# NUMERICAL DETERMINATION OF THE DYNAMIC KEY CURVES

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## ABSTRACT

'Dynamic Key Curve' (DKC) is the name of the method proposed by W.Böhme for determination of the dynamic stress intensity factor (DSIF) variation with time from the results of an instrumented impact test. According to this method, DSIF can be expressed as the product of a quasi-static SIF multiplied by a dynamic correction function, which normalized form is called DKC. In this article, numerical procedure for DKC calculations has been proposed. Commercial finite element program ADINA has been used for 2D (plane stress) modelling of the instrumented impact tests for a wide range of configurations of the specimen. The influence of the specimen geometry parameters on the shape and amplitude of DKC have been investigated.

## INTRODUCTION

Procedures of determination of dynamic fracture toughness of brittle materials are quite similar to their quasi-static counterparts. This similarity is based on the common theoretical background of linear fracture mechanics. Additionally, specimens used in dynamic and quasi-static tests are practically the same. This similarity has it positive and negative sides. On the one hand, it simplifies for a researcher or an engineer transition from the conventional and relatively simple quasi-static tests to more complicated dynamic ones. On the other side, it often encourage unjustified utilization of the quasi-static methods to process the results of a dynamic test.

Similarly as in statics, the aim of a dynamic fracture test (only conventional impact tests will be considered in this study) is to determine the critical value of dynamic stress intensity factor (DSIF) at the onset of crack growth. To compute this critical value, well-known relationships between the SIF and external loading are used in statics. This method utililizes an overall equilibrium of forces in the system which consists of the specimen and the testing machine. Due to this equilibrium, loading measured by the load gauge of the testing machine is equal to the internal forces in the specimen.

In general, it is impossible to apply this approach for dynamic tests. The word *dynamic* itself implies that inertia forces act on the specimen during a test together with the external forces. Thus, the internal forces within the specimen are not equal to the external ones. This difference is especially large at the very beginning of an impact test when inertia effects dominate. Of course, no purely quasi-static method can be used to determine DSIF during this stage of the specimen deformation.

Numerous attemts to modify the quasi-static procedure of DSIF evaluation using the load registed during a test are known. In practice, such procedures inevitably limit impact velocity either indirectly [1,2] or directly [3]. Searching for simple and sufficiently accurate methods for DSIF determination for high-velocity tests still remain an important and unsolved problem.

Perhaps one of the most reasonable ways to solve this problem was proposed by Böhme [4-6] as the 'Dynamic Key Curves' (DKC) method. It is based on the deep understanding of the fact that the specimen deformation during an impact test is rather striker *displacement* then striker *load* controlled process. Thus, it is preferable to evaluate the quasi-static part of the specimen response using practically linearly increasing deflection of the specimen than the highly oscillating tup force. According to the Böhme, DSIF $K_I(t)$ can be expressed as a product of a quasi-static part $K_I^{qs}(t)$ and dynamic correction function $k^{dyn}(t)$, which in normalized form is called *dynamic key-curve* (DKC):

$$K_I(t) = K_I^{qs}(t) \cdot k^{dyn}(t) \tag{1}$$

where $K_I^{qs}(t)$ can be computed using an analytic formula based on impact velocity and compliance of the specimen and the testing machine. In fact, this term represents SIF for the same but massless specimen under the same impact loading. The dynamic correction function was proposed to be determined once in model experiments.

The main advantage of this method is the simplicity of determination of critical DSIF values. If DKC for the given specimen geometry is known, one needs to measure the time-to-fracture only during a test to determine $K_{Id}$. Thus, the availability of DKC for a particular specimen configuration is the inevitable requirement for this method to use. Unfortunatelly, pure experimental method of DKC determination proposed by Böhme is too complicated and expensive. That is the reason why DKCs have been determined for four specimen configurations so far.

The main goals of this article are to propose the simple numerical procedure for DKC calculations for a wide range of specimen configurations used in impact tests and to investigate the influence of the specimen geometty parameters on the shapes of these curves.

