ENERGY DISSIPATION RATE AS A FRACTURE ANALYSIS TOOL

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ABSTRACT

The paper presents the case for predicting unstable ductile tearing by the energy dissipation rate approach. It is argued that stable tearing does not arise from an increase in toughness with crack growth (the J or G resistance curve). Instead it is proposed that stability is due to the material having an inherent crack growth dissipation rate, which is present even in small scale yielding (SSY), and which exceeds the energetic crack driving force available for crack advance under fixed system displacement or load. Full scaling laws to predict the geometry dependence of the dissipation rate term, D, and the energetic driving force terms, C_p and C_q, have still to be developed. However, the trends in these parameters are already clear, and the technology for developing the full scaling laws is available in the form of elastic-plastic finite element analysis. It is shown that J resistance curve instability predictions are potentially unconservative when extrapolating from a fully yielded test piece to predict the behaviour of a structure in SSY.

INTRODUCTION

In the early days of fracture mechanics there was much interest in energetic methods for describing fracture phenomena. Crack tip characterising methods are now more popular, aided by the increasing power of elastic-plastic finite element analysis. The local approach is the logical extension of this trend, but it abandons the original idea of characterising crack tip stresses by an easily accessible engineering parameter dependent only on remote stress and crack geometry. The nearest approach to a generalised fracture analysis tool is provided by the J integral, but this parameter has severe limitations. Most notably, once the crack starts to propagate, the pure definition of the J integral is lost. This means that, leaving aside the specialised case of cleavage, where cracks are inherently unstable after initiation because of rate effects, any prediction of crack stability based on J analysis must be treated with caution.

This paper outlines the alternative approach to predicting crack stability based on energy dissipation rate. The method is perfectly general, and can be applied to any material, in any thickness, and at any extent of crack growth. In recent years the chief advocates of the energy dissipation rate approach have been Turner and Kolednik [1], but similar ideas can be traced back to the early 1950s in the work of Irwin and Kies [2].

Irwin later abandoned the energy approach and concentrated instead on K as a characterising parameter. In 1961 Kraft, Sullivan and Boyle suggested the K (or G) resistance curve approach [3] as a method to explain stable tearing and crack stability in contained yield. This was followed in 1979 by the proposal by Paris [4] that the J resistance curve might be useful to characterise stable tearing in specimens containing extensive plasticity.

Both the G and J resistance curves approaches depend on the idea that cracks are stable because toughness increases with crack growth. Shear lip development is often cited, but in G_R curves for large thin panels, 'toughness' continues to increase well after fully slant fracture is established. Also increasing J_R curves can be seen in thick side-grooved specimens, where the fracture is completely flat.

On a microstructural scale it is known that material failure is governed by critical strain as a function of stress state. Many computational crack growth studies have been made which incorporate local failure criteria, such as the Gurson model [5]. These studies predict increasing 'toughness' with crack growth (in terms of J), even though the local failure criterion is invariant with crack growth. It thus appears that the idea of a tearing resistance curve (increasing toughness with crack growth) is inconsistent with physical reality.

A major advantage of the energy dissipation rate approach is that it explains stable tearing <u>without</u> the need to postulate an increase in toughness with crack growth.

CRACK STABILITY WITHOUT A CRACK GROWTH RESISTANCE CURVE – THE ENERGY DISSIPATION RATE APPROACH

In G and J theory the same symbol is used to mean both the toughness and the driving force causing crack instability. To avoid the confusion that this can cause, dissipation rate theory uses two separate symbols [1]:

1. a toughness (or material energy dissipation rate) term, designated D;
2. an energetic driving force term, designated C.

D represents the rate at which energy, U_{diss}, is dissipated to all sources as a function of crack propagation area (Figure 1).

