MODELLING OF A FATIGUE CUMULATIVE DAMAGE UNDER RANDOM LOADING CONDITIONS

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ABSTRACT

The basic problem in material research is to predict a life-time behaviour of structures. That is not only a material problem, but also loading conditions, technology design and environmental properties are very significant. A service loading has usually a random character, and a loading signal must be simulated. Traditional experimental investigation methods are very expensive and time-consuming in this case. Therefore, a computer-simulation method seems to be a very suitable way to this purpose. The simulation method of a fatigue damage evaluation is based on a computer modelling of a loading process, decomposition of the simulated process into rainflow cycles and subsequent cumulative damage calculation. The damage accumulation is evaluated according to a cumulative damage hypothesis, and the residual life is predicted with respect to a fatigue failure criterion. The estimation of cumulative damage under service loading is significantly influenced by a time loading history. The problem of the history effect is converted to the problem of investigation of equivalent loading with non-zero stress/strain mean value. The conversion method is based in specification of an equivalent amplitude of the harmonic cycle with zero stress and strain mean values. As an example, the method was applied for the most typical gaussian loading signal with various autocorrelation properties. For the experimental confirmation, various types of a macroblock of harmonic loading cycles were used. The predicted life-times according to the presented method were compared with life-times predicted by Miner´s cumulative damage hypothesis.

INTRODUCTION

In order to evaluate a fatigue damage under dynamic loading, the cumulative representation of a damage increase over a closed cycle is accepted. Under a harmonic loading with constant both amplitudes and mean values of cycles, stress-life (σ-N) and/or strain-life (ε-N) curves can be experimentally obtained. Then it is possible to specify a relative damage over a closed loading cycle according to some hypothesis (e.g. Miner, Corten-Dolan, etc.). The difficulties arise under varying mean value of cycles, because the relative damage accumulated over individual cycles cannot be simply added. The complete loading history must be respected, and it is a big problem especially under random loading.

The problem of an effect of the mean stress/strain loading is usually solved by considering of a cycle with the zero mean value and some equivalent amplitude, instead of the actual amplitude and mean value, whereby the damage effect must be identical. The equivalent amplitude depends on the actual stress/strain mean value and stress/strain amplitude and some other response parameter, e.g. strain/stress mean value, respectively. The serious difficulty is that we must know the complete time loading history until the actual loading cycle. Therefore, a new approach in order to suitably interpret the loading history has been created.

Knowledge of a fatigue damaging process is a necessary condition for an optimum structure design and for effective production with respect to material and energy saving. Especially, CAD and CAE technologies are very successful because they are by far more quick and less expensive in comparison with traditional theoretical and experimental methods. Moreover, an analytical solution is usually not possible regarding to a stochastic nature of a service loading.

For the optimum structure design, it is very important to estimate an elapsed time and/or number of cycles to failure. Because there is no deterministic principle to describe a cumulative damage mechanics in a material unambiguously, especially under stochastic loading, we must use some of cumulative hypotheses and failure criteria. The hypotheses should follow a physical nature of the cumulative mechanics in a material but they are usually originated according to experimental experience and practice.

CUMULATIVE DAMAGE FUNCTION

Hypotheses of a cumulative damage are usually based on a decomposition of a harmonic loading signal and proceed from the basic Wöhler´s curve for the given stress ratio R : $\sigma_a^q \; N_f(\sigma_a) = A$, where σ_a is the amplitude of a stress cycle, $N_f\,(\sigma_a\,)$ is the number of cycles to failure with the given mean stress σ_m and the stress amplitude σ_a , and A, q are material parameters of the Wöhler´s curve. Then we can introduce a cumulative damage function which evaluates an accumulated damage in a material after application of N closed loading cycles. The function can be analytically expressed as follows [1]

$$D(N) = \sum_{i=1}^{n} k(\sigma_{a_i}) \left(\frac{N(\sigma_{a_i})}{N_f(\sigma_{a_i})} \right)^{l(\sigma_{a_i})} \; ; \quad N(\sigma_{a_i}) < N_f(\sigma_i), \tag{1}$$

where n is a number of stress levels, $N(\sigma_{a_i})$ is a number of applied cycles with the amplitude σ_{a_i}, whereby $N = \sum_{i=1}^{n} N(\sigma_{a_i})$ is the total number of cycles and $k(\sigma_{a_i})$, $l(\sigma_{a_i})$ are some parameters whose are dependent on material properties and a loading nature.

