EFFECTIVE STRESS INTENSITY FACTOR AND CONTACT STRESS FOR PARTIALLY CLOSED OBLIQUE EDGE CRACKS

M . Beghini[1], *L. Bertini*[1], *V. Fontanari*[2]

[1] Dip di Ingegneria Meccanica, Nucleare e Produzione – University of Pisa
Via Diotisalvi 2 - 56126 Pisa (Italy)
[2] Dip. di Ingegneria dei Materiali - University of Trento
Via Mesiano 77 –38050 Trento (Italy)

KEYWORDS: weight function, oblique edge crack, Green function, partial closure, contact stress

ABSTRACT

The matrix like Weight Functions developed for determining the Stress Intensity Factors of an oblique edge crack was used to derive the Green Functions giving the components of the Crack Opening Displacement function for any loading condition. The computational efficiency of this method allowed for solving the non linear problem of a partially closed inclined edge crack. The condition of remote tension and bending loading was analysed. Taking into account the general properties of the Green Functions, a discretised formulation of the problem was developed and a numerical evaluation of the Crack Opening Displacement components was obtained. By adopting an iterative procedure the open portion of the partially closed crack was determined and, consequently, the contact pressure distribution, the effective Crack Opening Displacement components and the effective Stress Intensity Factors were calculated. The efficiency and the accuracy of this approach were confirmed by comparison with Finite Element solutions of the problem.

INTRODUCTION

The presence of an edge cracks inclined with respect to the normal of the surface may be quite often experienced in mechanical components subjected to variable surface loading in service, such as for instance in wear damaged rolling bearings and gears teeth flanks or fillets.
The lack of symmetry and the presence of both normal and shear nominal (i.e. acting in the similar uncracked body) stress components, generally induces a mixed mode fracture loading, which requires the evaluation of both the Stress Intensity Factors (SIFs) K_I and K_{II}.
Moreover, the nominal stress fields to which this defects typically undergo in service can be very complex with local peaks and high gradients being the superposition of stress due to external loading and residual stress fields. During the loading cycles, a fatigue growing crack may experience conditions of partial closure giving rise to contact phenomena which significantly affect the SIFs and ,consequently, the growth rate. In this case, also within the limits of the linear elastic behaviour for the material, the problem is typically non linear as the boundary conditions (in particular location and extension of the contact zone) are partially unknown and depend on the applied loads. Therefore, the fatigue life of the component can be quite difficult to predict on the basis of the fracture mechanics approach being the effective crack tip stress cycle very different from that evaluated on the basis of the external loading only [1]. The Weight Function (WF) approach can provide an effective tool for solving this type of problems [2].

The problem of partial crack closure in the case of symmetric crack configurations was studied in [2,3,4] by using the WF approach and carrying out an computationally efficient iterative procedure for the evaluation of the open region of the crack. The knowledge of the closed region of the crack enabled the evaluation of the contact stress distribution, the Crack Opening Displacement (COD) in the open region and finally the effective **SIF** by superimposing the external and residual stresses to the contact stress. **A** more general formulation of this method suitable to solve the problem of a partially closed crack in a non symmetric plane body, is proposed in this work.
The matrix like WF suggested by Fett and Muntz *[5]* and developed by the authors *[6]* for an oblique edge crack in a semi-plane is employed. By starting from the general properties of the WFs, suitable Green Functions (GS) giving the COD components for a general loading conditions were deduced and expressed in the form of symbolic integrals. In particular, by considering the power law expansions adopted for the WFs, a recursive procedure was set up for the evaluation of these integrals which reduced the computational effort for the COD evaluation.
In order to assess the usefulness of the proposed method, **SIF** and COD components for an oblique edge crack under remote tension and bending were compared to the results of an accurate FE analysis. The definition of the GF, the method for its calculation and the case study with the comparison are reported in the following sections.

