the critical layer [6], the stress gradient of Papadopoulos [7], the V90% method of Sonsino *et al.* [4] and the energy based approach of Palin-Luc and Lasserre using the concept of volume influencing fatigue crack initiation. Between these approaches the last one is the only one, according to the authors, able to predict experimental differences between endurance limits in plane bending and rotative bending. This criterion is presented here together with the concept of the volume influencing fatigue crack initiation V*.

In high cycle multiaxial fatigue many others criteria are not based on an energetic approach. Some of them established through a critical plane concept are derived from the mesoscopic approach proposed by Dang Van. For instance, Papadopoulos has developed a mesoscopic theory of fatigue crack initiation taking into account the mesoscopic plastic strain accumulated in all the crystallographic planes unfavourably oriented in an elementary volume. By using the concept of the volume influencing fatigue crack initiation together with this local approach this paper shows that the concept of volume influencing fatigue crack initiation can be seen as a general concept. The predictions of the two volumetric criteria are finally compared with experimental fatigue data.

ENERGY BASED HIGH CYCLE MULTIAXIAL FATIGUE CRITERION

All the existing criteria do not distinguish loading types because they consider only the tensor of stresses at the critical point and do not take into account the distribution of stresses around this point. This distribution at any moment of the loading cycle is the same in plane bending and in rotative bending. In rotative bending, however, all the points lying on a circle centred on the middle of the specimen cross-section support the same stress during a cycle ; in plane bending there is no axisymmetry, there are only two points supporting the greatest stresses [1]. That is why it is important to reason on a complete loading cycle as proposed by Tsybanev [9] and Froustey *et al.* [10].

As proposed by Froustey et al. [10], Palin-Luc and Lasserre [1] use the mean value on one cycle of the volumetric density of the elastic strain energy, *Wa*, defined by (1) whatever the point *M* in the mechanical part for a sinusoidal fully reversed loading. $\sigma_{ij}(M,t)$ and $\varepsilon_{ij}^{e}(M,t)$ are respectively the tensor of stresses and the tensor of elastic strains at the considered point *M* function of time.

$$Wa(M)=\frac{1}{T}\int_0^T\frac{1}{2}\sigma_{ij}(M,t)\,\varepsilon_{ij}^{e}(M,t)\,dt \qquad (1)$$

Usually the endurance limit is low enough to consider that the material remains elastic at the macroscopic scale [11]. Thus, *Wa* can be considered as the mean value on one cycle of the total strain energy density at the considered point. The *Wa* distributions on the cross-section of a smooth specimen loaded in traction, rotative bending and plane bending are very different such as shown in Figure 1. In order to take into account these differences the authors reason upon a volume around each critical point C_i . A point is critical with regard to fatigue if at this location *Wa* has a local maximum. From $\sigma *$ and by analogy with a fully reversed sinusoidal traction load the corresponding mean value of the strain energy volumetric density, *Wa**, can be calculated by (2), where *E* is the Young modulus of the material.

$$Wa^*=\frac{\sigma^{*2}}{4E} \qquad (2)$$

Around each critical point C_i , it is always possible to define the volume $V^*(C_i)$ by the set of points *M* where $Wa(M)$ is higher than $Wa^*(C_i)$ (see equation (3)). From $V^*(C_i)$, $\varpi_a(C_i)$ is defined by (4). $\varpi_a(C_i)$ is the volumetric mean value of the strain energy density around the critical point C_i .

$$V^*(C_i)=\left\{\text{points } M(x,y,z) \text{ around } C_i \text{ such that } Wa(M)\geq Wa^*(C_i)\right\} \qquad (3)$$

$$\varpi_a(C_i)=\frac{1}{V^*(C_i)}\iiint_{V^*(C_i)}\left[Wa(x,y,z)-Wa^*(C_i)\right]dv \qquad (4)$$

