MODELLING THE MIXED MODE FRACTURE OF CONCRETE

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ABSTRACT

This paper presents an efficient numerical procedure for mixed mode fracture of quasibrittle materials. The model is based on the cohesive crack approach, and extends it from mode I to mixed mode (modes I and II) of fracture. In contrast to more sophisticated models, this method offers a major advantage: a diminution in the number of parameters, all of which have a clear physical meaning. The numerical results agree quite well with two sets of experiments of mixed mode fracture of concrete beams; one by Arrea and Ingraffea and the other from a *nonproportional loading* by the authors.

INTRODUCTION

Considerable effort has been devoted in recent years to the development of numerical models to simulate the fracture behaviour of quasi-brittle materials, such as mortar, concrete, rock or bricks, used in civil engineering structures. Even in two dimensions, the modelling of the fracture behavior of these materials is a complex problem. Traditionally, the numerical methods based on the FEM are classified in two groups [1]: "smeared crack" and "discrete crack "; although some authors include a third group: the "lattice approach" [2].

In the smeared crack approach the fracture is represented in a smeared manner; an infinite number of parallel cracks of infinitely small opening are (theoretically) distributed (smeared) over the finite element [3]. The cracks are usually modelled on a fixed finite element mesh. Their propagation is simulated by the reduction of the stiffness and the strength of the material. The constitutive laws, defined by stress-strain relations, are non-linear and show a *strain softening*. This approach was pioneered by [4 to 9] and more elaborated models have also been proposed [10, 11]. However, the *strain softening* introduces some difficulties in the analysis [12 to 13], which have been tackled [14 to 16], but there is no general solution of the problem.

The discrete approach is preferred when there is one crack, or a finite number of cracks, in the structure. The cohesive crack model, developed by Hillerborg and coworkers [17] for mode I fracture of concrete, was shown to be very efficient to model the fracture process of quasi-brittle materials. It has been extended to mixed mode fracture (modes I and II) and incorporated into finite element codes [18 to 20], as well as in boundary element codes [21]. One of the difficulties with these codes is that they require an input of material properties that are difficult to evaluate, such as the fracture energy and the softening function in mode II.

As shown by [22, 23] for quasibrittle materials, under *global mixed mode* loading there is an important *local mixed mode* when the crack starts from the notch, but when the crack is growing in a stable manner under *global* *mixed mode* loading the *local mode I* growth is predominant. This leads to an easier formulation of the mixed mode fracture models, with fewer parameters. This work presents a numerical model for mixed mode fracture based on this idea, with the advantage that it only requires the input of material properties easy to measure and with fully physical meaning. The model has been incorporated in a commercial Finite Element Method code (ABAQUS®) and verified with a set of experimental records of mixed loading in concrete [22]. The numerical model predicts different crack paths and the experimental records of the load *versus* the displacement of several control points, for geometrically similar specimens of three sizes of the test beams. Another set of experimental data was used to verify the procedure: [24].

THE COHESIVE CRACK MODEL

The cohesive crack model, called *fictitious crack model* by Hillerborg and co-workers, has been used successfully in the analysis of the fracture of concrete, rock and cement based materials since its proposal [17]. Part of this success is due to its simplicity and physical meaning. A detailed review of this model is found in [3]. The softening function, $\sigma = f(w)$, is the main ingredient of the cohesive crack model. This function, a material property, relates the stress σ acting across the crack faces to the corresponding crack opening w (Figure 1). For mode I opening, the stress transferred, σ, is normal to the crack faces.

Two properties of the softening curve are most important: the tensile strength, f'_t, and the cohesive fracture energy, G_F. The tensile strength is the stress at which the crack is created and starts to open ($f(0) = f'_t$). The cohesive fracture energy, G_F, also called *specific fracture energy*, is the external energy supply required to create a full unit surface area of a cohesive crack, and coincides with the area under the softening function. The tensile strength and the specific fracture energy are material properties and may be measured experimentally in accordance with ASTM C 496 and RILEM 50-FMC. Many softening curves have been developed to model the experimental fracture behaviour of concrete in tension [3]. The bilinear curves are accepted as reasonable approximations of the softening curve for concrete.

