CRACKED BEAM ELEMENT APPLIED TO FATIGUE DAMAGE IDENTIFICATION OF STEEL BRIDGES

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ABSTRACT

Steel structures dominated by dynamic loads are vulnerable to failure by fatigue. Fatigue fractures are generally considered a special type of fracture in steel bridges simply because they can and do occur in normal service, without excessive overloads, and under normal operating conditions. The cost of development of a general-purpose of steel bridge reliability program is usually very high so that amortization of investment in every way possible is essential. When a crack is discovered by inspection, it is important to predict further growth in order to decide the convenient time to remedial action. A Cracked Beam Finite Element (CBE) is developed here aiming to realize a practical and quickly method to identify fractures due cyclic loads in steel beam bridges. This formulation takes into account Linear Fracture Mechanics concepts to describe the crack propagation. It has been shown that the beam stiffness reduction first obtained leads to changes in the natural frequencies of the structure. Thus, a nondestructive inspection method could be implemented by using frequency measurements to determine cracks presence in a steel element. The simulation model is extended to incorporate the elastoplastic fracture mechanic concepts. This simplified approach is significantly practical to use than a conventional finite element procedure, which models details of a cracked geometry. The performance of the simulation model is compared with the solutions achieved by using other well known computer software.

INTRODUCTION

Sometimes repeated loading and unloading may result in failure even if the yield stress is never exceeded. Fatigue fractures are generally considered serious problems of service failure for steel bridges because they are insidious. When the cracks are discovered by inspection, a vital need exists to rapidly and accurately determine the amount of damage sustained by bridge by direct measurement of their actual traffic patterns and the true stresses registered in their main load-carrying elements.

Frequently cracking has been developed in welded bridge structures and the most likely locations that lead to fatigue crack form and growth are the following (see figure 1): at details with the lowest fatigue strength, in zones of highest tension stress range and at defects like section loss due to corrosion or flaws [1].

There are several well known international codes available covering the subjects: security, maintenance, inspection and rehabilitation of bridges structures [2,3].

Cracking can sometimes be located visually in case the crack is large enough to be seen and is not covered by paint and debris. The majority of cracks in steel have been first detected by visual inspection. Other nondestructive testing measures can be used to inspect and to discover cracks such as dye penetrant, X-ray, magnetic particle (magnaflux), and ultrasonics.

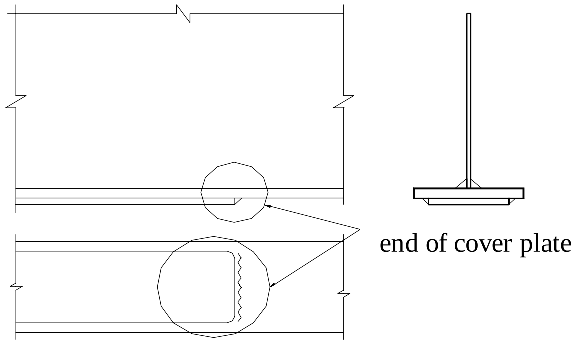


Figure 1: Schematic of cover-plated beam showing location of fatigue crack.

In order to treat the crack identification problem effectively a nondestructive evaluation technique (NDE) based on vibration principles is developed. The measurements of variations in dynamic characteristics of a steel bridge resulting from cracked-induced damages can provide a detailed mark of the nature of the damage, including crack location as well as to evaluate its grown size. It is used the fact that change of stiffness leads to changes in the natural frequencies of the steel bridge.

To this purpose, a finite element model is introduced here to simulate crack openings due fatigue. Initially, a fatigue crack growth prediction based on the linear elastic fracture mechanics (LEFM) is needed. Next in order, the model is extended to incorporate the elastoplastic fracture mechanic concepts. Thus, the change of stiffness is applied to determine the changes in the natural frequencies and identify the damage region.

THEORETICAL CRACKED MODEL

Let us consider an initially undamaged beam of length L, cross-section area A_o and uniform moment of inertia I_o. We begin by supposing that a crack forms in the section of maximum bending (Mode I) and the cracked section may be accounted for, utilizing an equivalent rotational spring of constant k (Figure 2) [4].