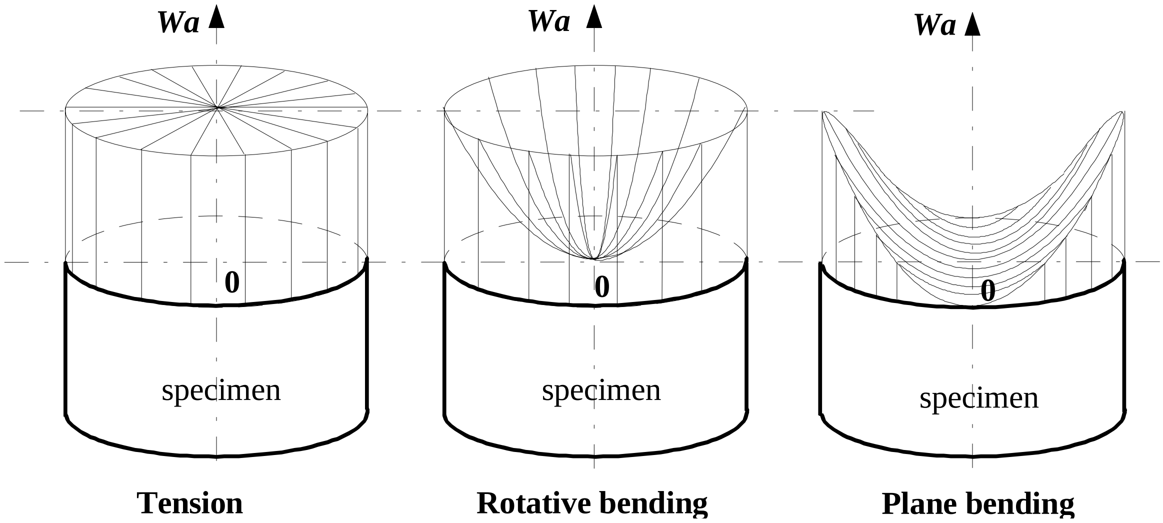

Figure 1: *Wa* distribution on the cross section of a smooth cylindrical specimen loading in tension, rotative bending and plane bending.

Uniaxial Stress State

At the endurance limit and at the critical point *C*, this new quantity $\varpi_a(C)$ is supposed to be constant, whatever the uniaxial stress state. If we note $\varpi_a^D(Uniax)$ its value at the endurance limit for any uniaxial stress state the volumetric energy based criterion can be written by inequality (5) where failure occurs if this inequality is not verified.

$$\varpi_a(C) < \varpi_a^D(Uniax) \tag{5}$$

Multiaxial Stress State

The influence of the triaxiality of stresses on the endurance limit has already been proven by several works. Palin-Luc and Lasserre propose to take into account this influence by defining the degree of triaxiality, dTa, as introduced by Froustey et al. [10]. They define dTa, for any fully reversed sinusoidal loadings, by expression (7) where *Wsa* and *Wda* are respectively the spherical part and the deviatoric part of the strain energy density (6). For any periodic loading it is easy to prove that $Wa = Wsa + Wda$.

$$Wsa = \left(\frac{1-2\upsilon}{6E}\right)\frac{1}{T}\int_0^T I_{1a}^2(t)dt \quad \text{and} \quad Wda = \left(\frac{1+\upsilon}{E}\right)\frac{1}{T}\int_0^T J_{2a}(t)dt \tag{6}$$

where $I_{1a}(t) = \sigma_{kk}(t)$ and $J_{2a}(t) = \frac{1}{2}s_{ij}(t)s_{ij}(t)$ by noting $\sigma_{ij}(t) = \frac{\sigma_{kk}(t)}{3}\cdot\delta_{ij} + s_{ij}(t)$

$$dTa = \frac{Wsa}{Wsa + Wda} \tag{7}$$

Based on many experimental data in high cycle fatigue, Froustey and Lasserre have proposed to relate the value of $Wa(C_i)$, whatever the loading and the stress state at the critical point, to the value of *Wa* in Torsion, *Wa(tors)*, by the function *F* (8) depending on the degree of triaxiality at the C_i point, $dTa(C_i)$, and a new material dependent parameter β.

$$F\big(dTa(C_i),\beta\big) \;=\; \frac{Wa(C_i)}{Wa(tors)} \;=\; \frac{1}{1-dTa(C_i)}\cdot\left\{1-\frac{1}{\beta}\cdot\ln\left[1+dTa(C_i)\cdot\left(e^{\beta}-1\right)\right]\right\} \tag{8}$$

The β parameter is representative of the material triaxiality sensitivity. β is equal to zero for a XC18 annealed steel and is around 3 for a spheroidal graphite cast iron. The identification of the β parameter has to be done by applying equation (8) with the endurance limits in rotative bending and in torsion.