DEFINITION AND EVALUATION OF THE GREEN FUNCTION

By considering the general properties of the elastic field the general SIFs for an edge crack having length a in a plane body under nominal stress components $\sigma(x),\tau(x)$ (in the local crack reference frame with **x** axis laying on the crack, as shown in Fig. 1) can be written as follows:

$$\begin{pmatrix} K_I(a,\theta) \\ K_{II}(a,\theta) \end{pmatrix} = \int_0^a \begin{pmatrix} h_{I\sigma}(x,a,\theta) & h_{I\tau}(x,a,\theta) \\ h_{II\sigma}(x,a,\theta) & h_{II\tau}(x,a,\theta) \end{pmatrix} \cdot \begin{pmatrix} \sigma(x,\theta) \\ \tau(x,\theta) \end{pmatrix} \cdot dx \qquad (1)$$

being 8 the angle between the crack and the normal to the body edge. By indicating with the subscript M (either I or II) to the opening mode and the subscript μ (either σ or τ) the component of the nominal stress, the following expressions were proposed in *[6]* for the WF components:

$$h_{M\mu}(x,a,\theta) = \sqrt{\frac{2}{\pi a}} \cdot \left[\left(1-\frac{x}{a}\right)^{-1/2} + \sum_{i=1}^{n} \alpha_i^{M\mu}(\theta) \cdot \left(1-\frac{x}{a}\right)^{i-1/2} \right] \quad for\, M\mu = I\sigma \text{ or } II\tau \qquad (2)$$

$$h_{M\mu}(x,a,\theta) = \sqrt{\frac{2}{\pi a}} \cdot \left[\sum_{i=1}^{n} \alpha_i^{M\mu}(\theta) \cdot \left(1-\frac{x}{a}\right)^{i-1/2} \right] \quad for\, M\mu = II\sigma \text{ or } I\tau \qquad (3)$$

The angular dependence of the $\alpha_i^{M\mu}(\theta)$ coefficients were approximated by truncated Fourier's expansions:

$$\alpha_i^{M\mu}(\theta) = \lambda_{i1}^{M\mu} \cdot \tan^2(\theta) + \sum_{j=2}^{m} \lambda_{ij}^{M\mu} \cdot \cos((j-2)\theta) \quad for\, M\mu = I\sigma \text{ or } II\tau \qquad (4)$$

$$\alpha_i^{M\mu}(\theta) = \lambda_{i1}^{M\mu} \cdot \tan^2(\theta)\sin(\theta) + \sum_{j=2}^{m} \lambda_{ij}^{M\mu} \cdot \sin((j-1)\theta) \quad for\, M\mu = I\sigma \text{ or } II\tau \qquad (5)$$

being $\lambda_{ij}^{M\mu}$ suitable constants calculated in [6] for an edge crack in a semiplane in the range $-70° < 8 < 70°$.
In matrix notation eqn. 1 can be written as:

$$[K(a,\theta)] = \int_0^a [W(x,a,\theta)] \cdot [S(x,\theta)] \cdot dx \qquad (6)$$

Considering the definition the WF matrix $W(x,a,\theta)$ [6], the COD components u and v, indicating the relative displacement of adjacent points on the crack edges in the **x** and y direction respectively, can be calculated:

$$v(x,a,\theta)=\frac{2}{H}\int_x^a\left[h_{I\sigma}(x,b,\theta)\cdot K_I(b,\theta)+h_{II\sigma}(x,b,\theta)\cdot K_{II}(b,\theta)\right]db \tag{7}$$

$$u(x,a,\theta)=\frac{2}{H}\int_x^a\left[h_{I\tau}(x,b,\theta)\cdot K_I(b,\theta)+h_{II\tau}(x,b,\theta)\cdot K_{II}(b,\theta)\right]db \tag{8}$$

where H is equal to E (Young Modulus) for plane stress and $E/(1-\nu^2)$ for plane strain.
For a particular angle, after introducing the expressions (1) for K_I and K_{II} , eqn. **7** and 8 become:

$$\begin{aligned} v(x,a)=&\frac{2}{H}\cdot\int_x^a\left[\int_0^b h_{I\sigma}(x,b)\cdot\left[h_{I\sigma}(x',b)\cdot\sigma(x')+h_{I\tau}(x',b)\cdot\tau(x')\right]dx'\right]db\\ &+\frac{2}{H}\cdot\int_x^a\left[\int_0^b h_{II\sigma}(x,b)\cdot\left[h_{II\sigma}(x',b)\cdot\sigma(x')+h_{II\tau}(x',b)\cdot\tau(x')\right]dx'\right]db \end{aligned} \tag{9}$$

