LOCAL VARIATIONS IN YIELD STRENGTH – A POSSIBLE MEANS FOR ENHANCING THE FRACTURE RESISTANCE OF STRUCTURAL COMPONENTS

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ABSTRACT

In materials with local variations in yield stress, the effective near-tip crack driving force can become different from the nominally applied far-field value. The near-tip crack driving force is enhanced, if the yield stress decreases in the crack growth direction, and vice versa. This effect, termed as the yield stress gradient effect, can be used for the design of fracture resistant structural components by applying materials where the yield stress (and/or the elastic modulus) is intentionally varied. A model has been developed that allows us to derive analytical expressions to quantify the effect. The analytical expressions can be used to optimize monotonically or cyclically loaded components with smooth gradients in yield stress as well as interface and interlayer transitions. It is demonstrated, for example, that both a soft or a hard interlayer may act as a crack arrestor. Similarly, the fracture resistance of components with hard surface layers can be improved by appropriate interlayers between the surface layer and the base material.

INTRODUCTION

It is known long since, e.g., from papers by Erdogan and co-workers [1, 2, 3], that in materials with local variations in the elastic modulus the local near-tip crack driving force can become different from the nominally applied far-field value,. Recently, Kolednik [4, 5] has been demonstrated analytically that local variations in the yield stress can have a similar effect: in materials with a gradient in yield stress in the crack growth direction an additional crack-driving force term appears, called the yield stress gradient term. The yield stress gradient term is positive, i.e., it enhances the effective crack driving force when the yield stress decreases in crack growth direction. On the contrary, an increase of the yield stress in the crack growth direction induces a negative yield stress gradient term which diminishes the effective crack driving force.

Experimentally, it was first shown by Suresh et al. [6] that a gradient in yield strength alone can influence the effective driving force of fatigue cracks. They conducted fatigue crack growth experiments on explosion clad bimaterial specimens consisting of austenitic and ferritic steel. For a crack approaching the interface from the softer ferritic steel (soft-hard transition), the crack growth rate dropped precipitously and the crack stopped before the interface. However, for a hard-soft transition, the crack advanced unimpeded through the interface; the crack growth rate remained roughly constant, except for some distance before the interface where a slight increase of the crack growth rate was noted. Subsequent experiments by Suresh et al. [7] and, recently, by Pippan and Flechsig [8, 9] further confirmed these findings. Finite element studies by Sugimura et al. [10] and Kim et al. [11] revealed that for a stationary, monotonically loaded crack the near-tip J-integral, J_{tip}, begins to deviate from the applied far-field value, J_{appl}, as soon as the plastic zone around the crack tip touches the interface: for a hard-soft transition, J_{tip} becomes higher than J_{appl}; for a soft-hard transition, $J_{tip} < J_{appl}$. J_{tip} can be interpreted as the intensity of the crack-tip field and approximated as the effective, or near-tip, crack driving force. The latter is true as long as small scale yielding conditions prevail.

It shall be outlined in this presentation that the yield stress gradient effect can be used to design more fracture resistant structural components by applying materials where the yield stress (or the elastic modulus) is intentionally varied. In the following section, the yield stress gradient effect is described.

A MODEL OF THE YIELD STRESS GRADIENT EFFECT

An energy based, analytical model of the yield stress gradient effect has been presented in Kolednik and Suresh [4], Kolednik [5]. The main idea of the model is the following:
Consider a pre-cracked body of thickness B subjected to an external load, Q. The body shall consist of a non-linear elastic material that has (during the loading) the same stress-strain response as an elastic-perfectly plastic material with a yield stress σ_y. σ_y has a gradient in the crack growth direction, x. The potential energy, P, of the body can be deduced from the load vs. displacement (Q - q) curve, Rice [12]. Thus, P depends on the load, Q, the crack length, a, the Young's modulus, E, the geometry and the yield strength,

$$P = P\left(Q, a, E, geometry, \sigma_y\right) \tag{1}$$

Note that E, a, and *geometry* determine the compliance of the body. σ_y determines the radius of the plastic zone, r_y, and, accordingly, the deviation of the Q - q curve from linearity. Note that a change of the yield stress alone would result in a variation of the potential energy. The effective crack driving force, C_{tot}, is given as the total change of the potential energy during the crack extension,