In order to take into account the triaxiality influence the authors postulate that *Wa** is stress state dependent and verifies equation (9). Thus *Wa** is defined whatever the loading, *V** is also defined.

$$\frac{Wa*(C_i)}{Wa*(Tors)} = F\left(dTa(C_i),\beta\right) \tag{9}$$

The influence of triaxiality has also to be taken into account in the definition of the limit value of ϖ_a^D. By analogy with the previous assumption, we postulate that for any loading and stress state at the endurance limit the value of ϖ_a^D, noted $\varpi_a^D(C_i, load)$, verifies (10). This quantity is dependent on the stress state at C_i and of the volumetric distribution of stresses inside V*. Then the criterion can be applied on any fully reversed loading.

$$\frac{\varpi_a^D(C_i, load)}{\varpi_a^D(Tors)} = F\left(dTa(C_i),\beta\right) \;\Rightarrow\; \frac{\varpi_a^D(C_i, load)}{\varpi_a^D(Uniax)} = \frac{F\left(dTa(C_i),\beta\right)}{F\left(dTa(Uniax),\beta\right)} \tag{10}$$

Criterion Application

To apply this energy based proposal $\sigma*$, β, the Young modulus, *E*, and the Poisson ratio, ν, of the material have to be known. Only three experimental endurance limits under fully reversed loadings are needed to identify $\sigma*$ and β : in traction, σ_{Trac}^D, in rotative bending, $\sigma_{RotBend}^D$ and in torsion, τ_{To}^D. These endurance limits can be considered as material parameters if the radius of specimen is larger than about 5 mm [7]. From equation (8) in case of rotative bending, the β material parameter can be identified since $\sigma_{RotBend}^D$ is supposed known.

At the endurance limit, this proposal predicts that the terms of the tensor of stresses are solutions of equation (10). In this equation $\varpi_a^D(C_i, load)$ is defined by :

$$\varpi_a^D(C_i, load) \;=\; \frac{1}{V*(C_i)} \iiint_{V*(C_i)} \left[Wa(x,y,z,load) - Wa*(C_i)\right] dv \tag{11}$$

$$\text{where} \quad V*(C_i) = \left\{\text{points } M(x,y,z) \text{ such that } Wa(M) \geq Wa*(C_i)\right\}$$

$$\text{and} \quad Wa*(C_i, load) = Wa*(Uniax)\frac{F\left(dTa(C_i),\beta\right)}{F\left(dTa(Uniax),\beta\right)}$$

APPLICATION OF THE CONCEPT OF VOLUME INFLUENCING FATIGUE CRACK INITIATION TO A MESOSCOPIC APPROACH

The aim of this part is to show that the concept of the volume influencing fatigue crack initiation proposed by Palin-Luc et al. [1] is not restricted to an energy based approach. Indeed, other criteria can be modified so as to consider the stress distribution around a critical point. Let us consider for example the fatigue criterion developed by Papadopoulos these last years [3], [12], [13] and based upon a mesoscopic approach.

Endurance criterion expressed at critical locations

In this model, crystals of a metallic aggregate are assumed to follow a combined isotropic and kinematical hardening rule when flowing plastically. It is long known that metal grains possess some preferred orientations (slip systems) along which plastic strain can develop. A slip plane and a slip direction on this plane constitutes each slip system. The author showed that the accumulated plastic strain along a slip direction on a slip plane induced by an external cyclic load becomes nearly proportional to the macroscopic resolved shear stress amplitude T_a, when the number of load cycles increases indefinitely:

$$\sum_{\infty} \Delta\gamma^{p} \propto T_a \qquad (12)$$

To avoid that the accumulated plastic strain exhausts the ductility of the crystal, a critical value can be defined as a material parameter. The limitation of this mechanical quantity leads to a condition precluding the creation of a micro-crack within an elementary material volume. Although a so defined fatigue criterion could be of interest to investigate the fatigue strength of single crystals, it is of no use within the engineering framework since in this context one has to prevent the creation of a fatigue crack of the same size as the elementary volume.