## CALCULATIONS

According to the definition, DKC can be determined as the ratio between the dynamic and quasi-static SIF determined for the same specimen deflection. Thus, in the computations the impact specimen deformation during a test has been modelled both as a transient dynamic and a quasi-static problem, respectively. All calculations have been performed using commercial finite element program ADINA for the two-dimensional (plane stress) model of the specimen (see Fig. 1). It has been assumed that:

1. The impact velocity $v$ does not change during a test
2. Striker and supports are perfectly stiff and have permanent curvatures of their tips $r_{str}$ and $r_{sup}$, respectively.

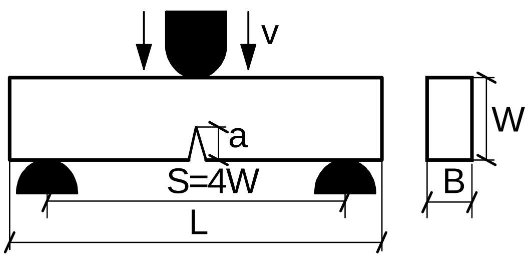

Figure 1. Scheme of the specimen

The first assumption simplifies calculations and has no influence on the accuracy of DKC determination. The second assumption is satisfied when brittle polymers are tested. For metallic materials, finite stiffness of the striker and supports affects contact stiffness between the specimen and the testing machine and. therefore, should be taken into account. For such a material, the perfect stiffness case may be considered as an overestimation of the real specimen/testing machine contact conditions. Accordingly, oscillations of the corresponding dynamic key-curves could be considered as the upper bond estimation of the oscillation of real DKCs.

To test the accuracy of the method proposed, numerical DKCs have been computed for the large scale Araldite B specimens used by Böhme [4-6] to determine DKC experimentally. Four configurations of the specimen ($L$=550 and 412 mm, $a$=30 and 50 mm) with $W$=100 mm, $B$=10 mm, $r_{str}$=8 mm and $r_{sup}$=10 mm have been considered. Material properties (Young modulus $E$=3.38 GPa, Poisson'a ratio $\nu$=0.33, density $\rho$=1216 kg/m$^3$) and impact velocity $v$=1 m/s were the same for all tests. Both dynamic and static SIF values were determined using standard methods implemented in ADINA.

To obtain the data for experimental DKCs, corresponding plots from Ref.[4] were scanned and 'digitized' by the freeware computer program Marisoft Digitizer 3.3. It is worth to note, that DKCs based on the experimental points in these plots as well as in the similar plots presented in more recent publications [5,6] do not approach the limit unity value. In fact, their limit values are notably smaller. This difference is more pronounced for stiffer specimens with shorter cracks. Perhaps, this phenomenon is caused by the underestimation of the specimen compliance in the procedure used by Böhme to determine $K_{\mathrm{I}}^{\mathrm{qs}}(t)$. He used the specimen compliance determined by Bucci's formula [7] and did not take into account the additional contact compliance arising from the interaction between the specimen and the testing machine. The total compliance of the specimen with contact compliance included can be easily determined from the results of the quasi-static analysis of a test. For the test arrangement considered, such a total compliance is greater than the 'pure' specimen compliance determined by Bucci's formula by 1.9% for $\lambda$=$a/W$=0.5 and by 6% or7% (larger value relates to shorter specimen) for $\lambda$=0.3.