$$\begin{aligned} u(x,a)=&\frac{2}{H}\cdot\int_x^a\left[\int_0^b h_{I\tau}(x,b)\cdot\left[h_{I\sigma}(x',b)\cdot\sigma(x')+h_{I\tau}(x',b)\cdot\tau(x')\right]dx'\right]db\\ &+\frac{2}{H}\cdot\int_x^a\left[\int_0^b h_{II\tau}(x,b)\cdot\left[h_{II\sigma}(x',b)\cdot\sigma(x')+h_{II\tau}(x',b)\cdot\tau(x')\right]dx'\right]db \end{aligned} \tag{10}$$

As suggested in *[5]*the order of integration can be changed as follows:

$$\begin{aligned} v(x,a)=&\frac{2}{H}\cdot\int_0^a\left[\int_{\max(x,x')}^a\left[h_{1\sigma}(x,b)\cdot h_{1\sigma}(x',b)+h_{2\sigma}(x,b)\cdot h_{2\sigma}(x',b)\right]\cdot\sigma(x')\cdot db\right]\cdot dx'\\ &+\frac{2}{H}\cdot\int_0^a\left[\int_{\max(x,x')}^a\left[h_{1\sigma}(x,b)\cdot h_{1\tau}(x',b)+h_{2\sigma}(x,b)\cdot h_{2\tau}(x',b)\right]\cdot\tau(x')\cdot db\right]\cdot dx' \end{aligned} \tag{11}$$

$$\begin{aligned} u(x,a)=&\frac{2}{H}\cdot\int_0^a\left[\int_{\max(x,x')}^a\left[h_{1\tau}(x,b)\cdot h_{1\sigma}(x',b)+h_{2\tau}(x,b)\cdot h_{2\sigma}(x',b)\right]\cdot\sigma(x')\cdot db\right]\cdot dx'\\ &+\frac{2}{H}\cdot\int_0^a\left[\int_{\max(x,x')}^a\left[h_{1\tau}(x,b)\cdot h_{1\tau}(x',b)+h_{2\tau}(x,b)\cdot h_{2\tau}(x',b)\right]\cdot\tau(x')\cdot db\right]\cdot dx' \end{aligned} \tag{12}$$

From eqns. 11 and 12 a matrix-like formulation of the GFs **G**(x,x') can be deduced, whose terms can be expressed as symbolic integrals:

$$G_{v\sigma}(x,x')=\int_{\max(x,x')}^a\left[h_{1\sigma}(x,b)\cdot h_{1\sigma}(x',b)+h_{2\sigma}(x,b)\cdot h_{2\sigma}(x',b)\right]db \tag{13}$$

$$G_{v\tau}(x,x')=\int_{\max(x,x')}^a\left[h_{1\sigma}(x,b)\cdot h_{1\tau}(x',b)+h_{2\sigma}(x,b)\cdot h_{2\tau}(x',b)\right]db \tag{14}$$

$$G_{u\sigma}(x,x')=\int_{\max(x,x')}^a\left[h_{1\tau}(x,b)\cdot h_{1\sigma}(x',b)+h_{2\tau}(x,b)\cdot h_{2\sigma}(x',b)\right]db \tag{15}$$

$$G_{u\tau}(x,x') = \int_{\max(x,x')}^{a} \left[h_{1\tau}(x,b)\cdot h_{1\tau}(x',b) + h_{2\tau}(x,b)\cdot h_{2\tau}(x',b)\right]\cdot db \tag{16}$$

These terms can be written in compact in matrix notation:

$$G(x,x') = \begin{bmatrix} G_{v\sigma}(x,x') & G_{v\tau}(x,x') \\ G_{u\sigma}(x,x') & G_{u\tau}(x,x') \end{bmatrix} = \int_{\max(x,x')}^{a} \left[W(x,b)\right]^{T} \cdot \left[W(x',b)\right] db \tag{17}$$

eventually giving the COD components for any loading condition:

$$\begin{bmatrix} v(x,a) \\ u(x,a) \end{bmatrix} = \frac{2}{H}\cdot\int_{0}^{a}\left[G(x,x')\right]\cdot S(x')\cdot dx' \tag{18}$$