$$C_{tot} = -\frac{1}{B}\frac{dP}{da}\bigg|_Q = -\frac{1}{B}\left\{\frac{\partial P}{\partial a}\bigg|_{Q,\sigma_y} + \frac{\partial P}{\partial \sigma_y}\bigg|_{Q,a}\frac{d\sigma_y}{da}\right\} \tag{2}$$

$$C_{tot} = J + C_y \tag{3}$$

The first term in Eqn. 2 results from the variation of the potential energy if σ_y were held constant during the crack extension; it corresponds to the conventional J-integral, J. The second term in Eqn. 2 results from the variation of the potential energy if the yield stress is changed according to the new position of the crack tip; it is referred to as the yield stress gradient term, C_y. In bodies where the yield stress varies in the direction of the crack extension, the effective crack driving force differs from the applied far-field value, i.e., the J-integral. The yield stress gradient term is negative, and the effective crack driving force, C_{tot}, is decreased when the yield stress increases in crack growth direction. This means that the fracture resistance is higher than it would be in a homogeneous material. Contrarily, C_{tot} is increased when the yield stress decreases in crack growth direction. This is similar to the effect of a modulus gradient [1, 2].

To quantify the plasticity gradient term and the total crack driving force for a given specimen geometry, an analytical expression for the potential energy, Eqn. 1, must be known. For small-scale yielding conditions, such expressions can be deduced from known equations of the specimen compliance [4, 5].

THE YIELD STRESS GRADIENT EFFECT AT BIMATERIAL INTERFACES

A crack grows perpendicular to an interface between materials of different yield strengths, Figure 1. A yield stress gradient effect is induced as soon as the crack tip plastic zone touches the interface because an incremental crack extension produces a change in the total plastic strain energy of the body. In Kolednik [5], analytical expressions for the variation of the yield stress gradient term and the total crack driving force at interfaces and interlayers have been derived. Hereby, the following simplifications were made: Both materials behave elastic – perfectly plastic; the plastic zones are circular with their centers located at the crack tip and a radius r_y according to Irwin's model [13],

$$r_y = \beta\frac{K^2}{\sigma_y^{\,2}} = \beta\frac{GE}{\sigma_y^{\,2}}. \tag{4}$$

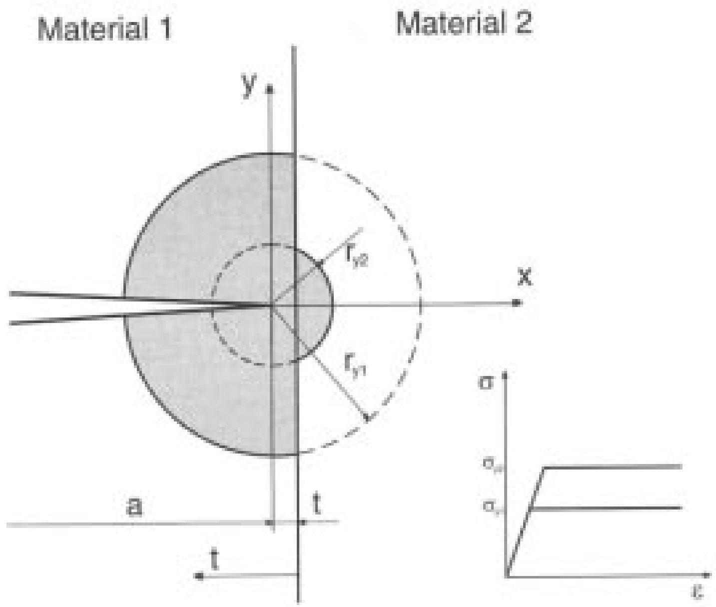

Figure 1: The plastic zone of a crack near a bimaterial interface dividing Material 1 with yield stress σ_{y1} from Material 2 with yield stress σ_{y2}.

σ_y is the yield strength at the crack tip, K is the stress intensity factor, G the elastic strain energy release rate, and the pre-factor β is $1/(6\pi)$ for plane strain conditions and $1/(2\pi)$ for plane stress conditions. The plastic strain energy exhibits a r^{-1} singularity; misfit strains at the interface are neglected.

