DEVELOPMENT OF EDGE CRACKS IN ROLLING BODIES

V.V. Panasyuk and O.P. Datsyshyn

Karpenko Physico-Mechanical Institute, National Academy of Sciences of Ukraine
5, Naukova St., 79601, Lviv, Ukraine

ABSTRACT

A calculational model for investigation of the fracture processes and durability of the rolling bodies surface contact zones has been proposed within the framework of the fracture mechanics concepts. The unidirection rolling in conditions of dry and boundary friction has been investigated. The model basis is the algorithm step-by-step construction of cracks development paths under complex stress state in the contact zone of two bodies, criteria of local fracture, equations of fatigue cracks growth and characteristics of material cyclic crack resistance. The paths calculation algorithm also takes into account redistribution of stresses, caused both by cracks growth and by mutual movement of contacting bodies. The stress-strain state at crack tip at each step of path construction is determined by the numerical solution of singular integral equations of the corresponding two-dimentional problem of the elasticity theory for a half-plane with edge curvilinear crack under efforts distributed according to elliptical law on the boundary part of the half-plane. In the case of rolling with lubrication, that can fill the crack, its action is modelled by stresses, additionally distributed on the crack faces. Paths of edge cracks growth have been constructed under rolling both under the dry friction and boundary lubrication conditions. The dependence of paths form on friction coefficient between contacting bodies, on the length and orientation of the initial crack, and in the presence of lubrication - on the intensity of its pressure on the crack faces, has been investigated. The experimentally established earlier regularity for the growth of a dominant crack branch from boundary into the material has been confirmed for a case of dry friction. Conditions, under which the branchings from the basic crack in the direction to body boundary appears, are also have been revealed. In the case of rolling under boundary lubrication theoretical solutions have been obtained that confirm a known Way`s hypothesis and other experimental data about the role of lubrication on the pittings appearance. The estimation of residual durability of the ШХ-15 bearings steel presurface layer under boundary lubrication has been made.

INTRODUCTION

Cracks, pittings and spallings are typical damages, which appear on the surface of the elements of rolling pairs and lead to the loss of their serviceability. In particular, it concerns the elements of roller bearings, pairs "rail-wheel", rolls of the rolling mills, elements of gears etc. Experimental investigations of the fracture surface under rolling in conditions of dry and boundary friction (lubrication) between the contacting bodies have been carried out by many researches. Theoretical investigations in this direction are not so numerous (see [1-3] and a summerizing paper of Murakami et.al [4]).

Experimental investigations showed [4, 5] that in unidirection rolling cracks appear and propagate more frequently in a follower body. In this case a macrocrack propagation included two stages: at the first stage of development the crack grows rectilinearly by mode II fracture at an acute angle to the surface in the direction of a counter-body motion, basically as a continuation of initial macrocrack; later a crack developed predominantly curvilinearly by mode I fracture. At the first stage the influence of a lubricant results in a accelerated crack propagation due to a reduced friction between the crack faces [4, 6]. This stage is controlled by a normal component of the contact loading when latter moves under the crack mouth. At the second stage the edge crack path in dry friction differs significantly from that in boundary lubrication conditions. A common feature is that crack propagation by this mechanism is possible due to formation of the area material tension in the crack tip vicinity. In dry friction conditions this is possible due to large tangential stresses (friction forces) on the area boundary, when a driver counter-body approaches to the open crack mouth, while in lubrication conditions - because of the lubricat, that penetrating into a crack causes its wedging, when a counter-body covers the crack mouth.

The majority of theoretical works [4, 6-9] are dedicated to the first stage of fatigue macrocrack growth, as to the second stage, they are limited to an evaluation of stress intensity factors (SIF) and initial deviation angles for rectilinear [6-9] or plane [4] cracks. On this basis the paths and durability of rolling bodies are predicted.

This paper deals with peculiarities of the edge crack propagation at the second stage (mode I fracture) and a method of assessing the rolling bodies durability by their crack growth resistance. A two-dimentional model suggested by the authors in [10-14] has been used for investigations. An algorithm of the step-by-step macrocrack propagation path construction is a key elements of the model. The algorithm is based on a singular integral equation for a half-plane with edge curvilinear crack, employing the local fracture criteria under complex stress-strain state, equations of fatigue crack growth and characteristics of the cyclic crack growth resistance of the material. This algorithm allows to account the change of the stress-strain state by crack growth and a counter-body motion. The paths of the edge crack propagation under rolling in the conditions of dry friction and boundary lubrication have been constructed. A dependence of the path geometry on the coefficient of friction between two contacting bodies, the initial crack length and orientation and also, when the lubricant is present, on lubricant pressure on the crack faces has been investigated. A residual durability of the near-surface leyer of bearing steel ШХ-15 under rolling in boundary lubrication conditions was assessed. A generalized mode I fracture criterion (σ_θ-criterion) has been used in calculations.