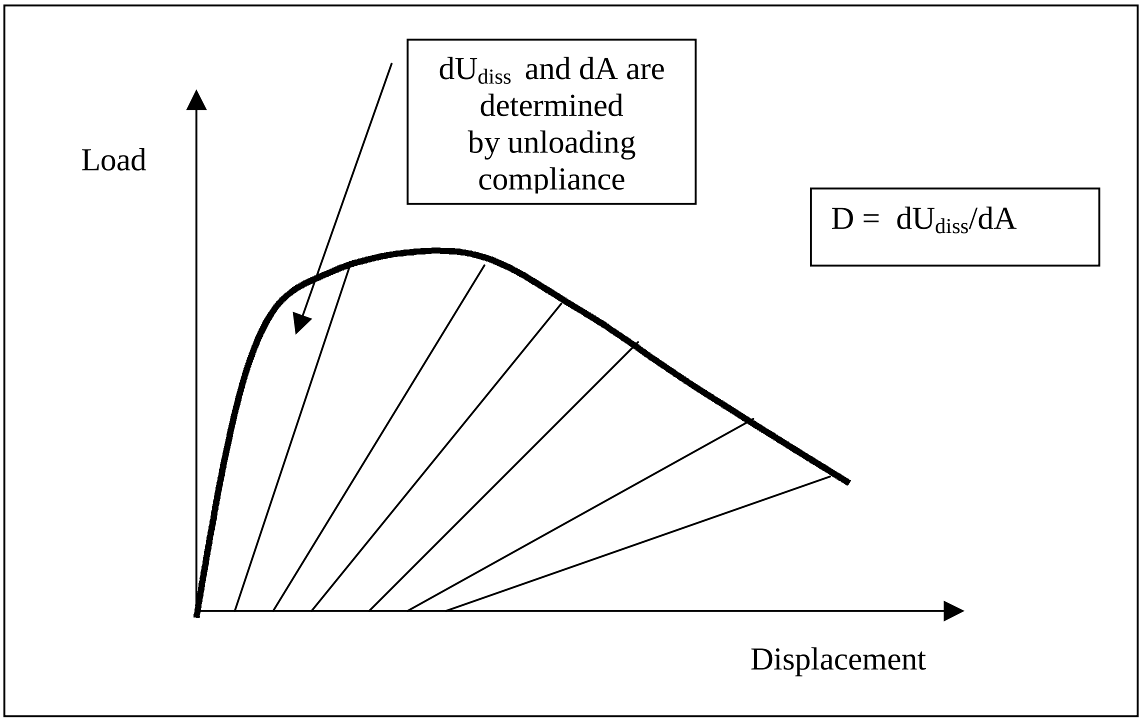

Figure 1: Experimental determination of D

Many authors have attempted to separate a local, material property term, usually designated as R or Γ, from the remote geometry dependent plasticity term. An example of this is provided by the 'essential work of fracture' approach of Cotterell and Reddell [6], which has gained popularity in recent years for characterising the tearing resistance of polymers [7], and sheet metals [8]. Atkins has also used this assumption in deriving scaling laws for elastic-plastic fracture [9]. The present author does not advocate this approach. The term D as used here contains all energy dissipation rate terms, local and remote from the crack tip. It is acknowledged that D defined in this way will be highly geometry dependent. In order to use D for crack stability predictions it has to be estimated for the particular geometry under consideration using scaling laws. The exact form of these scaling laws is not yet confirmed, but some known and suspected trends will be discussed later.

The definition of a stable crack is one that increases in length only when the remote load or boundary displacement is increased. Conversely, a crack is unstable if it increases in length even when the boundary displacement and load on the structure are held constant.

From the definitions it can be seen that the energy input from the system under fixed load and displacement are vital quantities in discussing crack stability. To clarify this in further discussion three terms are defined to describe the crack driving force term:

1. the generalised energy input term under increasing load or increasing displacement, designated C;
2. the energy input under fixed load (relevant to crack stability in a structure) designated C_p;
3. the energy input under fixed displacement (relevant to crack stability in a laboratory test piece) designated C_q.