The number of cycles to fracture with the amplitude σ_{a_i} can be expressed according to the Wöhler´s curve in the form: $N_f(\sigma_{a_i}) = N_0 \left(\frac{\sigma_C}{\sigma_{a_i}} \right)^q$, where σ_C is the endurance limit and $N_0 = N_f(\sigma_C)$ is the corresponding number of cycles to fracture for the endurance limit. After substituting into Eqn. 1, we get

$$D(N) = \sum_{i=1}^{n} k(\sigma_{a_i}) \left[\frac{N(\sigma_{a_i})}{N_0} \left(\frac{\sigma_{a_i}}{\sigma_C} \right)^q \right]^{l(\sigma_{a_i})} . \tag{2}$$

Supposing a long-time process, we can identify an occurrence probability of cycles with the stress amplitude σ_a as $p(\sigma_a) = \frac{N(\sigma_a)}{N}$. Substituting into Eqn. 2, it follows

$$D(N)=\sum_{i=1}^{n} k\left(\sigma_{a_i}\right)\left[\frac{N}{N_0}\,p\left(\sigma_{a_i}\right)\left(\frac{\sigma_{a_i}}{\sigma_C}\right)^q\right]^{l\left(\sigma_{a_i}\right)} . \tag{3}$$

According to an experience which has been confirmed by numerous experiments, the coefficient $l(\sigma_a)$ can be expected for most of used materials as $l\left(\sigma_{a_{ii}}\right)\approx 1$ for all amplitudes. Moreover, the dependence of the coefficient $k\left(\sigma_{a_i}\right)$ on amplitudes seems to be usually negligible, therefore $k\left(\sigma_{a_i}\right)$ can be substituted by some constant k (e.g. for the well-known Miner´s rule $k=1$). That makes for widely used linear cumulative hypotheses, and Eqn. 3 can be simplified as

$$D(N)=\sum_{i=1}^{n} \frac{k\,N}{N_0}\,p\left(\sigma_{a_i}\right)\left(\frac{\sigma_{a_i}}{\sigma_C}\right)^q . \tag{4}$$

The probability $p\left(\sigma_{a_i}\right)$ can be formally rewritten as $p\left(\frac{\sigma_{a_i}}{\sigma_C}\right)$, and the non-dimensional parameter $\xi_i=\frac{\sigma_{a_i}}{\sigma_C}$ can be introduced. In the case of a very long-time loading process and at the same time for a great number n, the amplitudes of closed cycles can be considered as a continuously distributed variable with the frequency probability density function $f\left(\sigma_a\right)$, respectively $f(\xi)$ in the non-dimensional representation. The range of permissible ξ is from zero to $\frac{R_m}{\sigma_C}$, where R_m is the ultimate strength. Then, it follows

$$D(N)=\int_0^{\frac{R_m}{\sigma_C}} \frac{k\,N}{N_0}\,\xi^q\, f(\xi)\,d\xi . \tag{5}$$

In the moment of a fracture, the cumulative damage function acquires the unit value. Then the number of cycles to fracture N_f can be expressed as

$$N_f=\frac{N_0}{k\int_0^{\frac{R_m}{\sigma_C}} \xi^q\, f(\xi)\,d\xi} . \tag{6}$$

EQUIVALENT AMPLITUDE OF CLOSED CYCLES AND RAINFLOW DECOMPOSITION

The described method is based on the presumption that the mean value is constant, at best zero. For the stochastic loading process, this presumption is not valid, and therefore the method must be modified. The mostly accepted procedure consists in a rainflow decomposition of the loading signal [2] and recalculation of actual amplitudes σ_a into equivalent amplitudes σ_a^*, whose make the same damage effect for reversed cycles ($R=-1$).