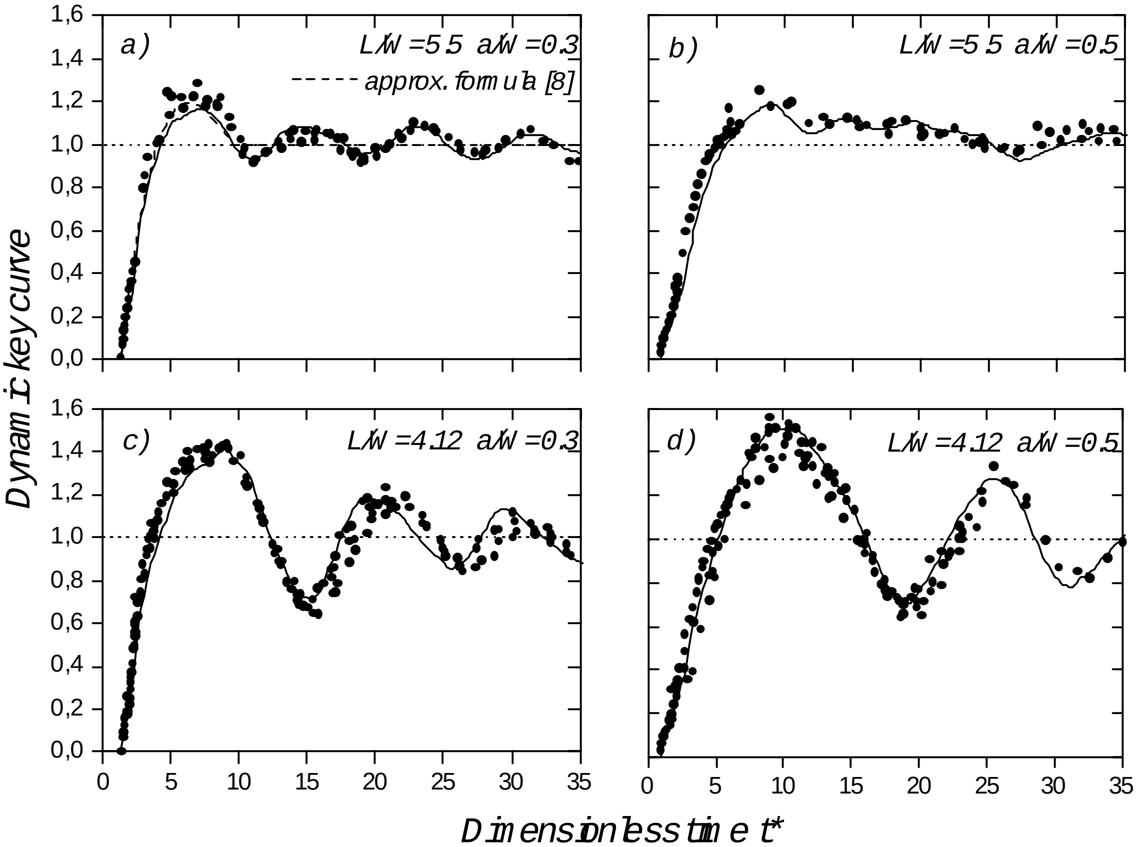


**Figure 2:** Comparison of the numerical DKCs (solid lines) with experimental data (points) for four different types of the impact specimen

Experimental data including the corrections mentioned above are compared with numerical results in the Fig. 2. In all subfigures of Fig. 2 proposed by Böhme dimensionless time $t^*=c_1t/W$ is used, where

$$c_1 = \sqrt{E/(\rho(1-\nu^2))} \tag{2}$$

is the longitudinal wave velocity for plane stress. Additionally, DKC that was computed using approximated formula proposed by Böhme [8] for the specimen with $L/W$=5.5, $\lambda$=0.3 is plotted in the Fig. 2a. In general, there is a good agreement between experimental and numerical data for all configurations of the specimen.

After these tests of the computational procedure, DKC were calculated for the following range of the specimen geometry parameters: $L/W$ = 4.5, 5.0, 5.5, 6.0, $\lambda$ = 0.3,0.4,0.5,0.6, $r_{str}/W = r_{sup}/W$=0.2. All computations were performed for a model material with $E$=$\rho$=1, $\nu$=0.3.

## RESULTS AND DISCUSSION

Preliminary plan of this study assumed to investigate the influence of Poisson's ratio on DKC variation with time. The results of calculations, however, have shown that this influence can be neglected if the proper normalization of the time scale is used.