Consider stable growth under load control, and assume for the moment that D is invariant with crack growth. The energy input, C, must match D by conservation of energy. But if the crack is also to be stable, C_p must be less than D. In small scale yielding (SSY) it is generally assumed that the linear elastic fracture mechanics (LEFM) term G gives the correct energy input term irrespective of load path. If this is true, $C = C_p = C_q$, and stable growth is impossible unless D increases with crack growth. This is the thinking that gave rise to the concept of the G_R resistance curve.

The first step in seeing the error in this viewpoint is to accept that very few metals fail without some crack tip plasticity. For instance, a critical crack tip failure strain for steel might be 30%. This is not achieved without the generation of a significant crack tip plastic zone. The plastic zone size can be small compared to structural dimensions (hence SSY), but its absolute size must be large to achieve 30% strain at the crack tip. The energy dissipation rate involved in recreating this plastic zone ahead of the advancing crack tip is not given by G. The elastic-plastic energy rate, C, that can be input during a crack growth increment depends on the compliance change, which in turn depends on the change in plastic zone size. If the load increases during the crack growth increment, there is a much larger increase in plastic zone size, and hence a larger compliance change, than if the load had stayed constant. It thus follows that in an elastic-plastic material, even one in SSY, $C > C_p > C_q$.

If this is accepted it is immediately apparent why cracks can be stable even when D does not increase with crack length. Consider a cracked structure under load control in SSY. The crack initiates when the critical crack tip strain is reached. In a tough material, this occurs when C_p (which to a first approximation is given by G in SSY) is well below D; so, the crack is stable. Crack growth can nevertheless be achieved under increasing load because this elevates C to match D. The growth will be stable until a load is reached where C_p matches D. At this stage the crack will go unstable as long as C_p continues to match or exceed D as the crack extends. An application of this approach to large aluminium panels is given in [10].

CHARACTERISTICS OF D

Energy dissipation rate can easily be measured by an unloading compliance, or multiple specimen method, of the type standardised for J resistance curve testing (Figure 1). Instead of plotting J against crack growth, dissipated energy, U_{diss}, is plotted against crack extension area. The energy dissipation rate, D, is the slope of this line.

There is no validity limit on D in terms of crack extension; but in a specimen with a fully yielded ligament, D will decrease as the crack extends. The reason for this is that the load is decreasing as the crack grows. Some normalisation can be achieved by dividing D by the current ligament length. Limited data presented in [11] shows that a normalised quantity D^*, given by D/b, where b is the current ligament length, is reasonably constant for crack growth across the first 50% of the ligament. Normalisation schemes for D in terms of specimen limit load are further discussed by Brocks and co-workers in [12,13].

Another feature of D is that it tends to start at a very high value for zero crack growth and subsequently decline [12,13]. There are several ways of understanding why this trend should occur. One explanation is that the crack is escaping from the initially heavily blunted crack tip region to assume its steady state form. Another way is to reason that, during the initial blunting phase of crack deformation, there is a lot of energy dissipated for a vanishingly small amount of crack growth. By this reasoning it makes sense that D should start at infinity for zero crack growth. A final link which can be made is with the shape of the J_R curve. This starts off steep, and then flattens with further growth. Because of their relative definitions D and dJ_R/da are linked approximately by the expression:

$$D = \frac{b}{\eta}\frac{dJ_R}{da} \tag{1}$$

where b is the current ligament length and η is the normal constant used for J determination. Hence, if the slope of the J resistance curve starts off high, D will also be high. Brocks and Siegmund [14] show an example of a dissipation rate curve for aluminium where D initially increases with crack growth. This trend is unusual, but is occasionally seen in materials which have a dramatic change from flat to shear fracture as the crack grows.