The equivalent amplitude σ_a^* is such an amplitude of harmonic cycle which has zero mean value of both stress and strain, whereby a service life under such a loading is the same as that under repeating application of the actual cycle. Generally, it is possible to proceed from the projections of Haigh diagram $\sigma_A = f(\sigma_M)$ for different $\Delta\varepsilon_m$, where $\Delta\varepsilon_m$ denotes the shift of strain mean value of the actual cycle from the mean value of the corresponding cycle on the cyclic stress-strain curve. Such a diagram can be obtained like a classical biaxial Haigh diagram (for $\Delta\varepsilon_m = 0$) but for the material with a plastic prestraining. Then, we can design the triaxial Haigh diagram using a composition of marginal dependencies (Figure 1). Supposing that the actual cycle is equivalent to the cycle with $\sigma_m = 0$; $\Delta\varepsilon_m = 0$, we can express the equivalent amplitude as follows

$$\sigma_a^* = \sigma_a \frac{\sigma_P}{\sigma_H} . \tag{7}$$

For small σ_m and $\Delta\varepsilon_m$ values (mainly in the case of a narrow-band random loading process), we can consider the relevant part of the area in Figure 1 as a plane, and the Eqn. 7 can be linearized. The equivalent amplitude can be effectively calculated using the description of the triaxial Haigh diagram according to Figure 2. Thus, we obtain the approximate relationship

$$\sigma_a^* = \sigma_a + \psi_\sigma \sigma_m + \psi_\varepsilon \Delta\varepsilon_m , \tag{8}$$

where $\psi_\sigma = \; cotg\, \varphi_\sigma$ and $\psi_\varepsilon = \; cotg\, \varphi_\varepsilon$ (the dimension of ψ_σ is [1] and of ψ_ε is [MPa]).

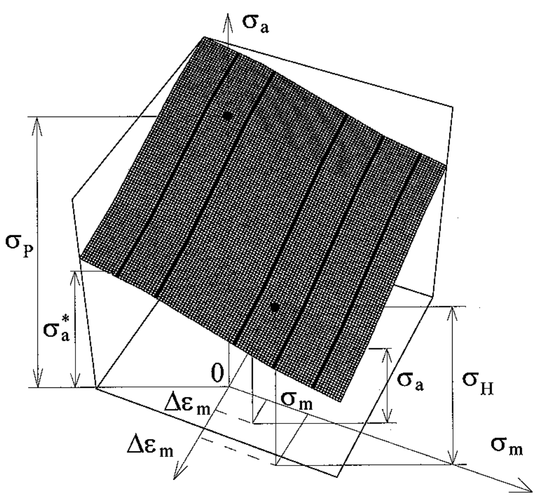

Figure 1: Identification of an equivalent amplitude using the triaxial Haigh diagram

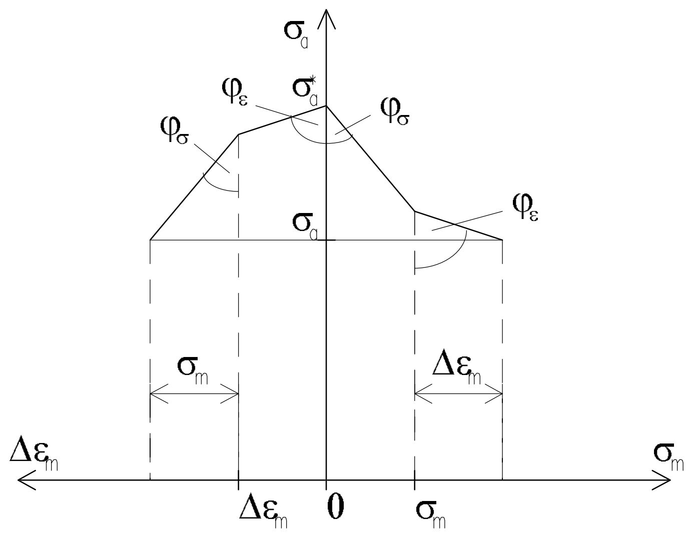

Figure 2: Description of a triaxial Haigh diagram to calculate the equivalent amplitude