By introducing the expression of the WFs, the following expressions hold:

$$G_{v\sigma}(x,x') = \frac{2}{\pi}\cdot\sum_{k,j=0}^{n}\left(\alpha_{(I\sigma)k}\cdot\alpha_{(I\sigma)j} + \alpha_{(II\sigma)k}\cdot\alpha_{(II\sigma)j}\right)\int_{\max(x,x')}^{a}\frac{1}{b}\cdot\left(1-\frac{x}{b}\right)^{\left(k-\frac{1}{2}\right)}\cdot\left(1-\frac{x'}{b}\right)^{\left(j-\frac{1}{2}\right)} db \tag{19}$$

$$G_{v\tau}(x,x') = \frac{2}{\pi}\cdot\sum_{k,j=0}^{n}\left(\alpha_{(I\sigma)k}\cdot\alpha_{(I\tau)j} + \alpha_{(II\sigma)k}\cdot\alpha_{(II\tau)j}\right)\int_{\max(x,x')}^{a}\frac{1}{b}\cdot\left(1-\frac{x}{b}\right)^{\left(k-\frac{1}{2}\right)}\cdot\left(1-\frac{x'}{b}\right)^{\left(j-\frac{1}{2}\right)} db \tag{20}$$

$$G_{u\sigma}(x,x') = \frac{2}{\pi}\cdot\sum_{k,j=0}^{n}\left(\alpha_{(I\tau)k}\cdot\alpha_{(I\sigma)j} + \alpha_{(II\tau)k}\cdot\alpha_{(II\sigma)j}\right)\int_{\max(x,x')}^{a}\frac{1}{b}\cdot\left(1-\frac{x}{b}\right)^{\left(k-\frac{1}{2}\right)}\cdot\left(1-\frac{x'}{b}\right)^{\left(j-\frac{1}{2}\right)} db \tag{21}$$

$$G_{u\tau}(x,x') = \frac{2}{\pi}\cdot\sum_{k,j=0}^{n}\left(\alpha_{(I\tau)k}\cdot\alpha_{(I\tau)j} + \alpha_{(II\tau)k}\cdot\alpha_{(II\tau)j}\right)\cdot\int_{\max(x,x')}^{a}\frac{1}{b}\cdot\left(1-\frac{x}{b}\right)^{\left(k-\frac{1}{2}\right)}\cdot\left(1-\frac{x'}{b}\right)^{\left(j-\frac{1}{2}\right)} db \tag{22}$$

All these terms can be calculated by solving the following $(n+1)$x$(n+1)$ integrals:

$$I_{kj}(x,x') = \int_{\max(x,x')}^{a}\frac{1}{b}\cdot\left(1-\frac{x}{b}\right)^{\left(k-\frac{1}{2}\right)}\cdot\left(1-\frac{x'}{b}\right)^{\left(j-\frac{1}{2}\right)} db \qquad k,j,=0,n \tag{23}$$

Considering the power law expansion proposed in [6] for the WFs, being n=4, 25 integrals have to be solved. A recursive approach was adopted in order to obtain a symbolic solution [7].

USE OF THE GREEN FUNCTIONS FOR SOLVING THE CONTACT PROBLEM

Knowing the GFs, the COD components can be determined at any location of the crack for any loading condition by eqn. (18) when the nominal stress components $\sigma(x)$ and $\tau(x)$ are known on the crack edge. Starting from this basis the problem of crack closure can be faced in an efficient way by using the procedure proposed in [3] for a symmetrical crack configuration. In the case of an oblique crack, the problem appears more complex as: a mixed cracking mode is expected, both u and **v** COD components have to be evaluated and a coupling effect is active between normal stresses and tangential displacements and viceversa.
In order to simplify the definition of the problem and to reduce the number of parameters, an ideal frictionless condition was assumed on the contact zone even though the method can take account of the friction in a more general case. Moreover the closed portion of the crack was assumed to be an interval