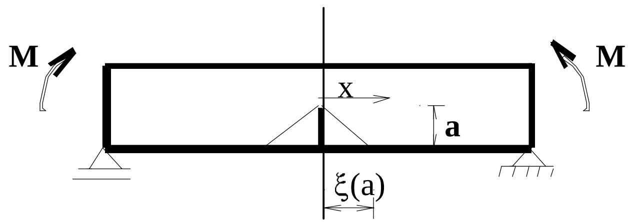
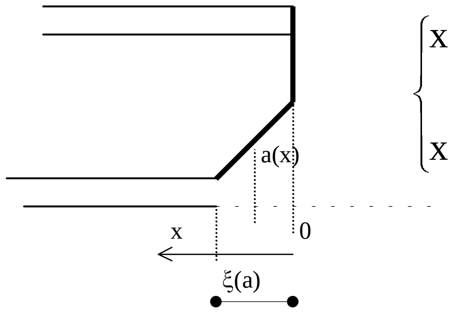


Figure 2: Schematic representation of beam with single crack (mode I).

If it is assumed that the portions of the beam under the stress diffusion lines do not contribute to the inertia of the corresponding sections (Saint-Venant's principle), the value elastic modulus × inertia product, EI, at any beam cross section is given by (Figure 3):

$$EI(x,\ a) = \begin{cases} EI_o[1 - \varphi(x,\ a)] & \text{for} \quad 0 \le x \le \xi(a) \\ EI_o & \text{for} \quad x > \xi(a) \end{cases} \tag{1}$$

where $\phi(x,a)$ is a function depending on the shape of cross section, such that $0 \le \phi(x,a) \le 1$.



$$\begin{cases} x = 0 \Rightarrow a(0) = a \\ x \Rightarrow a(x) \\ x = \xi \Rightarrow a(\xi) = 0 \end{cases}$$

Figure 3: Crack propagation.

To evaluate the complementary energy *U** of the beam between one of the supports and the cracked section, we can utilize :

$$U^* = U_o^* + \frac{M_i^2}{2EI_o}[\Theta_1 + 2l^{-1}(\kappa - 1)\Theta_2 + l^{-2}(\kappa - 1)^2\Theta_3] \tag{2}$$

where U_o* is the complementary energy, $M_e = \kappa.M_i$ (κ = constant), and Θ_i is written as:

$$\begin{cases} \Theta_1 = \int_0^{\xi(a)} \eta(x,\ a)dx \\ \Theta_2 = \int_0^{\xi(a)} x\eta\eta(x\ \ a)dx \\ \Theta_3 = \int_0^{\xi(a)} x^2\eta(x,\ a)dx \end{cases} \tag{3}$$

and

$$\eta(x,a) = \phi(x,a)[1 - \phi(x,a)]^{-1} \tag{4}$$

The complementary energy of the degrading spring with rotational stiffness placed in the equivalence of the cracked section is expressed as:

$$U_s^c = \frac{M^2}{2k} \tag{5}$$

And comparing the second term of (2) and (5) one obtains:

$$k = \frac{EI_o}{\Gamma(x,a)} \tag{6}$$

where $\Gamma(x,\ a) = [\Theta_1 + 2l^{-1}(\kappa - 1)\Theta_2 + l^{-2}(\kappa - 1)^2\Theta_3]$

Evaluation of the Stiffness Matrix

For the cracked beam the numerical model will be established following the finite element method (FEM). The k (spring stiffness) value has been here up to this time evaluated to a part of the beam, of length L/2. To evaluate the total spring stiffness we must do as follows (Figure 4):

$$\begin{cases} \dfrac{M_i}{k_1} = (\phi - \phi_1) \\ \dfrac{M_i}{k_2} = (\phi_2 - \phi) \end{cases} \quad \Rightarrow M_i = k^*(\phi_2 - \phi_1) \tag{7}$$

where k* is the equivalent spring stiffness defined as:

$$k^{*} = \left(\sum_{p=1}^{2} \left(k_p\right)^{-1} \right)^{-1} \tag{8}$$

Now, the global stiffness matrix accommodate the two sections of the beam in order to assemble the approached model:

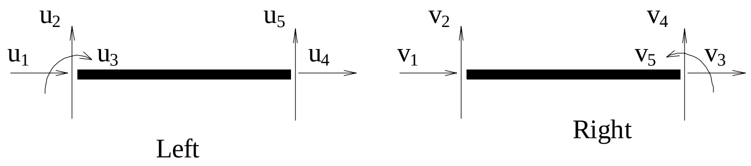


Figure 4: Elements Degrees of Freedom - Left and Right .