The yield stress gradient term is split into two components. C_{y1} is the part that originates from the plastic zone in Material 1 (if Material 2 would be elastic); C_{y2} originates from the plastic zone in Material 2:

$$C_{y1}(t) = \begin{cases} 0 & t > r_{y1} \\ -\dfrac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\dfrac{t}{r_{y1}}\right)^2}\right] \quad \text{for} & r_{y1} \geq t > -r_{y1} \\ 0 & t < -r_{y1} \end{cases} \tag{5a}$$

$$C_{y2}(t) = \begin{cases} 0 & t > r_{y2} \\ -\dfrac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\dfrac{t}{r_{y2}}\right)^2}\right] \quad \text{for} & r_{y2} \geq t > -r_{y2} \\ 0 & t < -r_{y2} \end{cases} \tag{5b}$$

The total yield stress gradient term results as the superposition of the two components,

$$C_y(t) = C_{y1}(t) + C_{y2}(t)\,. \tag{6}$$

In the equations above, t is the distance of the crack tip from the interface, Figure 2. The effective crack driving force is given by

$$C_{tot}(t) = J(t) + C_y(t)\,, \tag{7}$$

where J is a weak function of t, depending on the current size of the total plastic strain energy [5]. $J(t) \cong G = K^2/E$ could serve as a first approximation.

Figure 2 presents the situation near an interface with a soft-hard transition, $\sigma_{y2} = 2\sigma_{y1}$: The plastic strain energy, W_{pl} (Figure 2a), the effective J-integral (Figure 2b), the plasticity gradient term (Figure 2c), and the total crack driving force (Figure 2d) are plotted against t/r_{y1}, i.e., the distance between crack tip and interface, normalized by the plastic zone radius in Material 1. During the crack extension, the crack tip travels in the figure from the right-hand side (Material 1) to the left-hand side (Material 2).

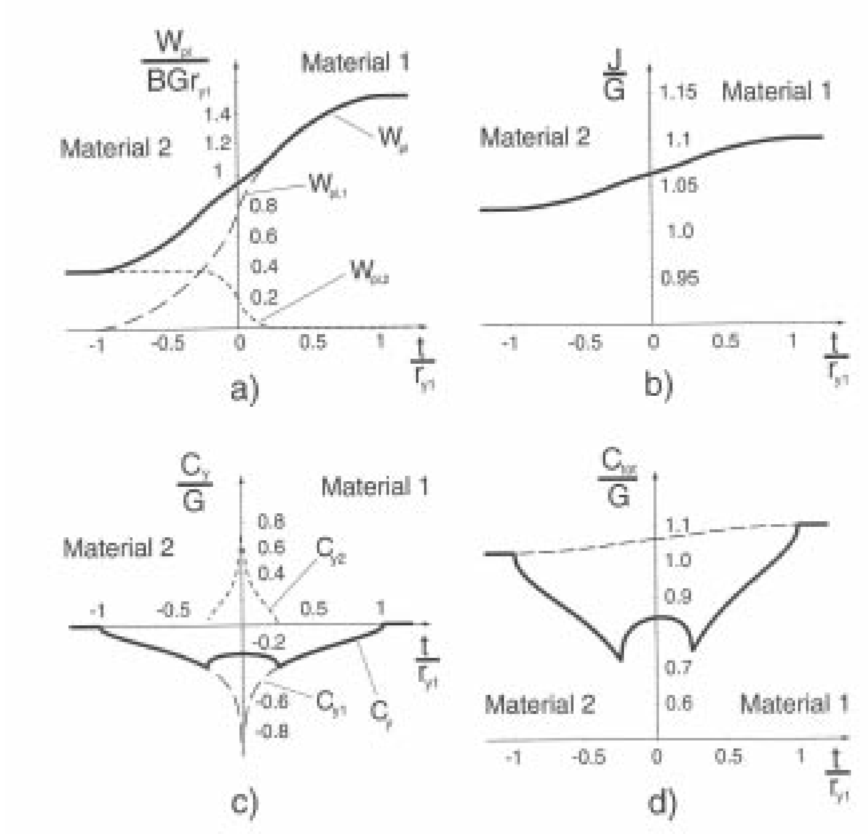

Figure 2: A soft-hard transition at a bimaterial interface with $\sigma_{y2} = 2\sigma_{y1}$. (a) Variation of the plastic strain energy, W_{pl}, as a function of the relative distance of the crack tip from the interface, t/r_{y1}. (b) The effective far-field J-integral, *J*. (c) The yield stress gradient term, C_y. (d) The total crack driving force, C_{tot}. *B* is the specimen thickness, *G* the applied strain energy release rate, and r_{y1} the plastic zone radius in Material 1.