In order to identify $\Delta\varepsilon_m$ without knowing of the total time loading history, we must express according to [3] the stress-strain response using a stress-strain response curve Φ as follows

$$\sigma = \pm 2\, \Phi\left(\frac{|\varepsilon|}{2}\right) \; ; \; for\ \varepsilon \gtrless 0 ,\ respectively. \tag{9}$$

The experimental results suggest that the cyclic curve can be expressed in the form: $\Phi(\varepsilon) = \pm K|\varepsilon - \varepsilon_e|^n$, for $\varepsilon \gtrless 0$, where ε_e is the elastic strain, and K and n are definite constants. Then, the Eqn. 1 can be adapted into the form

$$\sigma = \pm 2\, K \left(\frac{|\varepsilon - \varepsilon_e|}{2}\right)^n \; ; \; for\ \varepsilon \gtrless 0 ,\ respectively. \tag{10}$$

If we indicate the origins of hitherto unclosed hysteresis loops as $\sigma_1, \sigma_2, \ldots, \sigma_N$, we could use the method which was described in [4]. Then we can express $\Delta\varepsilon_m$ as follows

$$\Delta\varepsilon_m = \Phi^{-1}(\sigma_1) - \Phi^{-1}(\sigma_N) + 2\sum_{i=2}^{N} \Phi^{-1}\left(\frac{\sigma_i - \sigma_{i-1}}{2}\right). \tag{11}$$

or if we know an analytical expression of a stress-strain cyclic curve

$$\Delta\varepsilon_m = K^{-\frac{1}{n}} \left[\pm |\sigma_1|^{\frac{1}{n}} \mp |\sigma_N|^{\frac{1}{n}} \pm 2^{\frac{n-1}{n}} \sum_{i=2}^{N} |\sigma_i - \sigma_{i-1}|^{\frac{1}{n}} \right], \tag{12}$$

where for $\sigma_1 > 0, \sigma_N > 0$ and $\sigma_i > \sigma_{i-1}$, it holds the upper corresponding sign, otherwise the lower one.

Using Eqn. 8, we can define instead of ξ the new non-dimensional parameter ξ^* as $\xi_i^* = \frac{\sigma_{a_i}^*}{\sigma_C}$ and substituting into Eqn. 6, we can obtain the relative number of closed hysteresis cycles to fracture $\eta = \frac{N_f}{N_0}$ as

$$\eta = \left[k \int_0^{\frac{R_m}{\sigma_C}} \xi^{*q} f(\xi^*) \, d\xi^* \right]^{-1}. \tag{13}$$

Sometimes, the elapsed time to failure is relevant instead of a number of cycles. Then we can calculate a relative time to failure τ, i.e. an elapsed time to failure T_f related to a time unit T_1 , according to the relationship $\tau = \frac{T_f}{T_1} = \frac{N_f}{N_1}$, where N_1 is a number of closed cycles realised during a time unit. Thus we obtain a form

$$\tau = \eta \frac{N_0}{N_1} \tag{14}$$

APPLICATION EXAMPLE AND EXPERIMENTAL RESULTS

As an example, we have investigated a damage effect of a gaussian loading process with various autocorrelation properties. The process was simulated according to [5] using an autoregressive filtration model. The mean value of the process was zero, the standard deviation was *150 MPa*. The correlation coefficient ρ was varied from *-1* to *1*. The material properties was following: strength limit $R_m = 600$ *MPa*, endurance limit $\sigma_C = 180$ *MPa*, number of alternating cycles till failure for endurance limit $N_0 = 2.10^7$, the slope of the Wöhler´s curve $q = 3.5$, parameters of a triaxial Haigh´s diagram $\psi_\sigma = 0.1$ and $\psi_\varepsilon = 10^4$ *MPa*.

The simulated process was decomposed into closed cycles whose have been recalculated into equivalent cycles according to Eqn. 8. The probability density of relative equivalent amplitudes ξ^* for zero autocorrelation coefficient is drawn in Figure 3. It is evident that the significant density $f(\xi^*)$ is for $\xi^* \in (0;1)$, i.e. $\sigma_a^* \in (0;180)$ *MPa*. But from the fatigue life point of view, the multiplication $\xi^{*q} f(\xi^*)$ is decisive, and then the range $\xi^* \in (0.75;1.75)$ i.e. $\sigma_a^* \in (135;315)$ *MPa* is relevant.