BASIC POSITIONS OF THE CRACK GROWTH MODEL

Consider that one of the bodies of a rolling pair is damaged by a surface macrocrack. Assume that a characteristic size of the contact area and a crack length are small as compared to the curvature radius of the contact surface. Therefore, instead of a real body, in two-dimensional formulation of the problem, a half-plane with an edge cut is considered (Fig. 1*a*). An external loading, is assumed, to not cause significant plastic deformations around the crack tip, thus the apparatus of linear fracture mechanics can be used for calculation. It is also assumed that rolling is cyclic and unidirected with slipping. The contact action upond the body from the side of the other body (counter-body) of the pair is simulated by repeated translation motion in one direction along the half-plane boundary of Hertzian contact forces:

$$s(x) = (1+if)\ p(x) = -\frac{p_0}{a}(1+if)\sqrt{a^2-(x-x_0)^2}, \qquad |x-x_0| \le a, \tag{1}$$

where p_0 is maximum pressure in the contact center of the contact area, $2a$ is length of this area (Fig. 1*a*), f is the Amonton`s friction coefficient between bodies under contact. A center of a contact loading is at a distance x_0 from the crack mouth.

By simulation of boundary lubrication (friction) condition, assum that a lubricant does not separate the rolling bodies, that is the interlayer thikness is zero and a lubricant itself is uncompressed liquid. The action of a lubricant, that can be entrapped during rolling by an edge crack, is simulated by evenly distributed on its faces normal pressure q (Fig. 1*b*). Such a pressure appears when in the contact cycle a counter-body covers the crack mouth and liquid present in it began to press on the crack faces, causing

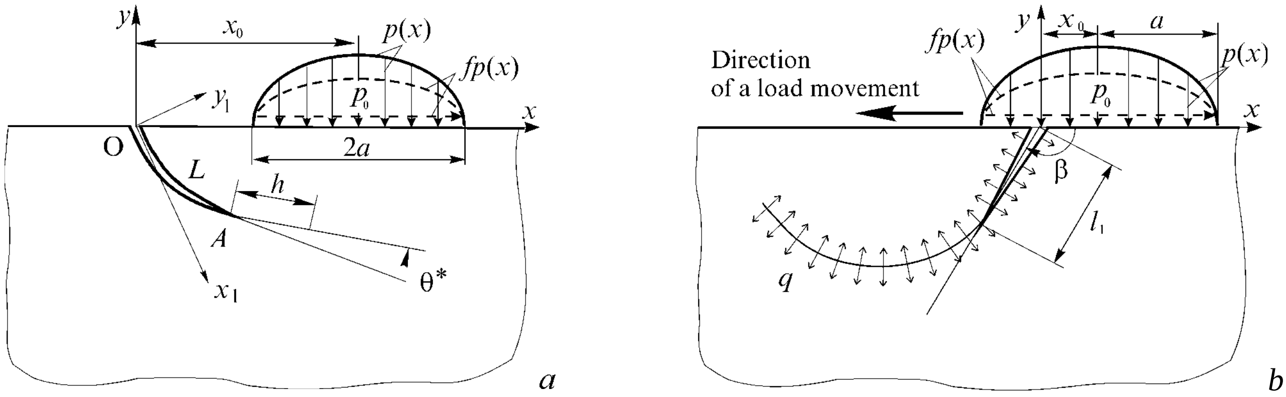


Figure 1. Calculational schemes of the problem

their wedging. The pressure over the crack mouth is $p_1 = p_0\sqrt{1-\lambda^2}$, where $\lambda = x_0/a$. The pressure on the crack edges is considered to be equal or lower than that in the crack mouth. For description of this situation, introduce a parameter $r = q/p_1$ ($0 \le r \le 1$) that show the ratio of the pressures on the crack edges and in its mouth under loading. Hence, the presure on the crack edges equals $q = rp_1 = rp_0\sqrt{1-\lambda^2}$. When a counter-body opens the crack mouth, the pressure on its faces falls down to zero.