The figure demonstrates that a soft-hard transition with $\sigma_{y2} = 2\sigma_{y1}$ produces a strong negative plasticity gradient term reduces the total crack driving force by up to 33 percent. For a hard-soft transition, C_y would have the same size, but would be positive and increase the total crack driving force, see also Figure 4a. If Material 2 were elastic, C_y would show a singularity and the total crack driving force would be zero at the interface. The maximum value of C_y, which occurs at $t = \pm r_{y,hard}$, depends on the ratio of the plastic zone radii in the two materials,

$$\left|C_{y,\max}\right| = \frac{G}{2\pi}\left[\operatorname{arctanh}\sqrt{1-\left(\frac{r_{y,hard}}{r_{y,soft}}\right)^2}\right]. \tag{8}$$

In Kolednik [5], the predictions of the model are compared to the results of the finite element computations by Sugimura et al. [10]. A good agreement has been found. The model also explains the results of the fatigue experiments on bimaterial specimens [6 - 9]. An analogy exists between the yield stress gradient effect and the effect of a modulus variation at an interface: a soft-hard transition of either the yield stress or the Young's modulus leads to a decreased effective crack driving force, i.e., an increased local fracture resistance.

THE YIELD STRESS GRADIENT EFFECT AT INTERLAYERS

Consider the case of an elastic interlayer of thickness *d* in an elastic - plastic matrix. $\sigma_{y,m}$ is the yield stress, and $r_{y,m}$ the plastic zone radius of the matrix material. *t* is the distance of the crack tip from the interlayer. The interlayer cuts off a strip of width *d* from the plastic zone. The yield stress gradient term becomes [5]

$$C_{y,m}(t)=\begin{cases} 0 & t>r_{y,m} \\ -\frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t}{r_{y,m}}\right)^2}\right] & r_{y,m}\geq t>r_{y,m}-d \\ -\frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t}{r_{y,m}}\right)^2}-\operatorname{artanh}\sqrt{1-\left(\frac{t+d}{r_{y,m}}\right)^2}\right] \quad for & r_{y,m}-d\geq t>-r_{y,m} \\ \frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t+d}{r_{y,m}}\right)^2}\right] & -r_{y,m}\geq t>-d-r_{y,m} \\ 0 & t\leq -d-r_{y,m} \end{cases} \tag{9}$$

The $C_y - t$ curve shows opposite singularities of equal size at the two interfaces. C_y is negative when the crack approaches the first interface; C_y is positive near the second interface. If its thickness is large, the interlayer acts like two consecutive interfaces. For a thin interlayer, the two singularities come close and interfere. Thus, the maximum value of C_y increases with increasing $d/r_{y,m}$ ratio, but only up to $d/r_{y,m} \cong 1$; no significant increase of $C_{y,max}$ is observed for a thicker interlayer.

Another limiting case is that of an elastic-plastic interlayer (thickness d, yield stress $\sigma_{y,L}$, plastic zone radius $r_{y,L}$) in an elastic matrix. The solution is similar to that of the foregoing case, however, the sign of the effect is different: The first interface attracts the crack, the second interface repels the crack. The equations are [5]

$$C_{y,L}(t)=\begin{cases} 0 & t>r_{y,L} \\ -\frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t}{r_{y,L}}\right)^2}\right] & r_{y,L}\geq t>r_{y,L}-d \\ -\frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t}{r_{y,L}}\right)^2}-\operatorname{artanh}\sqrt{1-\left(\frac{t+d}{r_{y,L}}\right)^2}\right] \quad for & r_{y,L}-d\geq t>-r_{y,L} \\ \frac{G}{2\pi}\left[\operatorname{artanh}\sqrt{1-\left(\frac{t+d}{r_{y,L}}\right)^2}\right] & -r_{y,L}\geq t>-d-r_{y,L} \\ 0 & t\leq -d-r_{y,L} \end{cases} \tag{10}$$

In general, both the base material and the interlayer deform plastically. The solution for the yield stress gradient term can be easily found as the superposition of the preceding two limiting cases:

$$C_y(t)=C_{y,m}(t)+C_{y,L}(t) \tag{11}$$

One example is presented in Figure 3 for a hard interlayer in a soft matrix with $r_{y,L}/r_{y,m} \cong 0.1$ and $d/r_{y,m} \cong 0.8$. The curve is similar to the case of the elastic interlayer in an elastic-plastic matrix, but the singularities at the interfaces are truncated because of the plasticity of the interlayer. It is seen that the interlayer thickness is not yet optimal; the maximum yield stress gradient term would be larger, if the interface were thicker.