This fatigue engineering criterion is then based on two average measures. The first one is related to the plastic strain accumulated in all the flowing crystals within the elementary volume (13).

$$\sqrt{\left\langle T_a^2 \right\rangle} = \sqrt{5}\sqrt{\frac{1}{8\pi^2}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{\psi=0}^{2\pi}\left(T_a\left(\varphi,\theta,\psi\right)\right)^2 d\psi \sin\theta d\theta d\varphi} \qquad (13)$$

The angle ψ varying from 0 to 2π covers all the gliding directions on a material plane whereas the angles φ and θ varying from 0 to 2π and from 0 to π, respectively, cover all the possible orientations of the material plane inside the elementary volume.
The second average measure is built according to the normal stresses acting on all the possible material planes:

$$\left\langle N \right\rangle = \frac{1}{4\pi}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} N\left(\varphi,\theta\right)\sin\theta d\theta d\varphi \qquad (14)$$

It is worth mentioning that this volumetric mean is nothing else but the hydrostatic stress Σ_H.

Finally, the multiaxial endurance criterion is defined as inequality (15) applied to a linear combination of the average quantities proposed above.

$$\sqrt{\left\langle T_a^2 \right\rangle} + a\,\Sigma_{H,max} \le b \qquad (15)$$

Endurance criterion taking into account the stress distribution [14]
The criterion described in the previous section is unable to differentiate two different stress distributions as far as the stress state at critical points remains unchanged (e.g. tension-compression and rotative bending).
To take into account all the potential crack nucleation sites, it seems natural to build a criterion by considering in a specimen all the points where a crack is likely to occur. According to the studied criterion, a threshold condition (16) applied to the parameter $\sqrt{\left\langle T_a^2 \right\rangle}$ is enough to describe the crack initiation at critical points.

$$\sqrt{\left\langle T_a^2 \right\rangle} \le q^* - p\,\Sigma_{H,max} \qquad (16)$$

where q* and p are two material constants.
By introducing an equivalent stress given by (17):

$$\sigma_{eq} = \sqrt{\left\langle T_a^2 \right\rangle} + p\,\Sigma_{H,max} \qquad (17)$$

it is possible to define a volume V* around a critical point C_i as composed by the points verifying $\sigma_{eq} \ge q^*$ that is :

$$V^* = \left\{ \text{M points around } C_i \text{ so that, } \sigma_{eq} \ge q^* \right\} \qquad (18)$$

This threshold value q* is a material parameter and can be seen as the equivalent stress necessary to initiate crack at critical locations.

To consider the stress distribution effect, it can be now proposed to carry out averages on the mechanical parameters used in the criterion:

$$\frac{1}{V^*}\iiint_{V^*}\left(\sqrt{\left\langle T_a^2\right\rangle}\right)dV \qquad \text{and} \qquad \frac{1}{V^*}\iiint_{V^*}\left(\Sigma_{H,max}\right)dV$$

Finally the new criterion is written as inequality (19).

$$\frac{1}{V^*}\iiint_{V^*}\left(\sqrt{\left\langle T_a^2\right\rangle}+p\,\Sigma_{H,max}\right)dV \le q \tag{19}$$

with $V^* = \{M \text{ points around } C_i \text{ so that, } \sigma_{eq} \ge q^*\}$ and $\sigma_{eq} = \sqrt{\left\langle T_a^2\right\rangle}+p\,\Sigma_{H,max}$

The crack initiation is then seen as the coalescence of many crack nuclei so that to form a crack of the same size order than the elementary material volume.
The use of this criterion requires the identification of three material constants : p, q and q*. It can be done by means of three fully reversed fatigue limits :