In dry friction rolling conditions between contacting bodies there occurs slipping with a great friction (f = 0,1÷0,4) [11, 13]. As a result in rolling bodies, in particular in the follower, tension zones arise and cracks develop in them as mode I cracks. Evidently, such a possibility will appear during rolling in boundary lubrication conditions when the level of lubricant pressure on the crack edges is rather high. Let parameter, $K_{I\theta}$, which at a given crack length and location of the loading, determines the mixed-mode stress intensity factor at the fatigue crack tip (point A in Fig. 1*a*) be related with SIF, K_I and K_{II} by relation of a generalized normal opening criterion (σ_θ-criterion)

$$K_{I\theta}(l, x_0, \theta) = \cos^3\frac{\theta}{2}\left[K_I(l, x_0) - 3K_{II}(l, x_0)\,\mathrm{tg}\frac{\theta}{2}\right]; \qquad (2)$$

here l is a curvilinear (or rectilinear) crack length; θ is a polar angle from the tangent to the crack at its tip A. In every rolling cycle during motion of contact loading along a half-plane boundary (at x_0 variation), parameter $K_{I\theta}$ changes assuming its maximum value, $K_{I\theta,\max} = K_{I\theta}(l,\ x_0^*,\ \theta^*)$ at corresponding $x_0 = x_0^*$ and $\theta = \theta^*$. It is assumed that in every cycle the crack increment occurs only at $x_0 = x_0^*$ (when parameter $K_{I\theta}$ acquired its maximum value), if a condition is satisfied:

$$K_{I\theta,\max} > K_{Ith}, \qquad (3)$$

where K_{Ith} is a fatigue crack growth threshold in normal opening conditions. Thus, based on σ_θ-criterion and the aforesuid assumptions the direction of the crack propagation is determined by the angle

$$\theta^* = 2\,\mathrm{arctg}\frac{K_I(l, x_0^*) - \mathrm{sgn}K_I\sqrt{K_I^2(l, x_0^*) + 8K_{II}^2(l, x_0^*)}}{4K_{II}(l, x_0^*)}. \qquad (4)$$

The crack path is constructed by steps. For this purpose let us introduce two types of steps: the main step, related with the crack growth and additional, related with motion of contact loading. A crack path increment step, h at every stage of its construction is put from the crack tip in direction $\theta = \theta^*$ (Fig. 1*a*). A additional step $\Delta\lambda$ ($\lambda = x_0/a$) are used for founding $K_{I\theta,\max}$ in contact cycle. At every path construction step (during a corresponding number of contact cycles) the values of θ*, λ*, $K_{I\theta,\max}$ are considered to be constant. Each path increment is approximated by a third degree polynomial, whose coefficients are obtained from the conditions of smooth contact of neighbouring path increments [10, 15]. Thus we receive a crack path, which has a kind of smooth curve.

SIF K_{I} and K_{II} at every step of crack growth path construction are found from a solution of the first main problem of the elasticity theory for a half-plane with an edge curvilinear crack (Fig. 1*a*). The boundary conditions of the problem can be written:
on the half-plane boundary as

$$\sigma_y(x) - i\tau_{xy}(x) = s(x), \quad |x - x_o| \le a, \qquad \sigma_y(x) - i\tau_{xy}(x) = 0, \quad |x - x_o| > a; \qquad y = 0 \tag{5}$$

and on the crack faces

$$N^{\pm}(t) + iT^{\pm}(t) = \begin{cases} 0 & \text{for dry friction} \\ -q(\lambda) & \text{for boundary lubrication} \end{cases} \qquad t \in L. \tag{6}$$

Indices showing the oridinal number of the step are omitted here. Thus, for example, at the first step we have an initial crack, then $L = L_1$. In equation (6) L iss a crack path which is known from the construction of crack growth increment at the previous steps; N and T are normal and tangential stresses on the crack edges, respectively: the positive direction of tangential stresses T coincides with the direction, enveloping the crack contour; upper indices "+"or "–" mean the boundary values if approaching the crack contour from the left or from the right. A problem is reduced to a complex singular integral equation which in a normalized form can be written as [10-12]:

$$\int_{-1}^{1} \left[R(\xi,\eta)\varphi(\xi) + S(\xi,\eta)\overline{\varphi(\xi)} \right] \mathrm{d}\xi = \pi P(\eta,\lambda), \qquad |\eta| < 1. \tag{7}$$