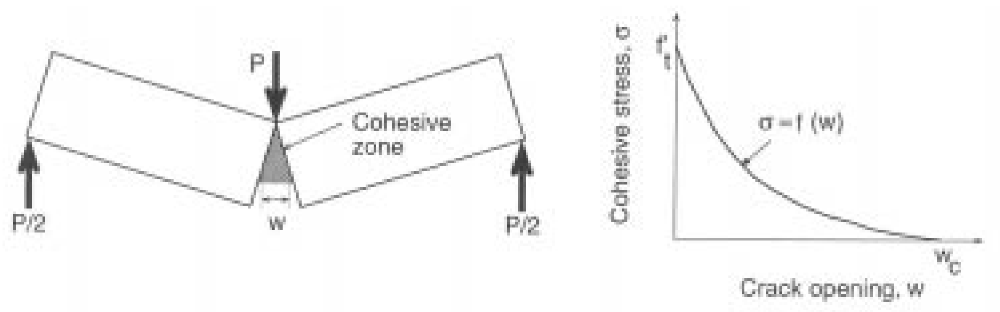

Figure 1: Cohesive crack and softening curve for mode I fracture of concrete

NUMERICAL PROCEDURE FOR MIXED MODE FRACTURE

The numerical simulation of the mixed mode fracture includes two main stages: 1) calculation of the crack path, and 2) incorporation of the cohesive model into the crack path. Linear Elastic Fracture Mechanics has proved its worth to predict the crack path [22, 23], which here is calculated according to the maximum tangential stress criterion [26]. Numerical details can be found in [23].

Cracking surface for mixed mode fracture

In a mixed mode (I and II) fracture, the interaction between normal stress, σ, and tangential stress, τ, should be taken into account. It is assumed that the crack grows when the combination of normal stresses, σ, and tangential stresses, τ, reaches a cracking surface $F(\sigma,\tau) = 0$, like a yield surface in classical plasticity. This work assumes the following hyperbolic expression [18]:

$$F = \tau^2 - 2c \tan\phi_f (f_t - \sigma) - \tan^2\phi_f (\sigma^2 - f_t^2) \quad (1)$$

where: c is the cohesion, ϕ_f the friction angle and f_t the tensile strength.

In accordance with the cohesive approach [17], the cracking surface evolves with the opening of the crack following softening curves of cohesion and tensile strength, defined from the softening parameter u^{ieff}, which is the integral norm of the vector of inelastic relative displacements between the crack faces, $\dot{\mathbf{u}}^i$, obtained by decomposition of the displacement vector, $\mathbf{u}$, into an elastic part, $\mathbf{u}^e$, and an inelastic part, $\mathbf{u}^i$. It is expressed: $\mathbf{u} = \mathbf{u}^e + \mathbf{u}^i$; $\dot{u}^{ieff} = \left\|\dot{\mathbf{u}}^i\right\| = \left(\dot{u}_x^{i\,2} + \dot{u}_y^{i\,2}\right)^{1/2}$, and the cracking surface $F = F\left(c, f_t, \phi_f\right)$, where $c = c\left(u^{ieff}\right)$ y $\sigma_t = \sigma_t\left(u^{ieff}\right)$. Figure 2 shows the softening curves.

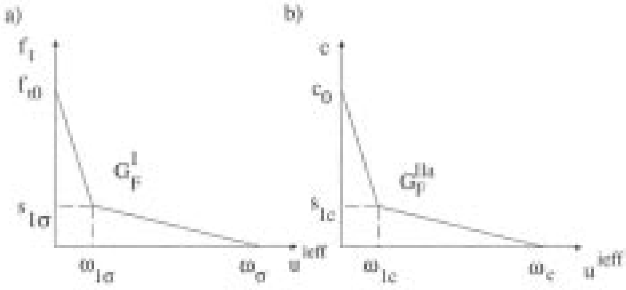

Figure 2: Softening curves: a) tensile strength, b) cohesion.

$G^I{}_F$ and $G^{IIa}{}_F$ are the specific fracture energy under modes I and IIa (mode II under large normal confinement); s_{1c}, ω_{1c}, $s_{1\sigma}$ and $\omega_{1\sigma}$, are the coordinates of the kink point in the softening curves. Figure 3 shows the cracking surface and its evolution.