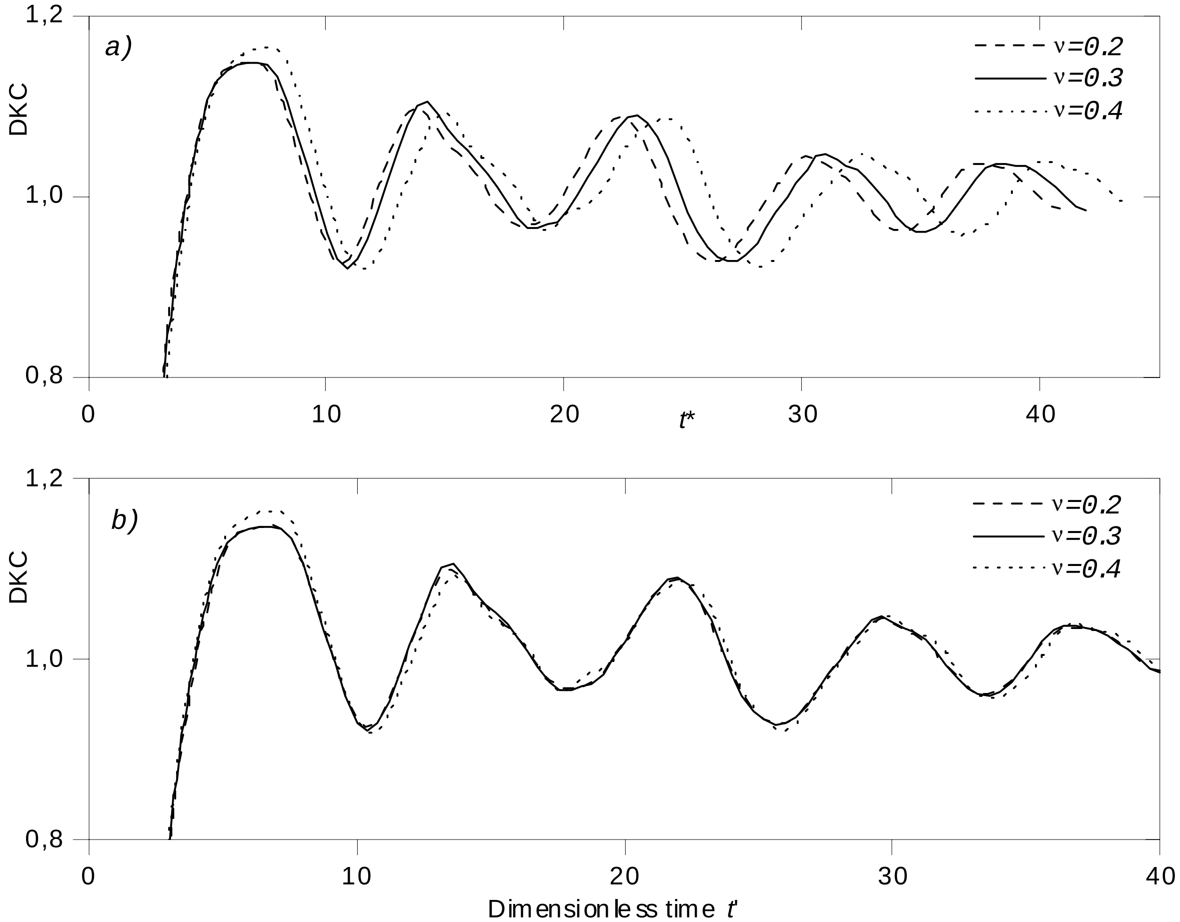


**Figure 3:** DKCs for different Poisson's ratios of the material.

In Fig. 3 dynamic key-curves computed for the same configuration of the specimen ($L/W$=5.5, $\lambda$=0.3) for the material with different values of $\nu$ are presented. Time axis in Fig. 3a is normalized by dimensionless time $t^*$. In this case, normalization formula contains longitudinal wave velocity $c_1$, which depends on Poisson's ratio of the material (see Eqn. 2). Due to this reason DKCs presented in Fig. 3a have different duration of their oscillations. Fortunatelly, this disagreement practically disappears when another form of dimensionless time $t'=c_0t/W$ is used (see Fig. 3b). Here $c_0=(E/\rho)^{1/2}$ is the bar elastic wave velocity. In such a case the maximum difference between DKC values determined for $\nu$=0.2 and $\nu$=0.4 is less than 2%. Thus, if the new form of dimensionless time is used, DKCs obtained for $\nu$=0.3 could be used for all values of Poisson's ratio within [0.2,0.4] range without any corrections.

All the rest calculations in this study have been performed for $\nu$=0.3, for the time range $0<t'<40$ and for the same dimensionless impact velocity $7.5\times10^{-4}\,c_0$, which corresponds to the velocity of 1.25 m/s for Araldite B or about 4 m/s for steels.