Its right-hand part is described by relationship:

$$P(\eta, \lambda) = P_1(\eta, \lambda) - q(\lambda), \tag{8}$$

where $$P_1(\eta,\lambda) = p_o \Big\{ \mathrm{Re}\big[(1 + if)(a(\eta,\lambda) - ib(\eta,\lambda) \big] \omega'(\eta) - e^{2i\beta}\,\overline{\omega'(\eta)} \times$$

$$\times \left[(1 - if)\left(\frac{\overline{b}(\eta,\lambda)}{\overline{a}(\eta,\lambda)} - i \right) i\,\mathrm{Im}(\varepsilon\omega(\eta)e^{-i\beta})\, / l - if\,(\overline{a}(\eta,\lambda) + i\overline{b}(\eta,\lambda)) \right] \Big\},$$

$$a(\eta,\lambda) = \sqrt{1 - b^2(\eta,\lambda)}, \qquad b(\eta,\lambda) = \varepsilon\omega(\eta)e^{-i\beta}\, / l_1 - \lambda;$$

function

$$t = \omega(\eta), \qquad t \in L, \qquad |\eta| < 1 \tag{9}$$

is the parametric equation of the crack contour L; $\varepsilon = l_1/a$ is a relative length of the initial crack $l = l_1$. An integral equation (7) is solved numerically by a method of Gaussian mechanical quadratures. At each consecutive step of the crack growth path construction the described procedure is repeated (every time for a new elongated crack contour).

The residual durability, that is a number of cycles for which a macrocrack length increases from an initial length l_1 to a critical l_c, is established using the known equation:

$$N_g = \int_{l_1}^{l_c} v^{-1}(\Delta K_{\mathrm{I}\theta}, C_1, \ldots, C_m)\, dl \approx \sum_{k=1}^{j_c} \frac{\Delta l_k}{v_k[\Delta K_{\mathrm{I}\theta}(l), C_1, \ldots, C_m]}; \tag{10}$$

here $v = dl/dN$ is the crack growth rate, function that described the fatigue fracture kinetic diagram of the material; and C_1, ..., C_m are characteristics of its cyclic crack growth resistance. Representing N_g as a sum is convenient in the step-by-step construction of a crack path; j_c is sum number of path increment steps necessary for growth of the crack to the critical length; Δl_k and v_k is the crack growth increment and the crack tip propagation rate at the k-th step of calculations.

CRACK GROWTH PATHS UNDER ROLLING IN DRY FRICTION CONDITIONS

Incorporating the above presented model a propagation path of an edge initially rectilinear crack, that appeared in a follewer body under conditions of dry friction rolling, has been constructed. Calculations were carried out for different relative lengths of the initial crack ($\varepsilon = l_1/a$= 0,2; 0,5; 1,0) and different crack orientation ($\beta = \pi/6$; $\pi/2$; $5\pi/6$; $8\pi/9$). Friction coefficient f varied in a rather wide range (f = 0,10÷0,40), that corresponded to different service conditions (dry or rainy weather) of the pair "wheel-rail". Among the chosen orientation of the initial crack an angle $\beta = 5\pi/6$ was preferred. The explanation is as follows. Experimental data show [4,5], that at a given direction of a counter-body motion (and respectively the direction of tangential contact forces) the prevailing shear macrocrack grows rectilinearly at angle $\beta = 5\pi/6$ to the tangential forces direction. This is also proved by the analysis of SIF K_{II} dependences on the angle β of crack inclination to the boundary at a given direction of tangential contact forces. It demonstrates [7, 13], that $\max|K_{II}(\beta, \lambda)|$, at recorded ε and f values, is reached at $\beta \approx 5\pi/6$ and relative distances of loading from the crack mouth $|\lambda| < 1$.

Before path calculations, using the analysed numerical values of SIF K_I the variation ranges of parameter $\lambda = x_0/a$, for which the initial crack was opened i.e., when $K_I(\lambda) > 0$, were established. It was noticed that for a given direction of the tangential contact forces action (see Fig. 1), the corresponding ranges of λ are on the right from a crack mouth ($\lambda \geq 1$). Just in these ranges the starting values of parameter λ^* for each combination of parameters ε, β, f were found.

The analyses of the paths (Fig. 2 and Fig.3) shows that for great f the propagation direction of a crack is practically independent of its initial orientation (Fig. 2*a*) and length (Fig.3*a*). After a short initial period, a crack propagates rectilinearly inside the material at the same acute angle γ to the boundary towards the action of tangential contact forces ($\gamma \approx 79^{O}$ for the friction coefficient f = 0,25). These numerical results agree well with the conclusions in paper [4], resulting from the analyses of the railway rails damaging, that in the dry friction conditions, a main crack under rolling propagates inside the material (rail). Note that in this case the paths calculations were done using a σ_θ-criterion.