The relative number of cycles to failure $\eta = 5.10^{-8} N_f$ was calculated according to Eqn. 13, and the dependence on the process autocorrelation is drawn in Figure 4. The elapsed time to fracture is considerably dependent on the frequency content of the loading process. It completely corresponds to the previous probability analysis. The evaluated maximum significant frequency of the frequency spectrum of the simulated process was $f_{max} \approx 40$ *Hz*. The calculated elapsed time according to Eqn. 14 was related to the *1 week* and the result for various autocorrelation is drawn in Figure 4, too.

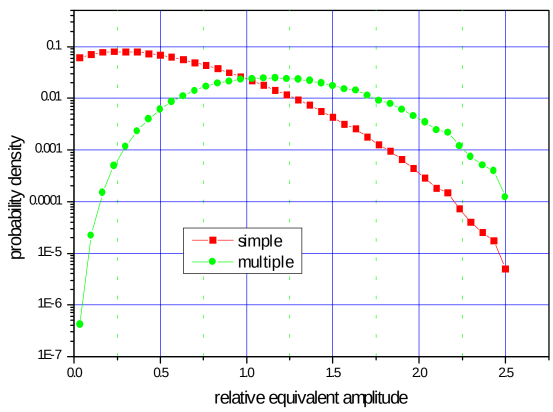

Figure 3: Figure 4. Probability density of ξ^* (simple) and multiplication $\xi^* q\, f(\xi^*)$ (multiple) for $\rho = 0$

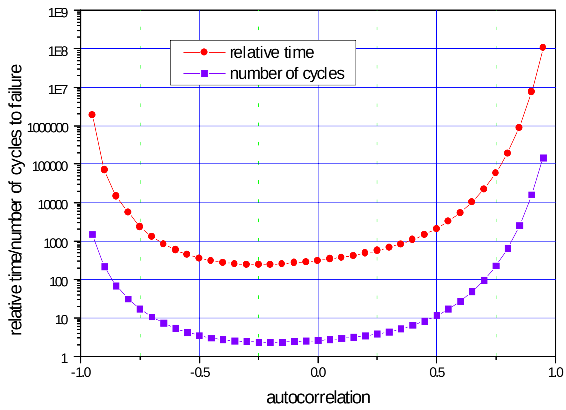

Figure 4: Dependence of elapsed time and number of cycles to failure on autocorrelation

For the comparison with experimental data, previously obtained results were used [6]. The investigated material was AISI 4130 steel under stress ratios of -1 and 0 for various macroblocks, see Figure 5. Blocks of harmonic amplitudes with defined stress levels and numbers of cycles were applied until failure. The predicted cumulative damage was calculated according to both Miner´s linear hypothesis and the submitted model. The expected final accumulated damage should be nearly unitary. Results are presented in Table 1.

CONCLUSIONS

The presented method enables to evaluate the cumulative fatigue damage and to estimate the fatigue life under random loading based on the knowledge of the stress/strain life curve, triaxial Haigh diagram and stress-strain response properties. According to the proposed procedure, the equivalent amplitude for any closed cycle in the loading history is specified, and the relative damage is calculated in the same way as in the case of operation under harmonic loading with zero mean value. The equivalent amplitude can be further calculated in order to respect the structure parameters (notches, surface treatment, welds, etc.), environmental effects (corrosion, radiation, high/low temperatures, etc.), loading mode (e.g. bending, torsion, complex and/or multiaxial loading) and other service conditions [7].

The submitted model enables much more correct evaluation of a cumulative damage under random loading than the traditional methods that are based on a macroblock representation of a service loading. The procedure is applicable for any complex random loading spectrum. Therefore the described method guarantees a structure reliability and enables a material saving at the same time. The material consumption can be significantly reduced in comparison with hitherto used design methods. That can effectively contribute to a competition capacity of engineering products in the business market.