Both a hard and a soft interlayer can act as a crack arrestor because they produce a region of negative yield stress gradient term where the local crack driving force is decreased and, consequently, the local fracture resistance is enhanced. This region lies in front of the first interface for a hard interlayer, near the second interface for a soft interlayer. The maximum size of the yield stress gradient term depends primarily on the ratio of the yield stresses in the two materials, as well as on the ratio of the interlayer thickness to the plastic zone radius in the softer material. For a large effect, $d/r_{y,soft} \geq 1$.

It has been noticed above that the ratio of the interlayer thickness to the plastic zone radius determines the size of the yield stress gradient term. If the plastic zone size is much larger than the interlayer thickness, C_y is negligible. Therefore, in many cases the yield stress gradient effect important in fatigue where the plastic zone sizes are smaller than in monotonically loaded structures. It is possible to extend the model to fatigue if the cyclic J-integral, ΔJ, is accepted as an engineering approach of the crack driving force for cyclic crack extension. The cyclic J-integral was introduced by Dowling and Begley [14] for cases

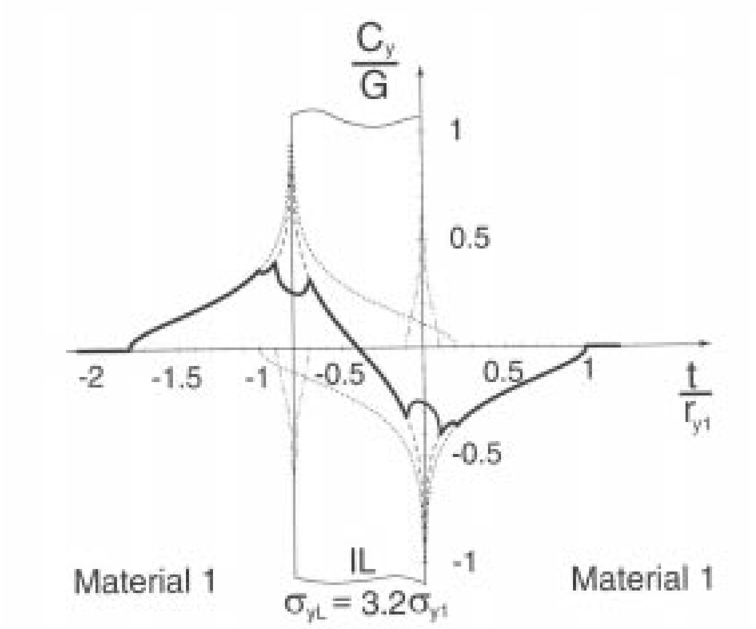

Figure 3: Variation of the yield stress gradient term, C_y, for a hard interlayer in a soft base material. The minimum crack driving force appears near the first interface.

where the description in ΔK is not valid any more; for comments see, e.g., Tanaka [15], Suresh [16]. To assess the crack driving force for cyclic loading, Kolednik [17], the same derivations can be used as for monotonic loading, but the J-integral must be replaced by ΔJ, the elastic strain energy release rate by ΔG, and the plastic zone radius by the cyclic plastic zone radius, $r_{y,cyc.}$

Pippan and co-workers [8, 9] conducted experimental studies on the behavior of fatigue cracks near interlayers of steel in Armco iron, and vice versa. The thickness of the interlayers was varied. The predictions of the model fit to the experimental results: The fatigue crack slows down and stops in front of a soft-hard bimaterial interface; depending on the interface strength, the crack may bifurcate into two (near-) interface cracks. It is noteworthy that Figure 3 depicts an experimental situation for $\Delta K = 25$ MNm$^{-3/2}$: the ratio of the cyclic plastic zones in the two materials is approximately *1/10*, and the thickness of the steel interlayer is a little bit smaller than the cyclic plastic zone radius in the ARMCO iron.