Next, we assume that the linear equations representing these elements ($\psi^{-1}.\mathbf{q} = \mathbf{Q}$) are given by:

$$\begin{gathered}
\begin{bmatrix} [K^{(1)}]_{5x5} & \{k^{(1)}\}_{5x1} \\ \{k^{(1)}\}^{T}_{1x5} & k^{(1)} \end{bmatrix} . \begin{Bmatrix} u^{(1)} \\ \phi^{(1)} \end{Bmatrix} = \begin{Bmatrix} P^{(1)} \\ k*(\phi^{(2)} - \phi^{(1)}) \end{Bmatrix} \\
\begin{bmatrix} k^{(2)} & \{k^{(2)}\}^{T}_{1x5} \\ \{k^{(2)}\}_{1x5} & [K^{(2)}]_{5x5} \end{bmatrix} . \begin{Bmatrix} \phi^{(2)} \\ u^{(2)} \end{Bmatrix} = \begin{Bmatrix} k*(\phi^{(1)} - \phi^{(2)}) \\ P^{(2)} \end{Bmatrix}
\end{gathered} \tag{9}$$

The stiffness elements ($[K^{(i)}]$, $\{k^{(i)}\}$, $k^{(i)}$, $i = 1,2$) are obtained from the matrix that corresponds to the normal beam element. For the left element the matrix K is described as:

$$K = \left[\begin{array}{ccccc:c}
\frac{EA}{L} & & & & & 0 \\
0 & \frac{12EI}{L^3} & & \text{Symmetric} & & -\frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{4EI}{L^2} & & & \frac{2EI}{L} \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & & 0 \\
0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\
\hdashline
0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L^2}
\end{array} \right] \tag{10}$$

From Equation (9) is obtained:

$$\begin{cases} \phi^{(1)} = \dfrac{1}{k*.\alpha} \{k^{(1)}\}^{T} \{u^{(1)}\} + \dfrac{1}{\alpha(k^{(2)} + k*)} \{k^{(2)}\}^{T} \{u^{(2)}\} \\ \phi^{(2)} = \dfrac{1}{k*.\beta} \{k^{(2)}\}^{T} \{u^{(2)}\} + \dfrac{1}{\beta(k^{(1)} + k*)} \{k^{(1)}\}^{T} \{u^{(1)}\} \end{cases} \tag{11}$$

Finally, the complete assembly corresponding to the cracked beam element is performed as:

$$\begin{gathered}
\left[[K^{(1)}] + \{k^{(1)}\}.\{k^{(1)}\}^{T}.\frac{1}{\alpha k*} \right] \{u^{(1)}\} + \left[\frac{1}{\alpha(k^{(2)} + k*)}.[k^{(1)}].[k^{(2)}]^{T} \right] \{u^{(2)}\} = \{P^{(1)}\} \\
\left[[K^{(2)}] + \{k^{(2)}\}.\{k^{(2)}\}^{T}.\frac{1}{\beta k*} \right] \{u^{(2)}\} + \left[\frac{1}{\beta(k^{(1)} + k*)}.[k^{(2)}].[k^{(1)}]^{T} \right] \{u^{(1)}\} = \{P^{(2)}\}
\end{gathered} \tag{12}$$

where $\{P^{(i)}\}$, i =1,2 is the applied load due each part (1,2) and

$$\alpha = \left(-\frac{k^{(1)} + k^*}{k^*} + \frac{k^*}{k^{(2)} + k^*} \right) \qquad \beta = \left(-\frac{k^{(2)} + k^*}{k^*} + \frac{k^*}{k^{(1)} + k^*} \right) \tag{13}$$

After jointing the elements the node unknowns for total representation for the cracked beam are (Figure 5):
$W = [w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8]^T$

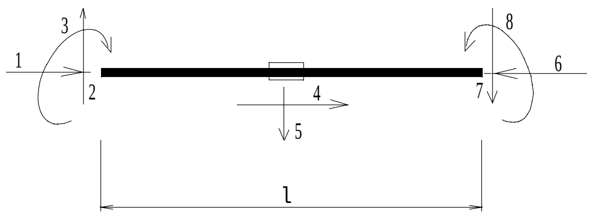


Figure 5: Cracked element degrees of freedom.