A key question is: ‘What happens to energy dissipation rate in SSY?’. Evidence on this is incomplete, but it should first be re-emphasised that, unless the material is intrinsically brittle, i.e. it has a failure strain which is close to the elastic yield strain, D does <u>not</u> reduce to G_i in SSY. Consider the trend of plasticity with increasing specimen size as the initial ligament length is increased. In a tough material there will be a range of ligament sizes over which the ligament will continue to be fully yielded. This will cause D to increase with specimen size. Eventually a specimen size will be reached where the crack begins to propagate before the ligament is fully yielded. Then, if the size increases still further, the crack will begin to propagate at lower and lower global stress levels. Eventually the crack will initiate and propagate with a plastic zone size equal to the Irwin SSY plastic zone size, R_{pi}.

After that point the plastic zone cannot get any smaller. An intuitive argument based on dimensional analysis is to say that the dissipation rate in small scale yielding can then be obtained in the following way:

1. determine the dissipation rate for the material in a small fully yielded bend specimen;
2. obtain the normalised dissipation rate, D*, as D/b where b is the current ligament length;
3. estimate the energy dissipation rate, D_{ssy}, for the structure in SSY as:

$$D_{ssy} = D^* R_{pi} \tag{2}$$

By inserting the expression for the Irwin plastic zone size at crack initiation, and rearranging:

$$\frac{D_{ssy}}{J_i} = \frac{D^* E}{n\pi\sigma_y^2} \tag{3}$$

Where: J_i is the J integral at crack initiation (= G_i in SSY); E is Young’s Modulus; and n is the constant in the Irwin plastic zone expression (= 1 in plane stress, = 3 in plane strain). This expression was given the name Crack Stability Index (CSI) in [11]. It is a material specific margin of safety against unstable tearing. For a very brittle material, with a crack tip strain to failure near the yield strain, the CSI will be 1 (Griffith material). However, for more typical structural materials, the CSI is likely to be in the region of 5 to 20, i.e. there is a large margin of safety against unstable tearing even in SSY.

The CSI as defined by (3) is identical to the ratio Γ^{ss}/Γ_o used by Tvergaard and Hutchinson (T&H) [15]. However, T&H incorrectly associate Γ with G at all stages of crack growth, and hence end up with curves which look like conventional G or K resistance curves. In reality, there is only equality between D and Γ for steady state crack growth. At this point, the crack is extending under constant applied G, and there has to be equality between all the terms G_{ss}, Γ^{ss}, and D_{ssy}. Up to this point crack growth has required increasing G, and, for reasons explained earlier, G underestimates D while the load is rising and the plastic zone size is increasing during the crack increment. It would be possible to determine D as a function of crack extension from computations of the type performed by T&H by considering changes in internal plastic energy, but results of this type have yet to appear in the published literature.

Boothman and Luxmoore performed some computations of D_{ssy} for a very large structure. These data are unpublished, but are contained in a University of Swansea report [16]. The results seem to support the estimation procedure for D_{ssy} in Equation (2). Moreover, D was found substantially independent of crack extension when crack initiation occurred in SSY. However, this is clearly an area where more computational evidence is required.

CHARACTERISTICS OF C

In order to apply energy dissipation rate theory in structures it will be necessary to develop full solutions for C_p as a function of applied stress. This might appear to be a daunting task, but it should be no more difficult than the generation of generalised J solutions. It can first be noted that C_p has the same limits as J as a function of applied stress over limit stress: for SSY both C_p and J tend to G; at plastic limit load both C_P and J tend to infinity.

This suggests that it will be possible to fit C_p analysis into a FAD (Fracture Analysis Diagram) framework using $(G/C_p)^{0.5}$ instead of $(G/J)^{0.5}$. However, the exact shape of the failure envelope will be changed because, in between the limits of G and infinity, C_p is not equal to J. This is illustrated in Figure 2 based on deformation plasticity definitions of J and C_p.