OPTIMUM YIELD STRESS TRANSITIONS

In a vast number of technical applications, a hard surface layer protects a softer matrix material. If the surface layer contains a crack, a large positive plasticity gradient term appears which promotes the crack propagation into the matrix, see Figure 4a. The following questions are relevant in this context: Can interlayers between surface layer and matrix reduce the total crack driving force? What are the optimum yield strength and the optimum thickness of such interlayers? The general question is how to design an optimum bimaterial transition between Material 1 and Material 2 where $\sigma_{y1} >> \sigma_{y2}$.

First, a single interlayer is considered. The optimum yield strength of the interlayer, $\sigma_{y,L}$, must be the geometric mean value,

$$\sigma_{y,L} = \sqrt{\sigma_{y1}\sigma_{y2}}\ , \tag{12}$$

to guarantee that the maximum values of C_y at both interfaces are equal in size. If the interlayer thickness, d, is smaller than the plastic zone radius of the soft material, r_{y2}, the interlayer brings no relief; $C_{y,max}$ may be even larger than without interlayer. However, $C_{y,max}$ is reduced significantly, if

$$d \geq r_{y1} + r_{y2}\ . \tag{13}$$

Figure 4b shows an example for $\sigma_{y1} = 4\sigma_{y2}$.

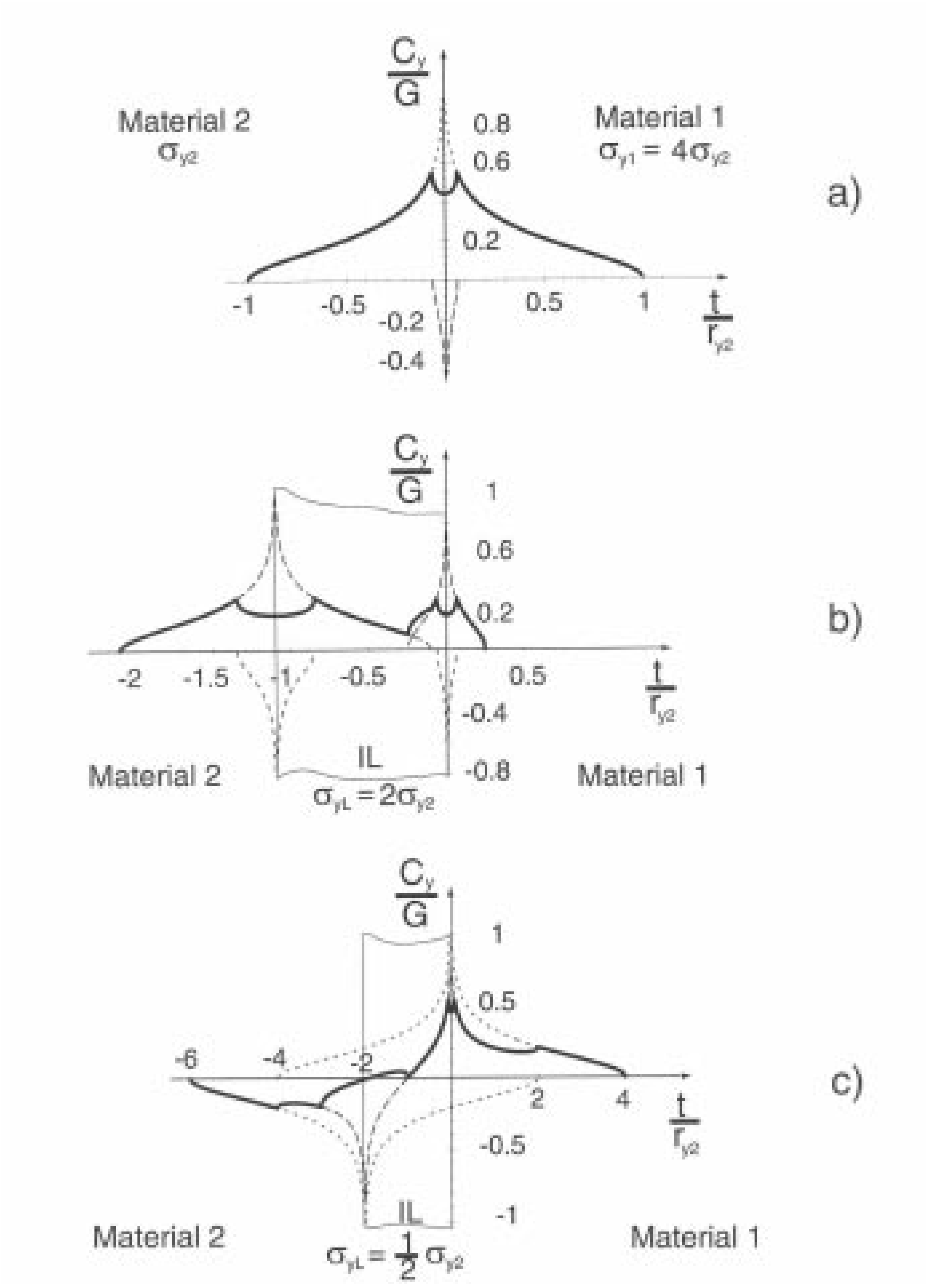

Figure 4: (a) A high, positive yield stress gradient term, C_y, appears at a bimaterial transition from a hard surface layer to a soft base material with $\sigma_{y1} = 4\sigma_{y2}$. (b) An interlayer with an intermediate yield stress reduces the maximum value of C_y. (c) Behind an interlayer that is softer than the base material, a beneficial zone of negative C_y appears.

The maximum plasticity gradient term can be further reduced, if several interlayers are inserted between surface layer and base material. The following equations give the optimum yield strength, $\sigma^i_{y,L}$, and the minimum thickness, d_i, of the i^{th} interlayer if n interlayers are introduced:

$$\sigma^i_{y,L} = \sigma_{y1}\left(\frac{\sigma_{y2}}{\sigma_{y1}}\right)^{\frac{i}{n+1}} \tag{14}$$

$$d_i = r_{y1}\left(\frac{\sigma_{y1}}{\sigma_{y2}}\right)^{2\frac{i-1}{n+1}}\left[1+\left(\frac{\sigma_{y1}}{\sigma_{y2}}\right)^{\frac{4}{n+1}}\right] \tag{15}$$

Equations 14 and 15 allow the design of optimum bimaterial transitions. It should be noted that optimum yield stress variations can be found for smooth variations in yield stress as well [5]. This solution agrees with the solution presented above if n becomes large.