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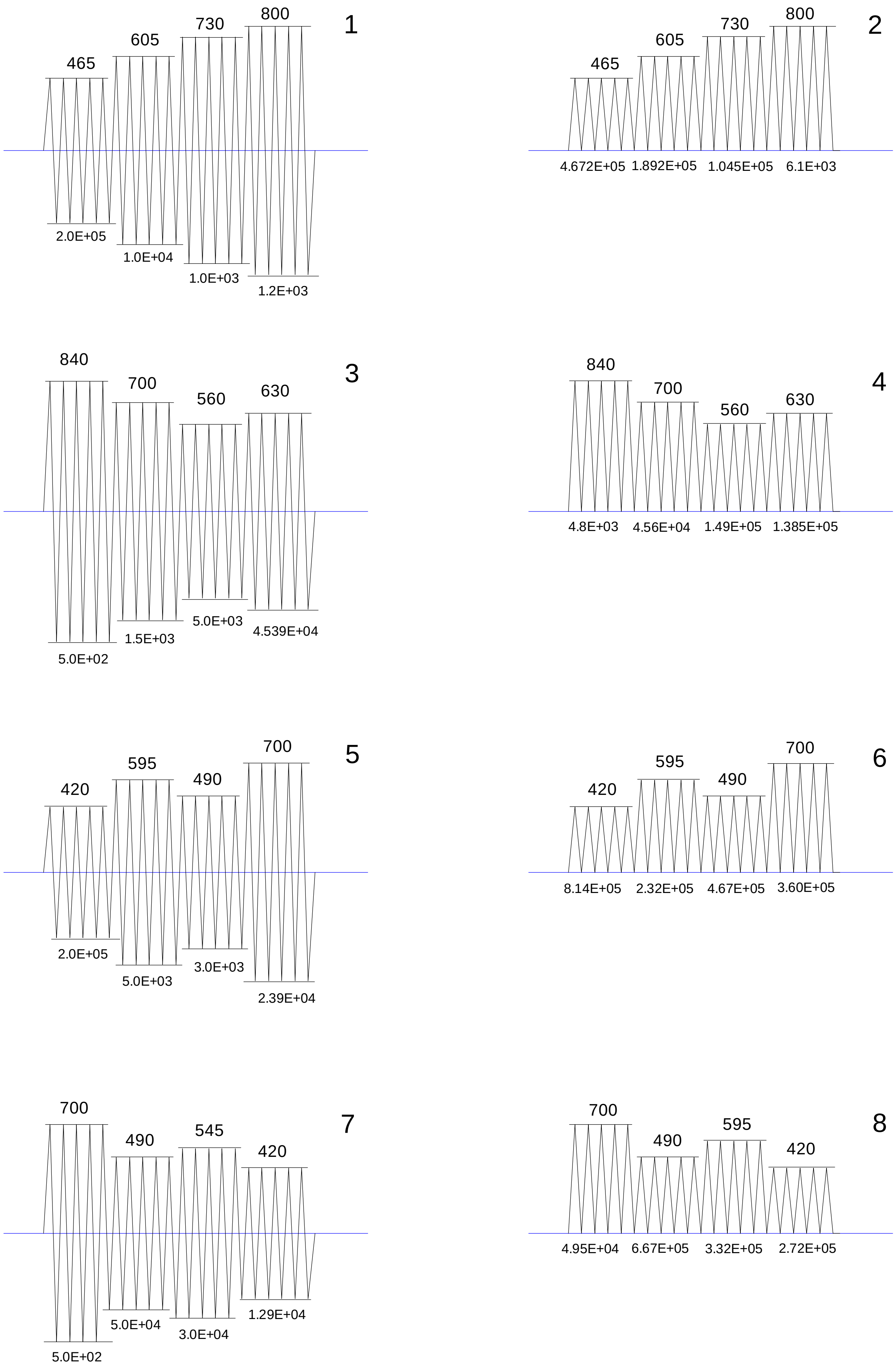

Figure 5: Loading block spectra (stress level in MPa / No. of applied cycles to failure)