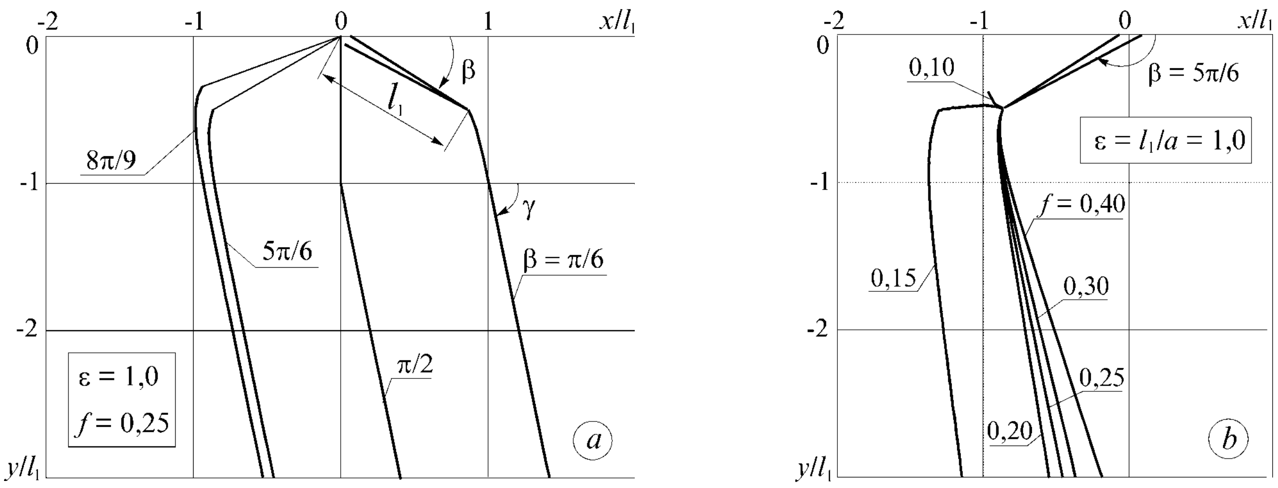
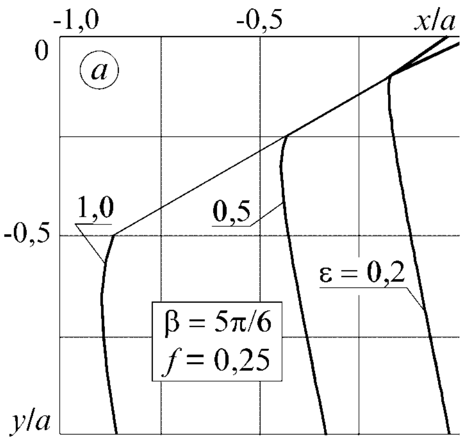


Figure 2. Paths of an edge, primarily rectilinear crack development vs. its orientation angle β (*a*) and friction coefficient f in the contact area between the rolling bodies (*b*)



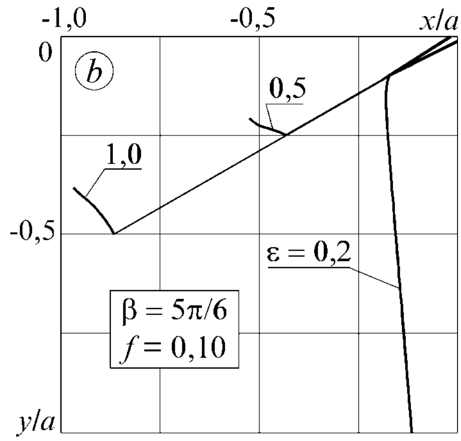


Figure 3. Crack propagation paths vs. its initial relative length $\varepsilon = l_1/a$ for friction coefficient $f = 0{,}25$ (a) and $f = 0{,}10$ (b)

Note another important peculiarity of the paths they depend significantly on the friction coefficient f. At large values of f (0,20 ÷ 0,40) it determines the direction (angle γ) of the crack growth into the material. The larger is the fraction, the smaller is angle γ. At low values of f (0,10 ÷ 0,18) the path geometry sharply changes (Fig. 2*b*). At $f \approx 0{,}10$ the crack propagates towards the boundary (see curve $f \approx 0{,}10$ in the Fig. 2*b*, and also curves $\varepsilon = 0{,}5$ and $\varepsilon = 1{,}0$ in Fig. 3*b*). This behaviour of the crack was noticed in paper [9] for small values of f under condition when rolling surfaces of the experimental discs were wetted with water, and also in paper [4] where the damaging of railway rails was analysed.