starting from the crack mouth and having (unknown) extension x_c. Also this hypothesis can be dropped for a general problem with several contact zones without requiring significant differences in the solving algorithm. The following properties for the COD components and the contact pressure $p(x,a)$ have to be fulfilled for the analysed condition:

$$COD(x,a,\theta):\begin{cases} v(x,a,\theta)=0 & for \quad 0\le x\le x_c \\ v(x,a,\theta)\ge 0 & for \quad x_, \le x\le a \\ u(x,a,\theta)=any\ value & for \quad 0\le x\le a \end{cases} \tag{24}$$

$$p(x,a,\theta):\begin{cases} \ge 0 & for \quad 0\le x\le x_c \\ =0 & for \quad x_c\le x\le a \end{cases} \tag{25}$$

The crack was subdivided in m (=100) contiguous intervals. A set of $2mx2m$ influence coefficients was calculated by applying eqn. (18) which represent the averaged COD on the any segment due to a unit normal stress and to a unit tangential stress applied at any other segment. A matrix [M] composed by ***4 mxm*** symmetric sub-matrices was defined as follows:

$$M=\begin{bmatrix} \begin{pmatrix} v\ component\ of\ COD \\ due\ to\ \sigma_{k(n)}=1 \end{pmatrix} & \begin{pmatrix} v\ component\ of\ COD \\ due\ to\ a\,\tau_{k(n)}=1 \end{pmatrix} \\ \begin{pmatrix} u\ component\ of\ COD \\ due\ to\ \sigma_{k(n)}=1 \end{pmatrix} & \begin{pmatrix} u\ component\ of\ COD \\ due\ to\ a\,\tau_{k(n)}=1 \end{pmatrix} \end{bmatrix} \tag{26}$$

As a consequence for any nominal stress distribution, represented by a vector $[\overline{S}]$ of stress components applied on the segments, the averaged COD vector $[\overline{COD}]$ can be obtained by:

$$[\overline{COD}]=[M][\overline{S}] \tag{27}$$

In order to determine the effective extension of the open zone, an iterative procedure can be carried out starting from a tentative value of the extension of crack closure ***(m '***of the m segment in which the crack has been subdivided). By applying the principle of superposition the following equations hold:

$$v(x_h)=\sum_{k=1}^{m'} p_k\cdot\int_{x_k}^{x_{k+1}} G_{v\sigma}(x_h,x')\cdot dx'+\sum_{k=1}^{m}\left[\sigma_k\cdot\int_{x_k}^{x_{k+1}} G_{v\sigma}(x_h,x')\cdot dx'+\tau_k\cdot\int_{x_k}^{x_{k+1}} G_{v\tau}(x_h,x')\cdot dx'\right] \tag{28}$$

$$h=1,..,m$$

$$u(x_h)=\sum_{k=1}^{m'} p_k\cdot\int_{x_k}^{x_{k+1}} G_{u\sigma}(x_h,x')\cdot dx'+\sum_{k=1}^{m}\left[\sigma_k\cdot\int_{x_k}^{x_{k+1}} G_{u\sigma}(x_h,x')\cdot dx'+\tau_k\cdot\int_{x_k}^{x_{k+1}} G_{u\tau}(x_h,x')\cdot dx'\right] \tag{29}$$

$$h=1,..,m$$

A system of $2m$ linearly independent equations was written, whose unknowns are: ***m'*** values of $p(x,a)$, ***(m-m')***values of v and m values of ***u.***
The system has to solved by changing ***m'*** until the obtained solution fulfil the condition ***(24)*** and ***(25).***

AN APPLICATIVE EXAMPLE

In order to demonstrate the usefulness of the described procedure, a typical problem of partial closure was faced: SIFs and COD components for oblique edge cracks in a semiplane under remote tension and bending were calculated (Fig. 1).