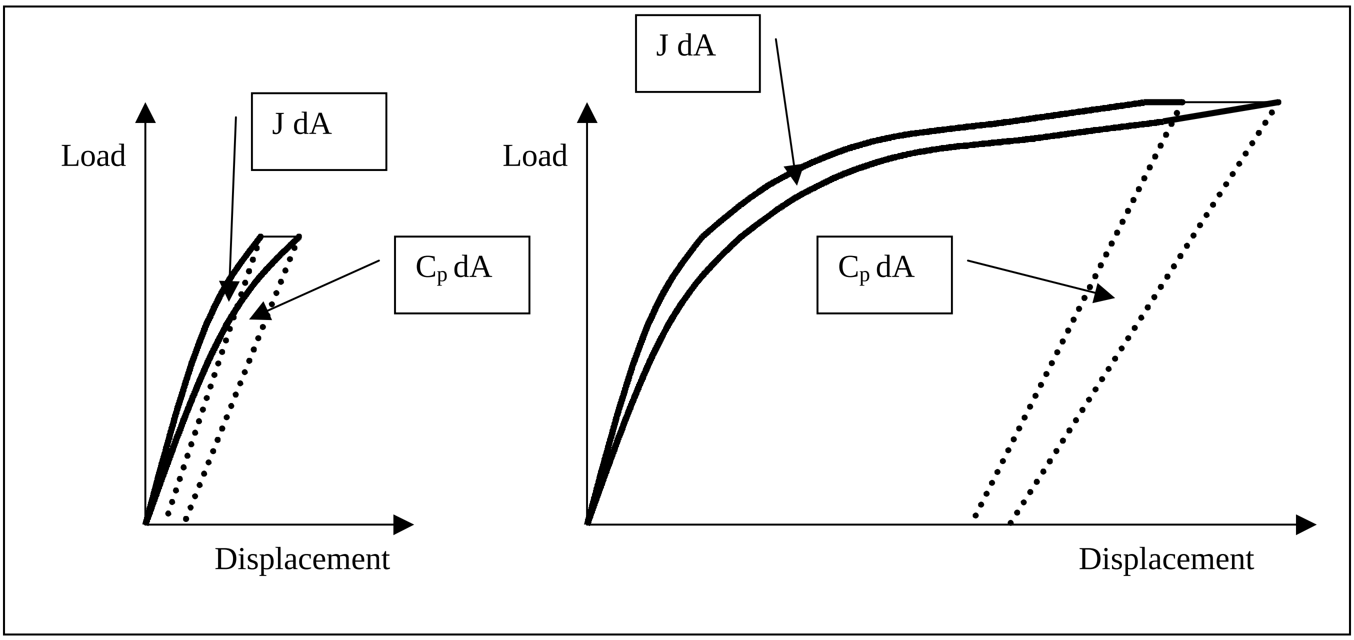

Figure 2: Definitions of J and C_p in SSY and after net section yield

In practice, C_p should be calculated for a real elastic-plastic material following the laws of incremental plasticity. However, computations in deformation plasticity are easier, and might be used to set an estimation scheme. Computations to determine C_p for both deformation and incremental plasticity are currently underway at the University of Illinois [17]. It is expected that work of this type will lead to easily usable estimates of C_p and C_q for structural analysis.

IMPLICATIONS FOR CRACK STABILITY

A typical value of D_{ssy} for a tough structural steel might be 5000 kJ/m^2. This is calculated from the procedures described above, and assumes plane stress, for a long through thickness crack. This immediately explains why unstable tearing is never encountered in structural steels under normal applied elastic stresses. A typical applied G for the size of crack which might appear in a structure would be about 250 kJ/m^2. A prediction of crack stability also results from application of J resistance curve theory to the same problem. Equivalent figures for the Paris tearing modulus, T [4], might be T_{mat} = 150, compared to a $T_{applied}$ = 5.

Both the J and energy dissipation rate approaches predict crack stability by a wide margin for this case, but there is a very important general difference between the two methods which can be seen by comparing the toughness and driving force terms.