Linear-elastic fracture mechanics has become a recognized tool to assess crack growth at welded details. The fatigue crack growth rate depends on the magnitude of the stress range $\Delta\sigma$, the crack length a and the material properties. Both, the stress range and the crack length, are used to compute stress intensity factor range ΔK. The Paris Power Law [5] is generally used to relate fatigue crack extension, da/dN, to ΔK, and is given by:

$$\frac{da}{dN} = C(\Delta K)^m \tag{14}$$

where N is the number of cycles and C, m are constants related with material properties and environments. This relationship will be used in the fatigue crack growth model.

To calculate the stress intensity factor K it will be applied the Irwin's relation [6]:

$$G = \frac{K^2}{E^*} \tag{15}$$

where, E* is equal to Young's modulus E in plane stress conditions and to $E/(1-\nu^2)$, with ν representing Poisson's ratio, for plane strain conditions. G is defined as energy release rate, which is related to the complementary energy U* of the structure by:

$$G = \frac{\partial U^*}{\partial A} \tag{16}$$

A designates the cracked area. Through the two above equations (15), (16) and also (2) one obtains K_I.

For an I-shaped section case, the damage effect into the beam should be verified following geometric changes during the crack propagation. Thus, if the crack is located into the web (Fig. 6), we can write:

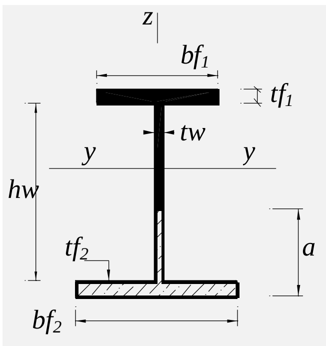


Figure 6: Damage effect into the cross section.

$$\overline{y}_1 = \frac{1}{A_1} \sum A_i y_i \tag{17}$$

where: $$\overline{y}_1 = \frac{[bf_1.tf_1.(\overline{s} + tf_1/2) + \overline{s}^2.tw/2]}{(bf_1.tf_1 + \overline{s}.tw)}$$ and,

$$I_{y_1} = \frac{1}{12} bf_1.tf_1^3 + bf_1.tf_1.\left(\frac{tf_1}{2} + \overline{s} - \overline{y}_1\right)^2 + \frac{tw}{12}\overline{s}^3 + tw.\overline{s}.\left(\frac{\overline{s}}{2} - \overline{y}_1\right)^2 \tag{18}$$

Hence, the damaged section stiffness is given by:

$$k_1 = \frac{EI_o}{\Gamma_1(a(x))} \tag{19}$$

Elastic Plastic Stiffness

The derivation of the elastic-plastic stiffness of the cracked element follows the classical plasticity theory, assuming a simple associated flow and isotropic hardening rule. The deformation increment is then decomposed into elastic and fully plastic parts as [7]:

$$d\mathbf{q} = d\mathbf{q_e} + d\mathbf{q_p} \tag{20}$$

The elastic strain rate will be given by:

$$d\mathbf{q_e} = \Psi.\mathbf{Q} \tag{21}$$

where $\mathbf{Q}$ is the generalized stress tensor, Ψ represents the elastic compliances and:

$$d\mathbf{q_p} = d\Lambda\{\partial f/\partial Q\} \tag{22}$$

is the plastic strain rate defined according to an adopted yield surface f($\mathbf{Q}$,α_y), being $d\Lambda$ the plastic multiplier.