Very interesting is the case of one interlayer which is softer than the base material. Figure 5c presents the variation of the plasticity gradient term for $\sigma_{y1} = 4\sigma_{y2}$, $\sigma_{yL} = (1/2)\sigma_{y2}$, $d = (1/2)r_{y,L} = 2r_{y,m}$. At the first interface, $C_{y,max}$ is almost the same as for the case without interlayer, but behind the second interface a zone of negative C_y appears which reduces the propensity of the crack to grow deeper into the base material. Note that the interlayer must be thick enough to be effective: the interlayer depicted in Figure 4c is too small for being most effective; the optimum thickness is $d \geq r_{y,L}$. The benefit of such a soft interlayer has been demonstrated experimentally by Suresh et al. [7] for a Cr_2O_3 coated steel with a Ni-5Al interlayer.

SOME REMARKS ON THE APPLIED MODEL

A simple energetically-based model has been developed which provides analytical expressions of the yield stress gradient term. The model has a few deficiencies that have been considered more extensively in [5]: (1) The model does not allow for the strain hardening of materials. An engineering approximation for considering different strain hardening exponents would be to insert into the equations an average flow stress instead of the yield strength. The mean value between the yield strength and the tensile strength could serve as a rough estimate. (2) The model assumes a circular plastic zone with its center at the crack tip. In reality, the plastic zone has a more complicated shape with a forward orientation, i.e., it extends much farther towards the crack growth direction. (3) The model does not consider misfit strains which will appear when the plastic zone intersects the interface between materials with different yield stresses. In spite of these deficiencies, the model is useful for the optimization of structural components with respect to their fracture properties.

SUMMARY

In materials with local variations in yield stress, the effective near-tip crack driving force can become different from the nominally applied far-field value. The near-tip crack driving force is enhanced, if the yield stress decreases in the crack growth direction, and vice versa. This effect, termed as the yield stress gradient effect, can be used for the design of fracture resistant structural components by applying materials where the yield stress (and/or the elastic modulus) is intentionally varied. It is demonstrated, for example, that both soft or hard interlayers may act as crack arrestors. Similarly, the fracture resistance of components with hard surface layers can be improved by appropriate interlayers between the surface layer and the base material.

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