TABLE 1

COMPARISON OF ENERGY DISSIPATION RATE AND J BASED INSTABILITY PRDICTION TERMS

	Toughness term	Driving force term
J based prediction	dJ_R/da or T_{mat}	dJ/da (dG/da for SSY) or $T_{applied}$
Energy dissipation rate based prediction	D (D_{ssy} for SSY)	C_p (approx G for SSY)

From Table 1 it can be seen that the toughness terms are rather similar in form. They are linked explicitly through Equation (1), and are both dependent on the rate of plastic energy dissipation in a fully yielded specimen. However, the SSY yielding driving force terms are fundamentally different. The J based approach uses dG/da whilst the energy dissipation rate approach uses G.

This has two important consequences. Firstly, dG/da has a very low absolute value. It is difficult to get $T_{applied}$ values in excess of 10, whilst on the other side of the equation, it is rare to find metals with T_{mat} less than 10. Consequently, it is very unlikely that crack instability would ever be predicted by J instability theory in SSY. Secondly, dG/da is independent of crack length unless there is a change in the geometric factor Y factor with crack length. Thus, considering a through crack in a large structure (for instance, a through crack in the deck of a ship) crack instability is not predicted to become any more likely with increase in crack length at a given stress level. This seems intuitively incorrect. By contrast, energy dissipation rate theory says that there is always a critical crack length for instability. For instance, assuming a D_{ssy} of 5000 kJ/m^2, the example given at the beginning of this section, the predicted critical crack length by energy dissipation rate theory is 16 metres at an applied stress of 200 MPa.

Although the prediction of critical crack lengths for steel this tough is likely to be of academic interest only, it is possible to envisage scenarios where the difference between the two theories will become of practical significance. It is not unknown for the upper shelf tearing resistance of irradiated steels to fall to a T_{mat} of around 30 or less. A steel of this toughness is still unlikely to be predicted to be unstable by J theory in SSY, because of the inherently low values of $T_{applied}$; but, the equivalent value of D_{ssy} using Equations (1) to (3), and assuming a plane strain plastic zone size is only 70 kJ/m^2, making a SSY instability a distinct possibility.

It is not even necessary to invoke an alternative theory to show that a prediction of tearing instability based on dJ/da must be incorrect for SSY. Figure 3 shows in schematic form the shapes of J resistance curve which are obtained by computational analysis of a propagating crack, firstly in the case of a fully yielded test piece [18], and secondly in the case of a SSY boundary value (elastic G field imposed on a ring of elements around the moving crack tip) [15]. In the former case it is shown that J rises steadily with crack extension, the familiar J_R resistance curve. However, in the SSY case a plateau is reached where the crack is increasing in length even although the boundary G is held constant. There are two problems here for J theory: firstly, the J resistance curve is not supposed to depend on size scale, the deeply notched, fully yielded bend specimen is supposed to constitute a 'high constraint', lower bound for SSY; secondly, within the confines of the SSY calculation itself, how can the crack be unstable when the applied dG/da is zero?