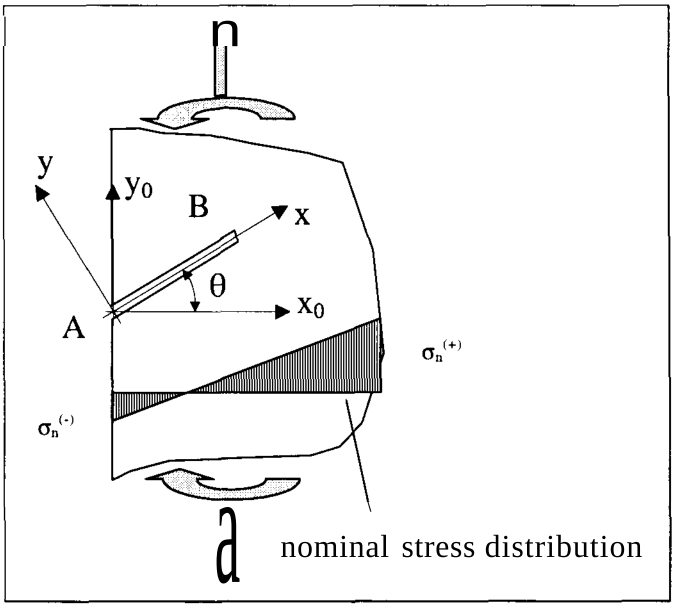

Fig. 1: Load systems used in the analysis

The nominal stress was chosen in order to produce a partial frictionless closure of the crack in the mouth region. The nominal stress resolved on the crack coordinate system can be written as follows:

$$\sigma = \left[\sigma_A + (\sigma_B - \sigma_A)\cdot \frac{x}{a)}\right]\cdot \frac{1+\cos(2\theta)}{2} \qquad \tau = \left[\sigma_A + (\sigma_B - \sigma_A)\cdot \frac{x}{a)}\right]\cdot \frac{\sin(2\theta)}{2} \tag{30}$$

COD (*v* component) and contact pressure along the crack are shown in figures 2 and 3 for two inclination angles. The dimensionless coordinate $\xi = x/a$ was introduced for the position and the nominal normal stress range resolved on the crack coordinate system $\Delta\sigma$ was uses for normalizing the COD components and the contact pressure.

Both the angular positions were studied by accurate FE analyses, using the model set up for the evaluation of the WFs reported in [7,8]. The model was developed by the Ansys5.3 computer program using linear elastic 8-nodes iso-parametric plane strain elements. In order to reproduce a virtually semi-infinite cracked plate a sub-modelling technique was adopted. Point to surface contact elements were introduced to simulate the crack closure.

The SIFs obtained by the FE analyses are compared to those obtained by the proposed approach in table 1. In any analysed condition and for any component a very good mutual agreement can be observed. In table 1 also the values calculated without taking into account of the contact pressure are reported (subscript *lin*) thus showing the strong effect produced on the effective SIFs by the contact pressure.

	θ = 0°		θ = 30°			
	$K_{Ilin}/(\Delta\sigma\sqrt{a})$	$K_{Ieff}/(\Delta\sigma\sqrt{a})$	$K_{Ilin}/(\Delta\sigma\sqrt{a})$	$K_{Ieff}/(\Delta\sigma\sqrt{a})$	$K_{IIlin}/(\Delta\sigma\sqrt{a})$	$K_{IIeff}/(\Delta\sigma\sqrt{a})$
FEM	0.1006	0.2883	-0.1265	0.1891	0.1629	-0.04550
WFs	0.1016	0.2888	-0.1271	0.1899	0.1622	-0.04588
diff. (%)	0.99	0.17	0.47	0.42	0.43	0.83

Tab. 1: SIFs obtained by FE analysis and by the proposed method.

The agreement between FE and the proposed method can be appreciated in the figure 4 and 5 too, in which a contact pressure and the COD components are compared respectively. Continuos lines refer to the proposed solution while symbols to the FE solutions. Relative differences are within a few per cent.