Following the standard plasticity theory, one can find the incremental expression of the plastic multiplier (satisfying also the Kuhn-Tucker and consistency conditions). The hardening rule is obtained by using the fact that the plastic work rate is equal to the continuum work rate over the plastic area A_{pl} of the cracked element:

$$dW_p = \int_{Apl} \sigma_{eq}.d\varepsilon_{p,eq}.dA \tag{23}$$

where σ_{eq} and $\varepsilon_{p,eq}$ are local values of tensile equivalent stress and plastic strain-rate in section A_{pl}. An approximation of equation (27) is given by Parks [8]:

$$dW_p = \chi.\sigma_y.d\varepsilon_{p.}.\eta^2 \tag{24}$$

The dimensionless factor χ is expected to be of order unity and η is the remaining plastic ligament (η=d–a). In consequence of the foregoing equations and also imposing the consistency condition the plastic multiplier is evaluated as:

$$d\Lambda = \frac{\left(\frac{\partial f^T}{\partial Q}.\psi\right)^T.dq}{\left(\frac{\partial f^T}{\partial Q}.\psi.\frac{\partial f}{\partial Q} - \frac{\partial f}{\partial \alpha_y}.\frac{E_p}{\sigma_y \chi \eta}.Q.\frac{\partial f^T}{\partial Q}\right)} \tag{25}$$

From Equation (25) together with Equation (20) the final expression of the actual effort increment can be found:

$$d\mathbf{Q} = \psi_{ep} d\mathbf{q} \tag{26}$$

It is then considered an approximation assuming an additive decomposition of the elastic and plastic stiffness, i.e. $k_e + k_p$ is the total stiffness of the considered section.

Crack Identification Problem

Let us consider the fact that damage in a structure produces changes of stiffness. The stiffness distribution in the damaged condition must differ from that in undamaged condition and it leads to changes in the natural frequency of the structure[9]. Thus it can be seen that, the measurement of the natural frequencies of a structure at two different stages of its life offers the possibility of locating damage in the structure and of determining the severity of the damage.

The crack position and evaluation can be obtained through the closed-form solution of the equation of motion. The displacements (u_1,u_2) on the two parts of the beam, left and right of the crack are described as:

$$\begin{cases} u_1(\beta) = A_1 \sin\lambda\beta + A_2 \cos\lambda\beta + A_3 \sinh\lambda\beta + A_4 \cosh\lambda\beta \\ u_2(\beta) = B_1 \sin\lambda\beta + B_2 \cos\lambda\beta + B_3 \sinh\lambda\beta + B_4 \cosh\lambda\beta \end{cases} \tag{27}$$

where A_i , B_i , i=1,...4, are integral constants, and

$$\lambda = \left(\frac{\rho A \omega^2}{EI} \right)^{1/4} \tag{28}$$

The quantity ω is the angular natural frequency and ρ is the material density. The natural frequency of the cracked beam will be modified resulting to: $\omega_f = \omega_n - \Delta\omega$, where ω_n is the natural frequency of the uncracked beam, and $\kappa = [\omega_f^2 (A\rho / EI)]^{1/4}$.

Taking into account the appropriate boundary conditions and the displacements equations resulted from the equation of motion, one can find the characteristic equation of the model, used to obtain the natural frequencies changes:

$$\Delta\omega_n = 2\omega_n . \Omega_n(\beta) . K^{-1} \tag{29}$$

where $\Omega_n(\beta)$ is a function that depends only on the crack position. Now, an inverse technique can be applied to recover the equivalent dimensionless stiffness K from the fracture mechanic theory. If Equation (29) is written for two vibration modes 1 and 2, and the ratio of frequency changes is arranged, it is seen that this ratio is a function of crack location only. Then:

$$\left(\frac{\Delta\omega_1}{\omega_1} \right) . \left(\frac{\Delta\omega_2}{\omega_2} \right)^{-1} = \frac{\Omega_1(\beta)}{\Omega_2(\beta)} \tag{30}$$

Finally, equation (30) is used to find the crack position. To monitor the condition of the beam natural frequencies should be measured periodically. One should say that each bridge has its *signature* that is a set of frequency-response functions. When the signatures are compared during an inspection program it may be possible to evaluate the bridge performance and detect the critical local failure.

Numerical Example

It is analyzed the case of a bridge cracked section to compute the changes of eigenfrequencies (see Figures 7 and 8) [10]. The geometric properties of the simulated simply supported beam are indicated in Table 1.