Summating the numerical data obtained in this paper and in [13], we can say that they allow us to explain the nature of the typical damages like "squat"("dark-spot" [4]) in the railway rails.

CRACK GROWTH PATHS UNDER ROLLING IN BOUNDARY LUBRICATION CONDITIONS

According to the assumptions of the model about the lubricant action under rolling, the paths of the edge initially rectilinear crack growth in a follower, have been calculated, depending on the crack initial length and orientation and also the lubricant pressure intensity on the crack faces and friction coefficient in the contact zone between the bodies. In calculations, like in dry friction case, the advantageous inclination angle of the crack to the boundary was $\beta \approx 5\pi/6$, bearing in mind that crack orientations in the direction of a counter-body motion are the most favourable to be filled with lubricant. Naturally, if lubricant is present, the contact friction is low. Therefore, the friction coefficient was taken as $f = 0{,}005 \div 0{,}10$.

Fig. 4 and also the results in [14] show that the angle of crack inclination to the boundary has significant influence on the crack growth path geometry. The sure acute is the angle of initial orientation ($\beta \approx \pi/6$, $\beta \approx 5\pi/6$), the more rapidly a crack moves to the body boundary. Cracks, that are initially oriented at angles $\beta \approx \pi/2$, grow deep into a material. The initial crack length slightly influences its path geometry. Still, it has a significant influence on the critical location of a counter-body over the crack mouth, that is on parameter λ^*, at which $\max K_{I\theta}$ in a contact cycle is reached. At the crack length decreases its critical location under a counter-body displacement to the contact loading edge. Hence, it follows that mutual position of rolling bodies, when a crack mouth is under a contact loading center and contact pressure is maximum ($p_1(\lambda) = p_0$), is not the most dangerous. Besides, Fig. 4a and 4b represent an initial crack for each value of relative length ε in such position, that corresponds to its inherent value λ^*. Moreover, the value of λ^* during the crack growth practically did not change for all the cases considered in the paper.

The crack paths in Fig. 5*a* illustrate that the decrease of friction in a contact area between rolling bodies, promotes crack approach to the boundary (a path becomes a more steep), however no effect of it on the path geometry was noticed. As expected, however the decisive influence on the edge crack growth path has lubricant pressure intensity on the crack faces (Fig. 5*b* and 5*c*). For a crack, initially inclined at an acute angle to the boundary, the pressure growth (parameter r increase) significantly accelerates a crack tip approach to the boundary (Fig. 5b). At the values of parameter r, lower than the presented, a crack does not propagate.

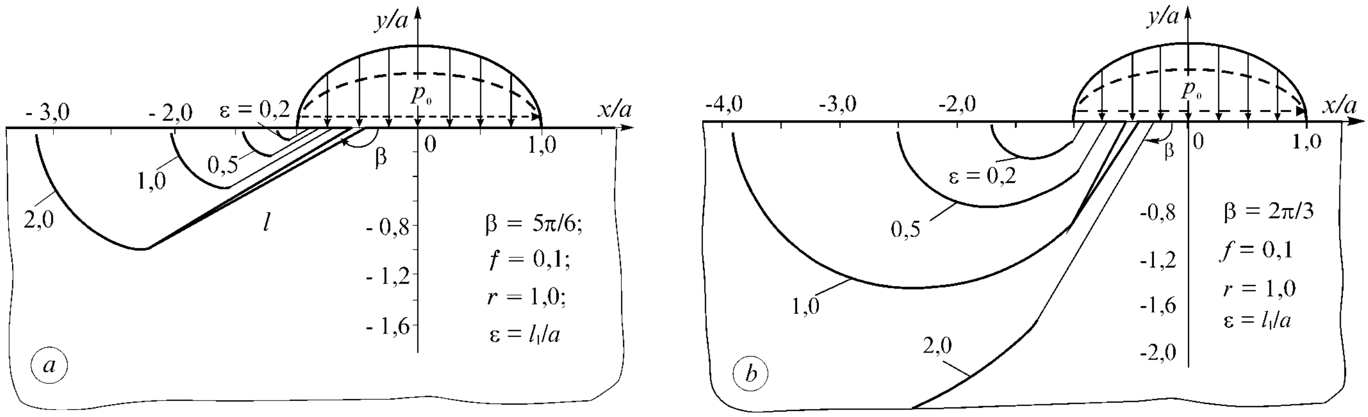


Figure 4. Edge crack growth paths, depending on its initial length $\varepsilon = l_1/a$ for different angles of a crack inclination to the boundary: $\beta = 5\pi/6$ (a) i $\beta = 2\pi/3$(b)