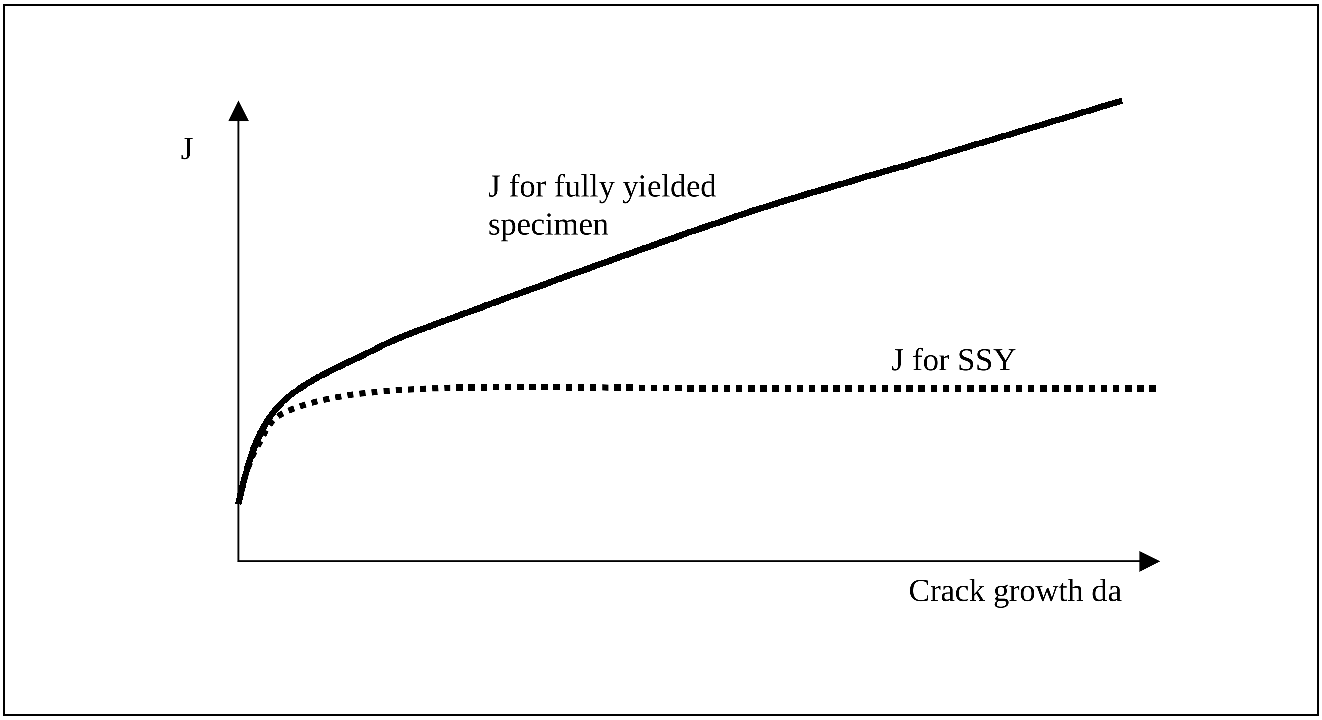

Figure 3: Trends in J for SSY and for a fully yielded specimen

There is no problem in explaining both these cases by energy dissipation rate analysis. Small specimens are tested under displacement control; hence, a C_q analysis applies. The specimen will be stable as long as C_q is less than the current value of D. This is very likely to be the case with a small specimen and a reasonably tough material. While the specimen is stable, accumulated work, and hence J will continue to increase without limit. However, in the SSY boundary value computation, a steady state is reached where the applied G matches D_{ssy}. The crack is then effectively unstable as far as the structure is concerned (i.e. it will increase in length for any non-negative dG/da).

CONCLUSIONS

Crack stability of ductile materials is best described by energy dissipation rate analysis. The toughness term D is not a material property, but may nevertheless be estimated for any geometry from its value in a small deeply notched test piece. The necessary estimation schemes are not yet in place, but their general form can be anticipated. The quantity D_{ssy} is of particular interest, since it is the lower limit of D for crack initiation in SSY (small scale yielding). This term is the same as the quantity Γ_{ss}, which appears in the work of Tvergaard and Hutchinson [15]. Cracks are initially stable, even in SSY, because D exceeds the energetic driving force term C_p from the outset of crack growth. By this analysis, it is not necessary to invoke an increase in toughness with crack growth to explain crack stability. The toughness term D can be constant, or decrease with crack growth. There will be no instability as long as D exceeds C_p. It is shown that the use of a J_R resistance curve derived from a small specimen may lead to unsafe predictions of crack stability for low toughness materials in SSY.

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