- tension-compression: s_{-1}
- torsion : t_{-1}
- rotative bending : f_{-1}

$$p = 3\cdot\frac{t_{-1}}{f_{-1}} - \sqrt{3}$$

$$q = \frac{t_{-1}\cdot s_{-1}}{f_{-1}}$$

$$q^* = \frac{t_{-1}\cdot s_{-1}}{f_{-1}}\cdot\left[\frac{3}{4}+\frac{1}{2}\frac{f_{-1}}{s_{-1}}\left(\sqrt{\left(\frac{3}{2}\frac{s_{-1}}{f_{-1}}+1\right)^2-4}-1\right)\right]$$

One main advantage of this new criterion is that for some stress distributions (e.g. combined rotative bending and torsion), analytical expressions of the mechanical parameters $\frac{1}{V^*}\iiint_{V^*}\left(\sqrt{\left\langle T_a^2\right\rangle}\right)dV$ and $\frac{1}{V^*}\iiint_{V^*}\left(\Sigma_{H,max}\right)dV$ can be achieved even for out-of-phase loadings. This criterion is also able to reflect the different fatigue strengths of specimens submitted to rotative bending and plane bending.

COMPARISON PREDICTIONS / EXPERIMENTS

The predictions of the two previous calculation methods are compared with experimental data in multiaxial fatigue on four materials : a 30NCD16 quenched and tempered steel [8], a XC18 annealed steel, a 35CD4 quenched and tempered steel and a spheroidal graphite cast iron close to the FGS800-2 (Afnor standard). For an objective and easy comparison the Relative Error of Prediction, *REP*, is defined by (20).

$$REP(\%) = \frac{\sigma^D_{Experiments} - \sigma^D_{Prediction}}{\sigma^D_{Experiments}} \times 100 \tag{20}$$

The Relative Error of Prediction, *REP*, is presented in Table 1. All the *REP* are between -10% and +10%.

This shows that the predictions of the proposals are in very good agreement with experimental data.

The good accuracy of these criteria proves that the concept of volume influencing fatigue crack initiation can be used as a general concept with a global energetic approach or with a mesoscopic criterion based on a critical plane approach. Of course other comparisons between predictions and experimental data have to be done on notched specimens.

TABLE 1.
EXPERIMENTAL RESULTS AND RELATIVE ERROR OF PREDICTION, *REP*, OF THE CRITERIA. THE ITALIC VALUES HAVE BEEN USED TO IDENTIFY THE MATERIAL PARAMETERS FOR EACH CRITERION (σ^*, β and p, q, q*).

Material and parameters	Loadings	σ^D (MPa)	τ^D (MPa)	σ^D/τ^D	φ (deg.)	*Energy REP* (%)	*Papadop. Modif. REP* (%)
30NCD16	*Traction*	*560*	-	-	-	-	
σ^*=441 MPa	*Rotative Bending*	*658*	-	-	-	-	
β=0.96	*Torsion*	-	*428*	-	-	-	
p=0.219	Plane Bending	690	-	-	-	-4.5	-4.6
q=364 MPa	Plane Bending + Torsion	519	291	1.78	0	-8.1	-6
q*=292 MPa	Plane Bending + Torsion	514	288	1.78	90	-9.1	-7
	Rotative Bending + Torsion	337	328	1.03	-	-8.3	-5.6
	Rotative Bending + Torsion	482	234	2.06	-	-9.3	-7.5
XC18	*Traction*	*273*	-	-	-	-	
σ^*=230 MPa	*Rotative Bending*	*310*	-	-	-	-	
$\beta \approx 0$	*Torsion*	-	*186*	-	-	-	
p=0.068	Plane Bending	332	-	-	-	-6.3	0
q=164 MPa	Plane Bending + Torsion	246	138	1.78	0	1.6	0.4
q*= 139 MPa	Plane Bending + Torsion	246	138	1.78	45	1.6	0.4
	Plane Bending + Torsion	264	148	1.78	90	8.3	7.2
35CD4	*Traction*	*558*	-	-	-	-	
σ^*=534 MPa	*Rotative Bending*	*581*	-	-	-	-	
β=1.33	*Torsion*	-	*384*	-	-	-	
p=0.251	Plane Bending	620	-	-	-	4.3	4.5
q=369 MPa							
q*= 353 MPa							
FGS 800-2	*Traction*	*245*	-	-	-	-	
σ^*=204 MPa	*Rotative Bending*	*280*	-	-	-	-	
β=3.09	*Torsion*	-	*220*	-	-	-	
p=0.625	Plane Bending	294	-	-	-	-9.2	-2.7
q=193 MPa	Plane Bending + Torsion	228	132	1.73	0	-7.2	-1.8
q*=162 MPa	Plane Bending + Torsion	245	142	1.73	90	0.2	5.3
	Plane Bending + Torsion	199	147	1.35	0	-10.2	-5