The model was used to reproduce the mixed mode fracture tests of concrete developed by Arrea and Ingraffea [24] and by the authors [22]. A detailed numerical study of the normal and tangential stress evolution along the crack path during the tests showed that under a global mixed mode the local mode I was predominant.

If local mode I is predominant, the tensile strength and tensile stress are very similar and it may be assumed that $\sigma \approx f_t$, and then $\sigma + f_t = 2 f_t$. Including this in equation (1):

$$F = \tau^2 - 2\tan\phi_f\left(c - f_t \tan\phi_f\right)\left(f_t - \sigma\right) \tag{2}$$

The evolution of the cracking surface depends on the softening curves of cohesion and tensile strength. It is assumed that these curves evolve in a similar manner, showing proportional softening curves. There is no experimental evidence to the contrary. Mathematically it is expressed as follow: $f_t = f_t\left(u^{ieff}\right)$, $c = c\left(u^{ieff}\right)$, and

$$\frac{c}{f_t} = \frac{c\left(u^{ieff}\right)}{f_t\left(u^{ieff}\right)} = \alpha, \quad \alpha \in \Re > 0 \tag{3}$$

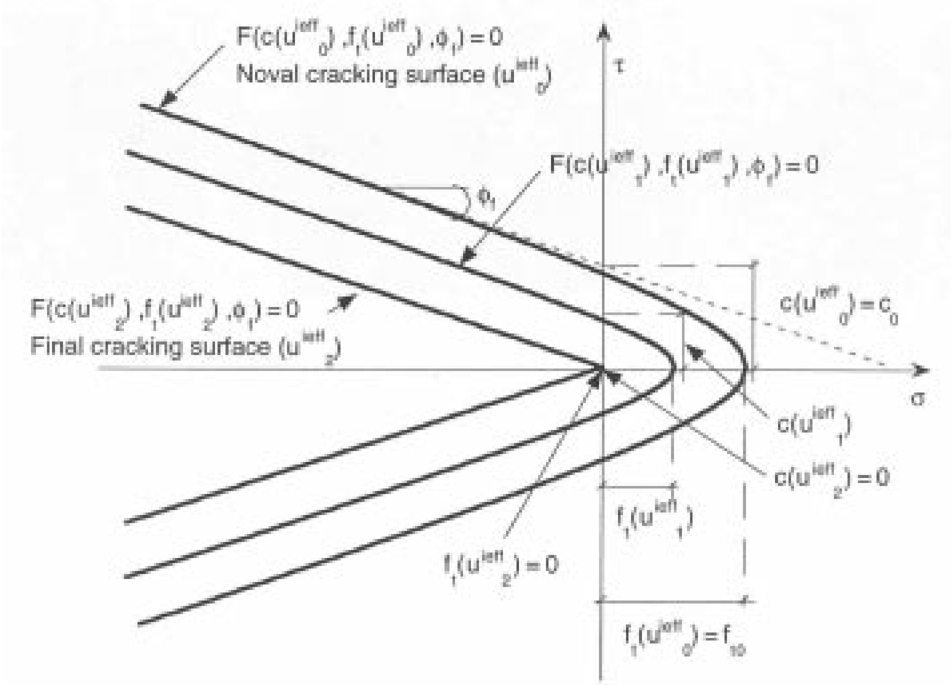

Figure 3: Cracking surface and its evolution

where α is positive real constant. Since the friction angle, ϕ_f, may be assumed constant, the following constant is defined:

$$C = 2\tan\phi_f\left(\alpha - \tan\phi_f\right) \tag{4}$$

and, finally the cracking surface is expressed as:

$$F = \tau^2 - Cf_t\left(f_t - \sigma\right) \tag{5}$$

Two major advantages are supplied by this equation: reduction of the parameters and simplicity. Figure 4 compares equations (1) and (5). Under predominant tension stresses the curves are practically equal, while under tangential stresses are quite different, but this behavior was no detected in the studied tests [22, 24].