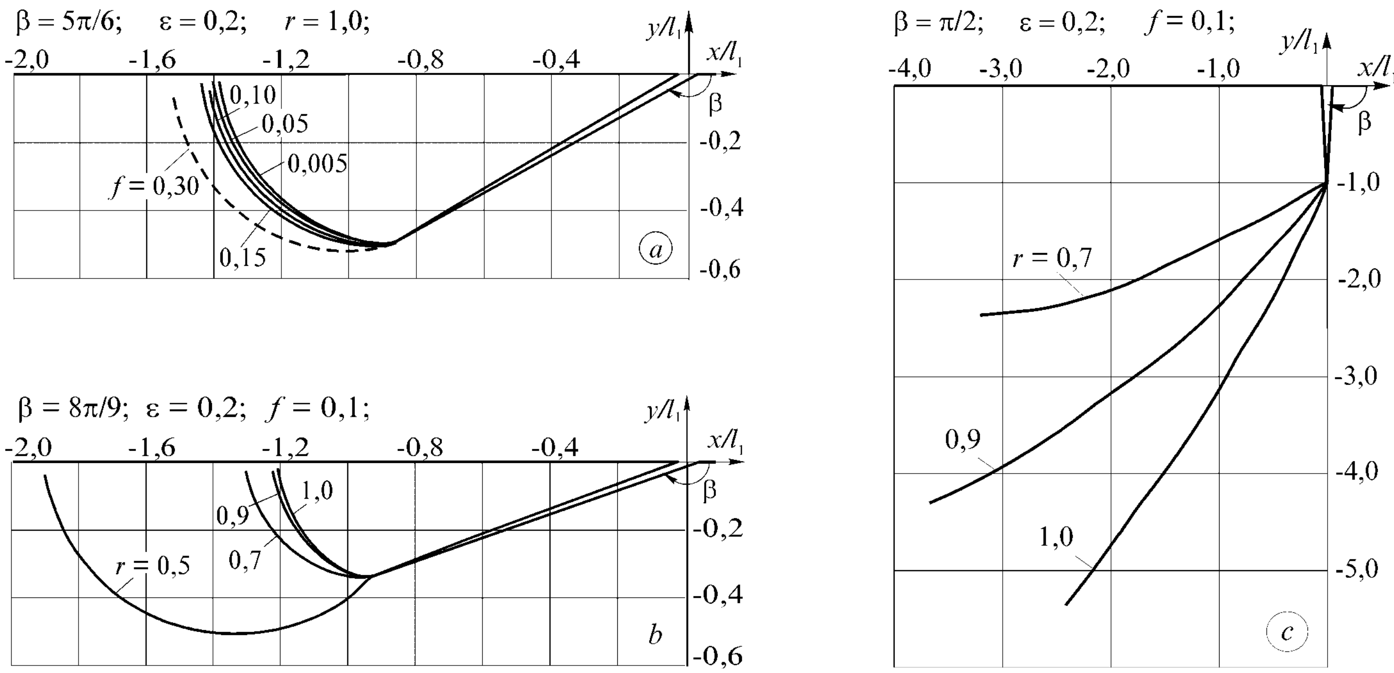


Figure 5. Edge crack growth paths, depending on a friction coefficient f (a) and parameter r of lubricant pressure intensity on the crack faces (b, c)

Results in Fig. 4 and 5 and in paper [14] prove that the edge cracks, inclined at small angles to the crack boundary with their edges subjected to lubricant pressure propagate to the boundary, cause its crumbling, i.e. to pitting formation. These results confirm the assumptions and experimental data, presented in paper [4, 6-8, 16] about the decisive role of the lubricant in pitting formation and also describe the process peculiarities.