CONCLUSIONS

The predictions of a global volumetric energy based criterion and a mesoscopic criterion modified by using the concept of volume influencing fatigue crack initiation V* proposed by Palin-Luc and Lasserre [1] are in very good agreement with experimental fatigue data on smooth specimens made with four metallic materials. Other comparisons between predictions and experiments have to be done especially on notched specimens to confirm the general meaning of the V* concept. Such comparisons have already been done on some experimental data for the energy based criterion, the results are also in very good agreement with fatigue tests [2]. Furthermore, one can show that in combined bending and torsion, predictions of the two volumetric proposals are closed to the Gough and Pollar ellipse quadrant [1], [14] and are phase independent as proved by long life fatigue tests. Under biaxial tension their predictions are phase dependent which is in agreement with experiments. These first generalisation of the volume influencing fatigue crack initiation concept is very promissing to progress in fatigue strength prediction for components with a complex geometry and a complex volumetric stress distribution.

REFERENCES

1. Palin-Luc, T. and Lasserre S. (1998) *European Journal of Mechanics, A/Solids*, **17**, No. 2, 237-251.
2. Palin-Luc, T., Lasserre S. and Bérard J-Y. (1998) In: *Proc. Fatigue Design 98*, pp. 73-84, Marquis G. and Solin J. (Eds).
3. Papadopoulos, I.V. (1995), *Fatigue Fract. Engng Mater. Struct.*, **18**, No. 1, 79-91.
4. Sonsino, C.M., Kaufmann, H. et Grubisic, V. (1997) *SAE Tech. paper series*, 22 p.
5. Palin-Luc, T., Kaufmann, H., Lasserre S. and Sonsino, C.M. (1999) In: SF2M, 18èmes Journées de Printemps de la commission Fatigue, 2 et 3 juin, Paris, France, pp. 30(1)-30(10).
6. Flavenot, J-F. and Skally, W. (1983) *Rev. Mécanique Matériaux et Electricité*, **397**, 15-25.
7. Papadopoulos, I.V. and Panoskaltsis, V.P. (1994) In: *Proc. 4th Int. Conf. Biaxial/Multiaxial fatigue*, Vol. 1, 461-476, SF2M Ed., France.
8. Froustey, C. (1987) Ph. Thesis, E.N.S.A.M. CER de Bordeaux, France.
9. Tsybanev, G.V. (1994) In: *Problemy Prochnosti*, Ukrain. Ac. Sci., Plenum Publishing Corp., No. 2, 19-27.
10. Froustey, C., Lasserre, S. and Dubar, L. (1992) In: Mat-Tech 92, pp. 79-85, IITT-International, France.
11. Lemaitre, J. and Chaboche, J-L. (1988) *Mécanique des matériaux solides*, Bordas, Paris.
12. Papadopoulos I.V. (1993) Technical Note N° I.93.101, Commission of the European Communities, Joint Research Centre, ISEI/IE 2495/93.
13. Papadopoulos I.V., Davoli P., Gorla C., Filippini M., Bernasconi A. (1997) *Int. J. Fatigue*, **19**, No.3, 219-235.
14. A. Ledren, F. Morel (1998), Influence du gradient de contrainte en fatigue multiaxiale polycyclique, Rapport de DEA, LMPM-ENSMA, Futuroscope.