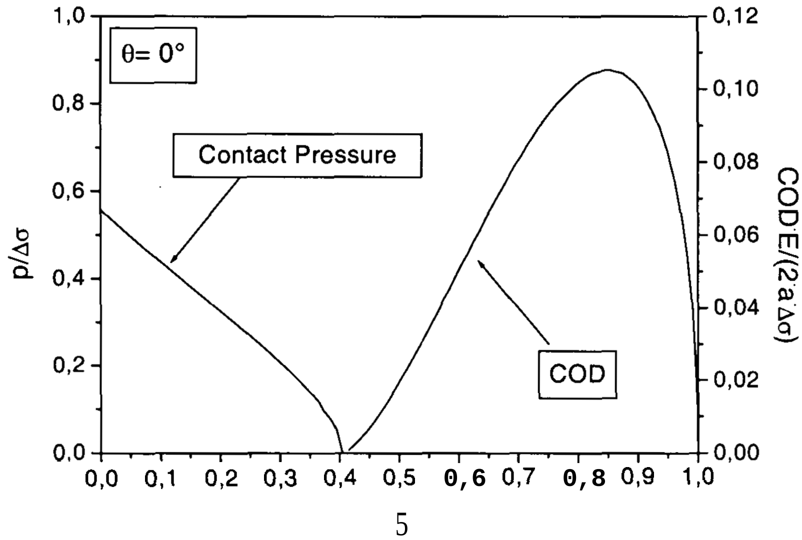

Fig. 2:Contact stress and COD function versus the position for a partially closed crack with θ=0°

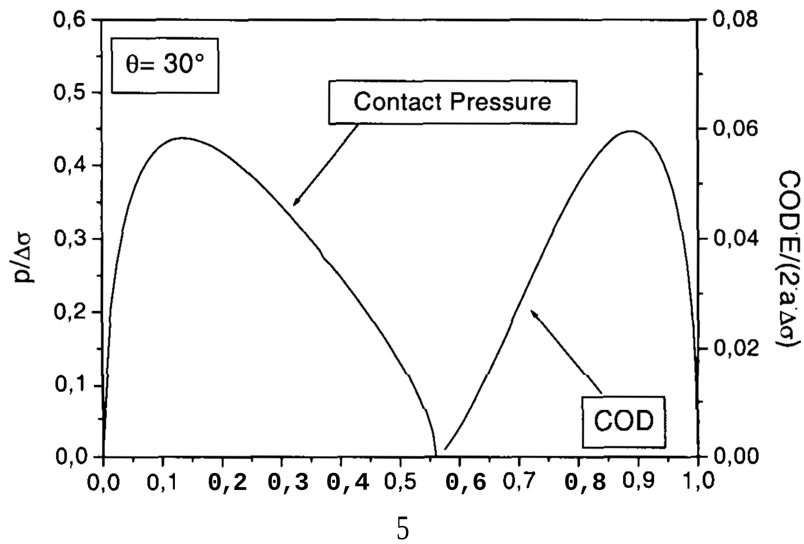

Fig. 3:Contact stress and COD function versus the position for a partially closed crack with θ=30°

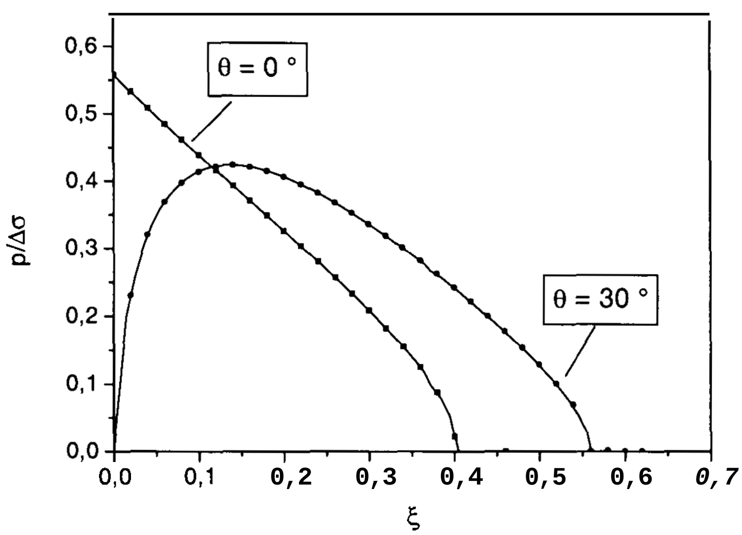

Fig. **4:** comparison between contact pressure calculated by FEM (symbols) and WF method (solid line)

The proposed applicative example proved the usefulness of the method based on the WFs and GFs approach. This method appears very effective for the calculation of contact pressure distribution, COD components and effective SIFs, as the accuracy is comparable to that obtainable with an accurate FE analysis and the computational effort is much smaller The frictionless contact hypothesis can be easily removed and more complex problems can be faced without introducing significant modifications.