TABLE 1

CALCULATED FINAL ACCUMULATED DAMAGE

1	Miner	$\frac{2.0E+05}{8.20E+05}+\frac{1.0E+04}{5.08E+04}+\frac{1.0E+03}{6.98E+03}+\frac{1.2E+03}{2.65E+03}=0.243+0.197+0.143+0.453=1.036$
1	Čačko	$\frac{2.0E+05}{8.20E+05}+\frac{1.0E+04}{5.08E+04}+\frac{1.0E+03}{6.98E+03}+\frac{1.2E+03}{2.65E+03}=0.243+0.197+0.143+0.453=1.036$
2	Miner	$\frac{4.672E+05}{1.63E+07}+\frac{1.892E+05}{1.04E+06}+\frac{1.045E+05}{1.46E+05}+\frac{6.1E+03}{5.61E+04}=0.029+0.182+0.716+0.109=1.036$
2	Čačko	$\frac{4.672E+05}{1.68E+07}+\frac{1.892E+05}{1.07E+06}+\frac{1.045E+05}{1.51E+05}+\frac{6.1E+03}{5.80E+04}=0.028+0.177+0.692+0.105=1.002$
3	Miner	$\frac{5.0E+02}{1.58E+03}+\frac{1.5E+03}{1.09E+04}+\frac{5.0E+03}{1.15E+05}+\frac{4.539E+04}{3.30E+04}=0.316+0.138+0.043+1.375=1.872$
3	Čačko	$\frac{5.0E+02}{1.58E+03}+\frac{1.5E+03}{1.47E+04}+\frac{5.0E+03}{3.08E+05}+\frac{4.539E+04}{7.90E+04}=0.316+0.102+0.016+0.575=1.009$
4	Miner	$\frac{4.8E+03}{3.37E+04}+\frac{4.56E+04}{2.27E+05}+\frac{1.49E+05}{2.32E+06}+\frac{7.385E+05}{6.81E+05}=0.142+0.201+0.064+1.084=1.491$
4	Čačko	$\frac{4.8E+03}{3.38E+04}+\frac{4.56E+04}{3.07E+0.5}+\frac{1.49E+05}{4.15E+06}+\frac{7.385E+05}{1.13E+06}=0.142+0.149+0.036+0{,}654=0.981$
5	Miner	$\frac{2.0E+05}{2.4E+06}+\frac{5.0E+03}{6.05E+04}+\frac{3.0E+03}{4.73E+05}+\frac{2.39E+04}{1.09E04}=0.083+0.083+0.006+2.193=2.365$
5	Čačko	$\frac{2.0E+05}{2.4E+06}+\frac{5.0E+03}{6.05E+04}+\frac{3.0E+03}{1.87E+06}+\frac{2.39E+04}{2.79E+04}=0.083+0.083+0.002+0.857=1.029$
6	Miner	$\frac{8.135E+05}{4.71E+07}+\frac{2.323E+05}{1.23E+06}+\frac{4.672E+05}{9.3E+06}+\frac{3.602E+05}{2.27E+05}=0.017+0.189+0.050+1.587=1.843$
6	Čačko	$\frac{8.135E+05}{4.73E+07}+\frac{2.323E+05}{1.27E+06}+\frac{4.672E+05}{2.90E+07}+\frac{3.602E+05}{4.92E+05}=0.017+0.183+0.016+0.732=0.948$
7	Miner	$\frac{5.0E+02}{1.09E+04}+\frac{5.0E+04}{4.73E+05}+\frac{3.0E+04}{1.54E+05}+\frac{1.289E+04}{2.4E+06}=0.046+0.106+0.195+0.005=0.352$
7	Čačko	$\frac{5.0E+02}{1.09E+04}+\frac{5.0E+04}{1.68E+05}+\frac{3.0E+04}{6.03E+04}+\frac{1.289E+04}{9.50E+04}=0.046+0.298+0.498+0.136=0.978$
8	Miner	$\frac{4.95E+04}{2.27E+05}+\frac{6.672E+05}{9.39E+06}+\frac{3.323E+05}{1.23E+06}+\frac{2.724E+05}{4.71E+07}=0.218+0.071+0.270+0.006=0.565$
8	Čačko	$\frac{4.95E+04}{2.24E+05}+\frac{6.672E+05}{3.76E+06}+\frac{3.323E+05}{5.78E+05}+\frac{2.724E+05}{1.05E+07}=0.221+0.177+0.575+0.026=0.999$

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