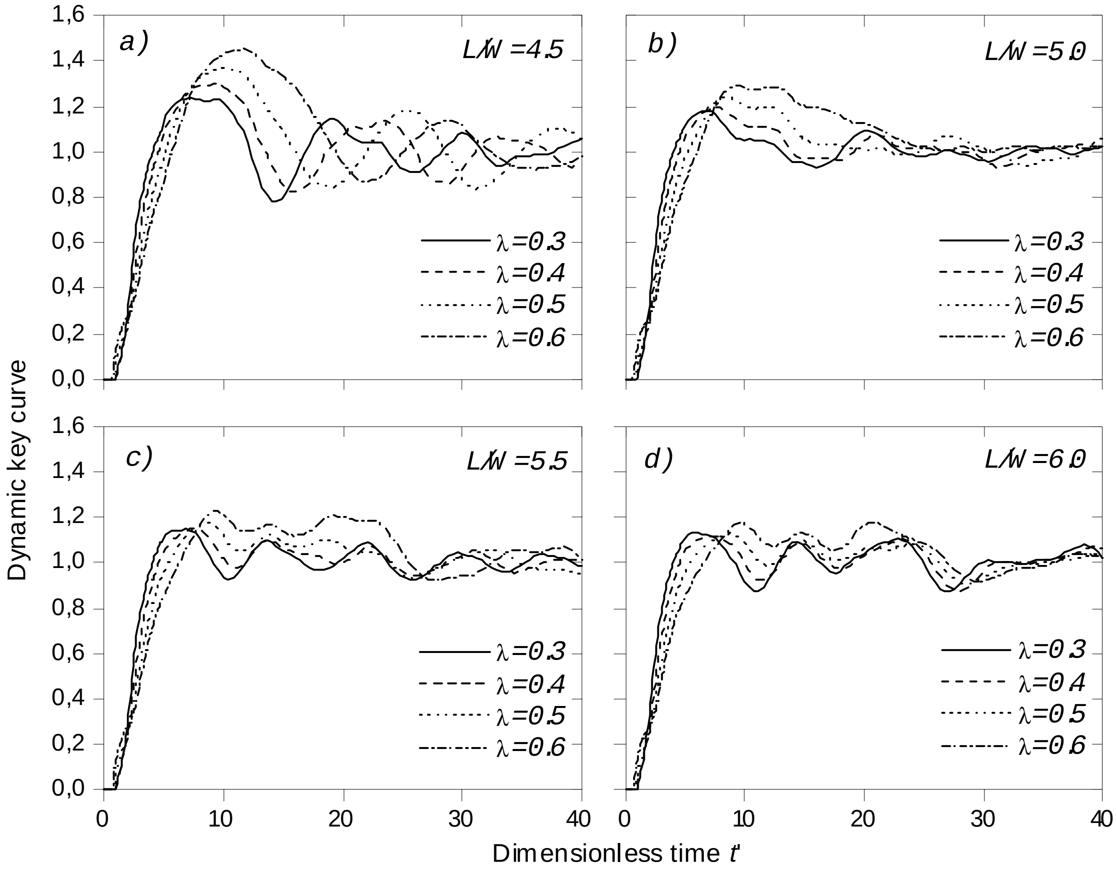


**Figure 4:** DKC variation with time for different specimen configurations

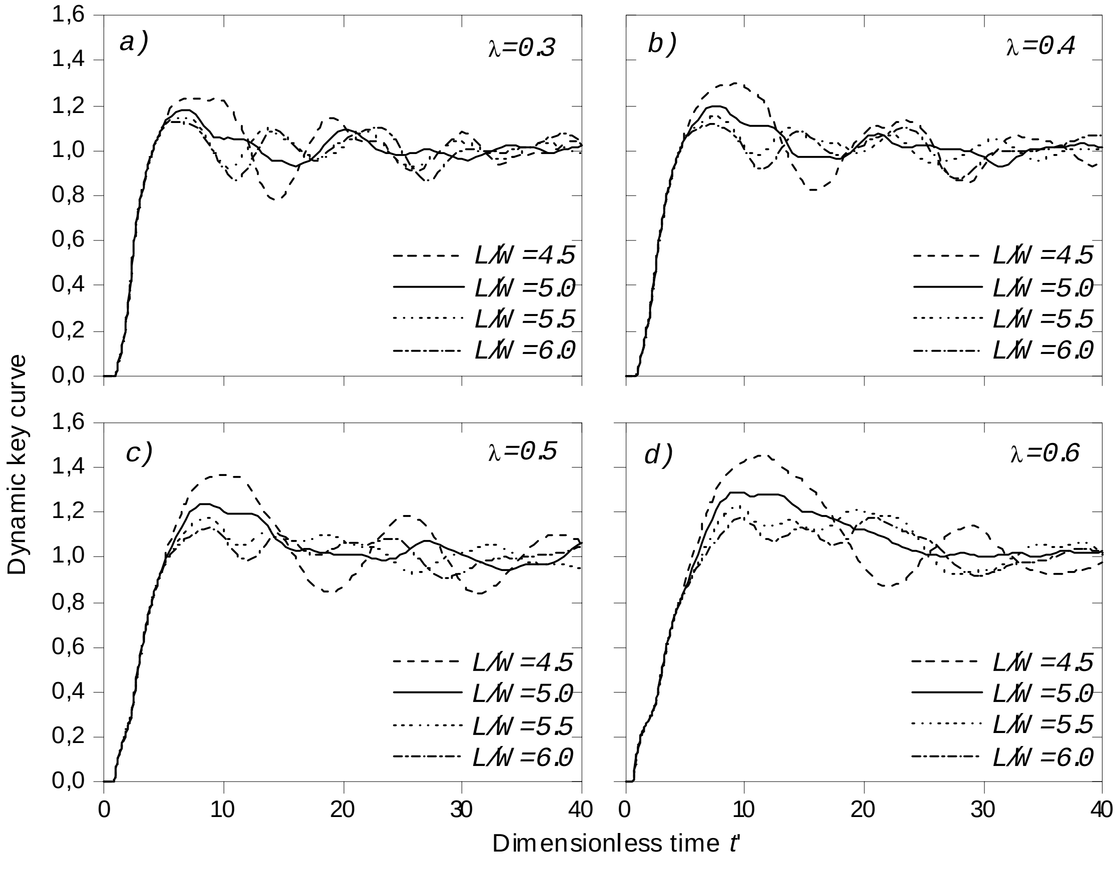


**Figure 5:** DKC variation with time for different specimen configurations

In each of four subfigures of Fig. 4 DKCs for the same relative specimen length *L/W* and different relative crack lengths are presented. In general, results obtained agree with the observations presented by Böhme and Kalthoff [9]. Namely, oscillations of DSIF (and, therefore, oscillations of DKC) are larger for shorter specimens. For the range of the specimen configurations considered in this study, DKC with the smallest and quickly vanishing oscillations are observed for the specimens with *L/W*=5.0-5.5 and $\lambda$=0.3. More detailed analysis shows that the optimum specimen configuration from this point of view is *L/W*=5.1-5.2, $\lambda$=0.3 [10].

Contrary to Fig. 4, in Fig. 5 DKCs for the same relative crack length and different *L/W* are collected in corresponding subfigures. It is easy to see that for the short initial period (usually associated with so-called one-point bend stage of specimen deformation) DKCs are independent of the relative specimen length. When the relative crack length increases, the specimen becomes more flexible and lager oscillations of DKC with longer time for their decay are observed.

## CONCLUSIONS

Results presented in this paper show that dynamic key curves for different configurations of the impact specimen can be determined numerically with the accuracy sufficient for practical purposes. A new procedure for normalization of DKCs is proposed to eliminate their artificial dependence on the Poisson's ratio. Results obtained show that oscillations of DKCs for the specimens with *L/W*=5.0-5.5 and $\lambda$=0.3 decay the most quickly.

All DKCs obtained in this study have been implemented into the version 2.1 of the free program *DSIFcalc* created for processing the data of an instrumented impact test. Current version of the program is available for downloading at `http://www.kielce.pl/~rokach/dsifcalc.htm.` This program allows a user to use several (five in the current version) different methods for DSIF computations and to compare the results with her or his own independent experimental or numerical data.

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