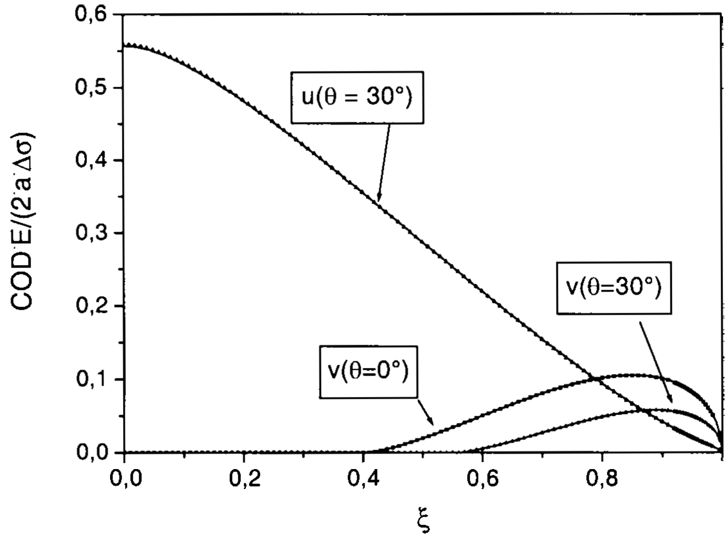

Fig. *5:* comparison between u and v COD components calculated by FEM (symbols) and WF method (solid line)

CONCLUSION

By means of the WF technique, the non linear problem of a partially closed inclined edge crack was faced. Starting from the theoretical definition of the COD components, a matrix like expression of the Green functions in terms of symbolic integrals was determined. A recursive procedure was set up in order to obtain the GFs analytically, thus providing an efficient method for the calculation of the COD components at any position along the crack and for any loading condition.

To test the technique, a typical problem of a partially closed crack was considered. An iterative procedure was carried out for evaluating of the closed region of the crack. The knowledge of the closed region allowed the contact pressure, the effective COD components and the effective SIFs to be efficiently calculated.

The results were compared to those obtained by an accurate FE analysis and a fairly good accuracy was demonstrated.

REFERENCES

1. Beghini M., Bertini L., Vitale E.: 'Fatigue crack growth in residual stress fields: experimental results and modelling', (1994) *Fatigue andfracture of engineering materials and Structures* **12,** 1433.
2. Beghini M., Bertini L., Vitale E.: 'Weight functions applied to fatigue crack growth analysis', (1997) *Fatigue andfracture of engineering materials and Structures* **20** (8), 1093.
3. Beghini M., Bertini L.: 'Contact stress surface crack: exact solution and assessment of approximate evaluations'. (1995) In: *Contact mechanics II*, Comp. Mech. Pu., pp. 445-452.
4. Beghini M., Bertini L., Fontanari V.: 'Fatigue life evaluation by weight functions for orthotropic bridge decks'. (1997) *Theoretical and appliedfracture mechanics* **28**, 41.
5. Fett T., Muntz D.(1997). *Stress intensityfactors and weightfunctions,* Comp. Mech . Pu.
6. Beghini M., Bertini L., Fontanari V.: 'Weight function for an inclined edge crack in a semiplane', (1999) *International Journal offracture*, **99**, 28 1.
7. Beghini M., Bertini L., Fontanari V.: 'COD evaluation by weight function' in preparation
8. Beghini M., Bertini L., Fontanari V.: 'Stress intensity factors for an inclined edge crack in a semiplane' (1999). *Engineeringfracture mechanics* 62,607