The technique set forth in this study for cracked steel beam evaluation is accomplished applying two numerical algorithms to be implemented in microcomputers. The first one, **RAST** (*Rissausbreitung in Stahlträger*), was developed to model the cracked beam element and determine its stiffness K (see Figure 9). Then, the code **EIGFRISS** (*Eigenfrequenz für Rissmodelle*) has been implemented to identify the crack location and size (Figure 10).

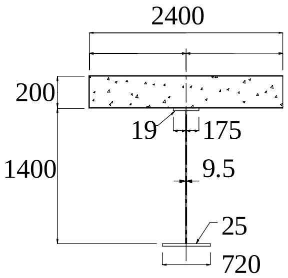


Figure 7: Composite beam section.

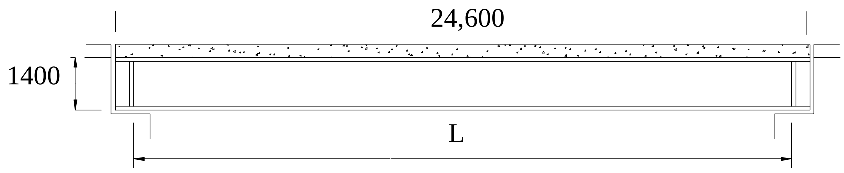


Figure 8: Beam elevation

TABLE 1

GEOMETRIC PROPERTIES

I	**A** (mm^2)	**L** (mm)	**I** (mm^4)	**E** (kN/mm^2)	F_y (kN/mm^2)	**Z** (mm^3)	ρ (N/mm^3)
1400 x 295	38,332	24,000	0.121 x 10^{11}	205	0.350	13589379.5	7.8 x10^{-6}

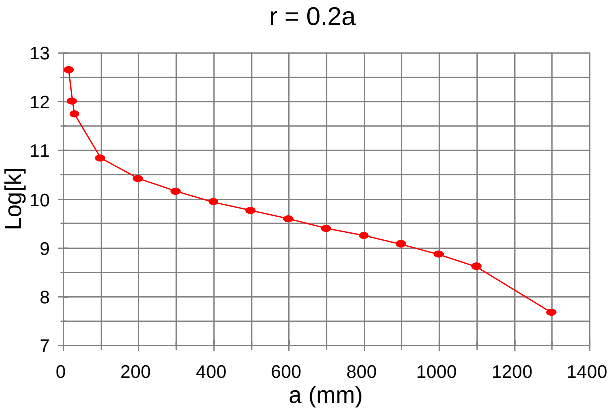


Figure 9: Crack length(a) x flexural stiffness obtained with RAST- $\xi(a) = r = 0.2a$.

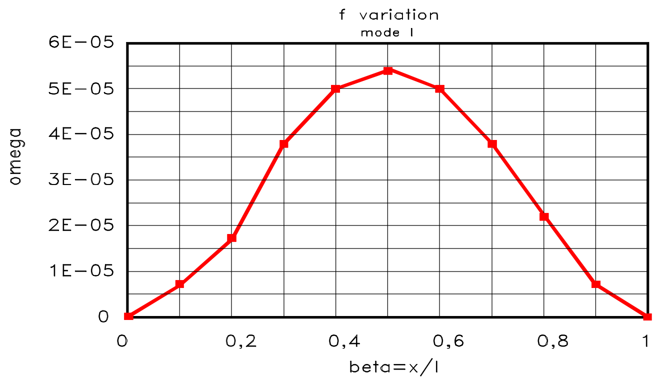


Figure 10: Function $\Omega(\beta)$ versus position β=x/L for simply supported beam.

CONCLUSION

A *cracked beam element (CBE)* is adopted aiming to represent the local flexibility introduced by the crack in *steel bridge beam*. It has been also derived a simple procedure to find cracks during bridges inspections; Hence, the natural frequency variation was used to estimate crack position and size. Two algorithms were written (RAST and EIGFRISS) and its validity was established by a comparison with other well known computer software (Ansys). It was observed that the methods proposed have been showed good accuracy and the results obtained were approximated with the practice.

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