RESIDUAL DURABILITY ASSESSMENT

As an example, a durability of the near-surface layer of a rolling body of bearing steel ШХ-15, operating in boundary lubrication conditions, was calculated. Equation (10), in which the fatigue crack growth rate v is described by a Paris low, was utilized

$$v = C(\max K_{I\theta})^n \tag{11}$$

Cyclic crack growth resistance characteristics of ШХ15 are the following: $C = 6{,}805\text{x}10^{-10}$, $n = 2{,}5$. The Paris region boundaries on the fatigue fracture curve are $K_{1\text{-}2} = 2{,}71\text{MPa}\sqrt{m}$, $K_{2\text{-}3} = 10{,}21\text{MPa}\sqrt{m}$. They were used in calculations instead of a threshold K_{Ith} and critical K_{Ifc} values, respectively. A rolling body is assumed initially damaged by an edge rectilinear crack, inclined at $\beta = 5\pi/6$, friction coefficient in contact between the bodies $f = 0{,}01$, intensity of lubricant pressure on the crack faces characterized by parameter r value $r = 0{,}7$, the length of the contact area be $2a = 2$ mm. Conditions of the crack start (3) are assumed as:

$$K_{I\theta\max} = p_0\sqrt{\pi a}\,\max F_{I\theta}^0(\varepsilon, \beta, f, r, \lambda^*) = K_{1-2}, \tag{12}$$

where $F_{I\theta}^0$ is a normalized values of SIF $K_{I\theta}$ for an initial rectilinear crack. A starting pressure values p_0 is obtained from equation (12).

Table 1

l, mm (ε)	0,2	0,5	1,0	5,0	10,0
N_g	$4{,}90\times10^3$	$1{,}01\times10^4$	$2{,}04\times10^4$	$9{,}90\times10^4$	$1{,}92\times10^5$
p_0, MPa	645,84	145,24	53,40	6,29	2,78

The table 1 presents durability (a number of rolling cycles) during which SIF $K_{\text{I}\theta\max}$ at the crack tip increases from the initial value $K_{1\text{-}2}$ to the final $K_{2\text{-}3}$ and the surface contact crumbling occurs. As data shows the durability N_g depends significantly on the initial crack length.

1. Kolesnikov Yu.V. and Morozov E.M.(1989). *Contact Fracture Mechanics*. Nauka, Moskow.
2. Sosnovskiy L.A., Mahutov N.A. and Shurinov V.A. (1992). *Zavod. lab*., **58**, №11, 44.
3. Li Y.C. (1994). *Handbook of fatigue crack propagation in metallic structures,* vol. 2, pp. 1497-1537. Ed. A. Carpinteri. Oxford: Elsevier Science Ltd.
4. Murakami Y., Sakae C., Hamada S. (1999). In: *Engineering against fatigue*, pp. 473-485. Ed. J.H. Beynon, M.W. Brown, T.C. Lindley et al. Rotterdam: A.A. Balkema Publ.
5. Bold P.E., Brown M.W., Allen R.J. (1991). *Wear*, **144,** 307.
6. Keer L.M. and Bryant M.D. AJ. (1983). *Lubric. Technol., Trans. ASME*, **105,** 198.
7. Bower A.F. J. (1988). *Tribol., Trans. ASME,* **110**, 704.
8. Kaneta M. and Murakami Y. (1987). *Trib. Int*., **20(4)**, 210.
9. Tyfour W.R., Beynon J.H. (1994). *Trib. Int.* **27**, №4, 273.
10. Panasyuk V.V., Datsyshyn O.P., Marchenko H.P. (1995). *Eng. Fract. Mech*. **52**, №1, 179.
11. Datsyshyn O.P., Marchenko H.P. (1995). In: *Contact Mechanics II. Computational Techniques,* pp. 377-383. Ed.: M.H.Aliabadi, C.Alessandri, Comp. Mech. Publ., Southampton Boston.
12. Datsyshyn O.P., Panasyuk V.V. (1996). In: *ECF-11*. *Mechanism and mechanics of damage and failure*, **II**, pp. 1411-1416. Ed. J. Petit, EMAS LTD., Warley, UK.
13. Panasyuk V.V., Datsyshyn O.P., Marchenko H.P. (2000). To be published in: *Phys.-Chim. Mech. Mater.*
14. Panasyuk V.V., Datsyshyn O.P., Pryshliak R.Ye., Terleckiy A.B. (2000). To be published in: *Problemy prochnosti*.
15. Savruk M.P. (1981). *Two-dimentional Elasticity Problem for Bodies with Cracks*. Naukowa Dumka, Kyiv.
16. Way S. J. (1935). *Appl. Mech., Trans. ASME,* **2**, A49.