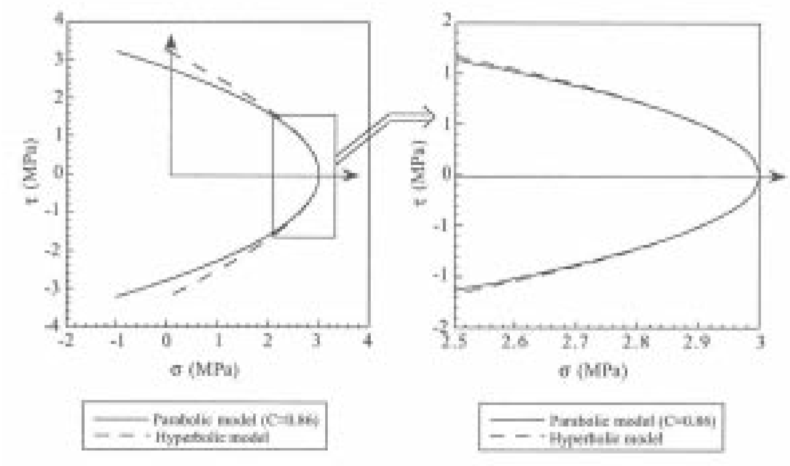

Figure 4: Comparison of parabolic and hyperbolic models of cracking surface.

Flow rule

In conjunction with equation (5) a flow rule is necessary to define the plastic displacements. For non-associative plasticity the flow rule is obtained from a potential function:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\left(\frac{\partial g}{\partial \sigma_{ij}}\right) \tag{6}$$

where $\dot{\varepsilon}^{p}$ is the incremental plastic deformation, $\dot{\lambda}$ is a positive constant usually referred to as "plastic strain-rate multiplier", g is the potential function and σ_{ij} the component *ij* of the stress. Based on equation (6) the evolution of the stresses from a point of the cracking surface is given by:

$$\sigma = \mathbf{E}\left(\dot{\mathbf{u}} - \dot{\lambda}\mathbf{b}\right) \tag{7}$$

where **E** is the stiffness matrix, $\mathbf{E}\dot{\mathbf{u}}$ the elastic predictor and $\mathbf{E}\dot{\lambda}\mathbf{b}$ the inelastic corrector. As shown in Figure 5, when mode I is predominant, the potential is not defined, and in this work the return to the origin (σ=0, τ=0) direction is assumed for the plastic correction, and $\mathbf{E}\dot{\lambda}\mathbf{b}$ is proportional to $\boldsymbol{\sigma}$. Since $\dot{\lambda}$ is a proportional factor, the direction of the plastic deformation is defined as:

$$\mathbf{Eb} = \sigma \Rightarrow \mathbf{b} = \mathbf{E}^{-1}\sigma \tag{8}$$

In this manner the flow rule is defined . It is worth noting that the **b** vector depends exclusively on the stresses and not on the history of displacements.

Integration of the rate equations

If we begin at the *n* step, where the cracking surface equation is satisfied $F(\sigma_n, \tau_n) = 0$, we pass to the *n+1* step:

$$\sigma_{n+1} = \sigma_n + \dot{\sigma} = \sigma_n + \mathbf{E}\left(\mathbf{u} - \dot{\lambda}\mathbf{b}\right) \tag{9}$$

substituting **b** by expression (8) particularized for σ_{n+1}, we obtain:

$$\sigma_{n+1} = \frac{1}{\left(1+\dot{\lambda}\right)}\left(\sigma_n + \mathbf{E}\dot{\mathbf{u}}\right) \tag{10}$$

where $\dot{\lambda}$ is the unknown quantity, which is found since the cracking surface equation should be satisfied in the *n+1* step:

$$F_{n+1} = \tau_{n+1}^{2} - Cf_t\left(f_t - \sigma_{n+1}\right) = 0 \tag{11}$$

Since the value of f_t is no constant and depends on the plastic displacements an iterative process is necessary.

Incorporation into a finite element code

The proposed model was incorporated into a finite element code by interface elements. The commercial code was ABAQUS® and the interface elements were defined by a user subroutine. The interface elements were incorporated into the crack path, previously obtained by a linear fracture elastic calculation. The FRANC [27] finite element code was used to calculate the crack path.

EXPERIMENTAL VERIFICATION

Comparison with the authors' experimental results

The proposed model was checked with two sets of experimental data of mixed mode fracture tests of concrete developed by the authors [22]. The two sets of the testing procedure were developed under proportional and nonproportional loading for two different families of crack paths. Three sizes of quasi homotetic beams were tested. Figure 5 shows the geometry, forces and boundary conditions of the tests. The material properties were: