AVERAGING OF THE DIFFRACTION WAVES IN COMPOSITES WITH RANDOM STRUCTURES BY GENERALIZED SELF-CONSISTENT METHOD

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Abstract

Developed is a self-consistent statistical mechanics approach for determining the effective dynamic elastic and diffraction properties of the composites with random structures. The problem is reduced to the model of a single inclusion with a inhomogeneous elastic neighbourhood in a medium with the effective dynamic elastic properties. The inhomogeneous elastic properties and size of the neighbourhood are defined by probabilistic laws for the inclusion distribution in a representative domain. Numerical results are given for the effective dynamic elastic properties and for the effective section of the scattering of the composite with unidirectional fibres.

1. Introduction

The superior mechanical dynamic properties of composites are achieved by combining different constituents such that their interaction would yield the desired response. Because of the complex character of the composite structure, the statistical mechanics approach could be applied to determine the effective properties. Unlike the known methods of singular approximation of random-function theory or the traditional self-consistent methods [3-10], the present method [1,2] could reflect the salient features of the composite by application of the probabilistic laws for the inclusion distribution in a representative domain $\mathbf{V}$.

2. Micro and macrolevels of the composite

We shall consider the propagation and effects of the scattering and the damping of the harmonic waves in matrix composites. For example, when all fibres have the identical geometrical form, orientation and size. The conditions of ideal contact on interphase surfaces are realized. Let the unidirectional fibres statistically homogeneous and isotropically are distributed on plane $r_1 O r_2$ in representative domain $\mathbf{V}$ with boundary Γ of the unidirectional fibre composite. Let structure of the unidirectional fibre composite on the plane $r_1 O r_2$ (Fig.1) is simulated by random placement of the centres of the disk cross-sections of the fibres in knot of the ideal periodic hexagonal lattice. The minimum guaranteed interlayer of matrix between fibres is 4 % of the external fibre radius r_F. A distribution of the unidirectional fibres in domain $\mathbf{V}$ is given by the fields of indicator functions

$$\omega_F(\mathbf{r})=\begin{cases}1, & \mathbf{r}\in\mathbf{V}_F\\ 0, & \mathbf{r}\notin\mathbf{V}_F\end{cases}, \tag{1}$$

where $\mathbf{V}_F$ is a domain of the fibres in the $\mathbf{V}$. Let the mass densities of the fibres ρ_F and

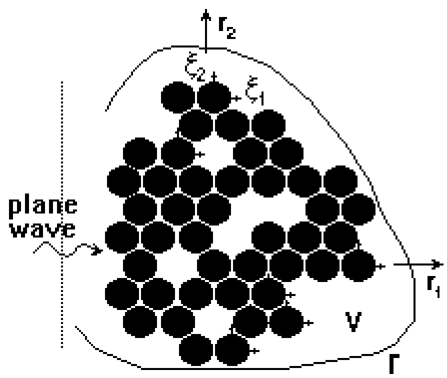

Fig.1. The representative domain

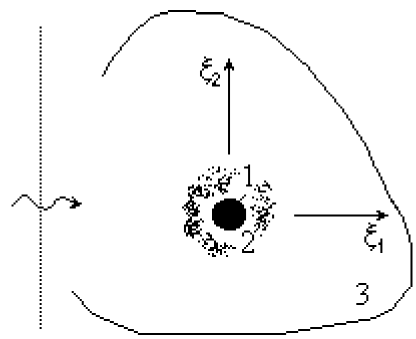

Fig.2. The calculated scheme; 1 is fibre, 2 is inhomogeneous neighborhood, 3 is homogeneous effective medium

of the matrix ρ_M are equal

$$\rho_F=\rho_M=\rho\,.$$

In domain $\mathbf{V}$ on microlevel of the composite we have the wave equations

$$\sigma_{ij,j}(\mathbf{r},t)=\rho\frac{\partial^2 u_i(\mathbf{r},t)}{\partial t^2} \tag{2}$$

or we have the equations

$$\left(C_{ijmn}(\mathbf{r})u_{(a)m,n}(\mathbf{r})\right)_{,j}+\rho\omega^2 u_{(a)i}(\mathbf{r})=0 \tag{3}$$

for complex amplitudes $\mathbf{u}_{(a)}(\mathbf{r})$ of the displacements $\mathbf{u}(\mathbf{r},t)$, which are associated with the decompositions

$$u_i(\mathbf{r},t)=u_{(a)i}(\mathbf{r})e^{-i\omega t}, \qquad \sigma_{ij}(\mathbf{r},t)=\sigma_{(a)ij}(\mathbf{r})e^{-i\omega t}.$$

The stress and strain fields are

$$\sigma_{ij}(\mathbf{r},t)=C_{ijmn}(\mathbf{r})\varepsilon_{mn}(\mathbf{r},t), \quad \varepsilon_{ij}(\mathbf{r},t)=u_{(i,j)}(\mathbf{r},t), \tag{4}$$

where the symmetric part of the tensor $\nabla\mathbf{u}$ on (ij) is $u_{(i,j)}$, ω is a circle frequency and t is a time. The field of the elastic properties $\mathbf{C}(\mathbf{r})$ in the Eqs.(3), (4) is

$$C_{ijmn}(\mathbf{r})=\omega_F(\mathbf{r})C^F_{ijmn}+\left(1-\omega_F(\mathbf{r})\right)C^M_{ijmn}, \tag{5}$$

where $\mathbf{C}^{\mathrm{F}}$ and $\mathbf{C}^{\mathrm{M}}$ are tensors of the elastic properties of the fibres and matrix, the indicator functions $\omega_F(\mathbf{r})$ in the Eq.(1).

We shall consider a calculation of the tensor $\mathbf{C}^*$ of the effective dynamic elastic properties of the composite. Tensor $\mathbf{C}^*$ is included in the generalized law of Hook on macrolevel of the composite

$$\sigma^*_{ij}(\mathbf{r},t)=C^*_{ijmn}\varepsilon^*_{mn}(\mathbf{r},t), \tag{6}$$

where macrostress $\sigma^*_{ij}(\mathbf{r},t)=<\sigma_{ij}(\mathbf{r},t)>$ and macrostrain $\varepsilon^*_{ij}(\mathbf{r},t)=<\varepsilon_{ij}(\mathbf{r},t)>$, $<...>=\frac{1}{|\mathbf{V}|}\int_{\mathbf{V}}...d\mathbf{r}$ is a operator of volumetric averaging on domain $\mathbf{V}$. From the Eqs. (5), (6) we shall receive equation

$$C^*_{ijmn}=C^M_{ijmn}+v_F(C^F_{ijpq}-C^M_{ijpq})N^F_{(a)pqmn} \tag{7}$$

for finding of the tensor $\mathbf{C}^*$, where v_F is a relative volumetric content of the fibres in the composite, $\mathbf{N}^{\mathrm{F}}_{(\mathrm{a})}$ are tensors of the concentrations of the average complex amplitudes of the strains on the fibres. The tensors $\mathbf{N}^{\mathrm{F}}_{(\mathrm{a})}$ are found on the basis of the decompositions

$$<\varepsilon_{(a)ij}(\mathbf{r})>_F=N^F_{(a)ijmn}\varepsilon^*_{(a)mn}, \tag{8}$$

where $<...>_F$ is a operator of volumetric averaging on domain $\mathbf{V}_{\mathrm{F}}$ of the fibres in the composite.

Thus, the problem of the calculation of the tensor $\mathbf{C}^*$ of the dynamic effective elastic properties of the composite is reduced to the calculation of the tensors $\mathbf{N}^{\mathrm{F}}_{(\mathrm{a})}$ of the concentrations of the average complex amplitudes of the strains on the fibres (Eqs. (7), (8)). It problem can be solved on the basis of the solution of the average boundary – value problem of the generalized self-consistent method [1, 2].

3. Generalized self-consistent method

In the local coordinate system ξ_1,ξ_2 (Fig.1) of a kth fibre we shall enter into reviewing the fields $\mathbf{u}^{(k)}_{(a)}(\xi)$, $\varepsilon^{(k)}_{(a)}(\xi)$ and $\sigma^{(k)}_{(a)}(\xi)$, which connected with corresponding fields in the global coordinate system r_1, r_2 on the basis of the equations

$$u^{(k)}_{(a)i}(\xi)=u_{(a)i}(\mathbf{r}_{(k)}+\xi),\qquad \varepsilon^{(k)}_{(a)ij}(\xi)=\varepsilon_{(a)ij}(\mathbf{r}_{(k)}+\xi),\qquad \sigma^{(k)}_{(a)ij}(\xi)=\sigma_{(a)ij}(\mathbf{r}_{(k)}+\xi),$$

the radius vector $\mathbf{r}_{(k)}$ refers to the centre of the disk cross-section of the kth fibre on plane $r_1 O r_2$. The Eq. (2) is changing to

$$\sigma^{(k)}_{(a)ij,j}(\xi)+\rho\omega^2 u^{(k)}_{(a)i}(\xi)=0\,. \tag{9}$$

We shall receive the average wave equations

$$\bar{\sigma}_{(a)ij,j}(\xi)+\rho\omega^2\bar{u}_{(a)i}(\xi)=0\,, \qquad (10)$$

after averaging the Eq.(9) by the operator

$$\frac{1}{N}\sum_{k=1}^{N}\ldots\ .$$

The average fields in Eq.(10) of the complex amplitudes of displacements and stresses are

$$\bar{u}_{(a)i}(\xi)=\frac{1}{N}\sum_{k=1}^{N}u^{(k)}_{(a)i}(\xi)\,, \qquad \bar{\sigma}_{(a)ij}(\xi)=\frac{1}{N}\sum_{k=1}^{N}\sigma^{(k)}_{(a)ij}(\xi)\,. \qquad (11)$$

We shall enter into reviewing the average field of the strain amplitudes

$$\bar{\varepsilon}_{(a)ij}(\xi)=\frac{1}{N}\sum_{k=1}^{N}\varepsilon^{(k)}_{(a)ij}(\xi)$$

by analogy with Eq.(11). It is possible to calculate value of the tensor $\bar{\mathbf{N}}^F_{(a)}$ from Eq.(8) and

$$<\varepsilon_{(a)ij}(\mathbf{r})>_F=\frac{1}{\pi r_F^2}\int_0^{2\pi}\int_0^{r_F}\varepsilon^F_{(a)ij}\xi_o d\xi_o d\theta\,,$$

where i,j=1,2, for example, when the plane of polarisation of waves coincides with the transverse plane $r_1 Or_2$ of the unidirectional fibre composite; r_F is radiuse of the fibre, the $\xi_o\equiv|\xi|$ and θ are polar local coordinates. The tensor $\mathbf{C}^*$ of the effective dynamic elastic properties of the composite can be calculated on the basis Eq.(1.9) and the found values of the tensor $\bar{\mathbf{N}}^F_{(a)}$.

Thus, fields $\bar{\mathbf{u}}_{(a)}(\xi)$, $\bar{\varepsilon}_{(a)}(\xi)$ and $\bar{\sigma}_{(a)}(\xi)$, where

$$\bar{\varepsilon}_{(a)ij}(\xi)=\bar{u}_{(a)(i,j)}(\xi)\,, \qquad \bar{\sigma}_{(a)ij}(\xi)=a_{(a)ijmn}(\xi)\bar{\varepsilon}_{(a)mn}(\xi)$$

can be defined from solution of the average boundary – value problem

$$\left(a_{(a)ijmn}(\xi)\bar{u}_{(a)m,n}(\xi)\right)_{,j}+\rho\omega^2\bar{u}_{(a)i}(\xi)=0\,, \qquad (12)$$

where the field of the elastic properties is

$$a_{(a)ijmn}(\xi)=\left[\bar{\beta}(\xi)C^M_{ijqp}+\left(\bar{\alpha}_F(\xi)C^F_{ijdb}-\bar{\beta}(\xi)C^M_{ijdb}\right)v_F N^F_{(a)dbqp}\right]k^{-1}_{qpmn}(\xi)\,, \qquad (13)$$

where $\mathbf{k}^{-1}(\xi)$ is a tensor field, which return to

$$k_{ijmn}(\xi)=\bar{\beta}(\xi)E_{ijmn}+\left(\bar{\alpha}_F(\xi)-\bar{\beta}(\xi)\right)v_F N^F_{(a)ijmn}\,. \qquad (14)$$

unit tensor is $\mathbf{E}$. The structural functions in the Eqs.(13), (14) are

$$\bar{\alpha}_F(\xi)=\frac{\bar{\varpi}_F(\xi)}{v_F}\,, \qquad \bar{\beta}(\xi)=\frac{1-\bar{\varpi}_F(\xi)}{1-v_F} \qquad (15)$$

which taking into account the distribution of the fibres in the composite by the average indicator functions

$$\varpi_F(\xi)\equiv\frac{1}{N}\sum_{k=1}^{N}\omega_F(\mathbf{r}_{(k)}+\xi),$$

where $\mathbf{r}_{(k)}$ is radius-vector for centre of the disk cross-section of the kth fibre in the $\mathbf{V}$.

The calculated scheme of the average boundary – value problem is a diffraction of the incident longitudinal and transversal waves on the single fibre with inhomogeneous elastic neighborhood in a homogeneous medium with the effective dynamic elastic properties (Fig.2).

4. Numerical results

The numerical values of the components C^*_{1111} and C^*_{1212} of the tensor $\mathbf{C}^*$ of the effective dynamic elastic properties for the unidirectional fibre composite in a plane $r_1\,Or_2$ of isotropy are represented in the Table when the circle frequency $\omega=600c^{-1}$ and the relative volumetric content of the fibres in the composite $v_o=0{,}5$. The Young modulus and Poisson's ratio for isotropic matrix are E_M =1 GPa and ν_M =0,15 and for fibres are E_F =100 $\mathrm{E_M}$, ν_F =0,1 when $E_M/\rho=10^6$. If the circle frequency ω is less than 20 c^{-1} when the numerical values of the component C^*_{1111} or C^*_{1212} which defined by this dynamic and static [1,2] approaches differ less than on 3%; the numerical values of the static approach are placed in the Table.

Table

Numerical results

Effective constants of the fibre composite	Static [1]	Dynamic
C^*_{1111}/C^M_{1111}	3,09	1,39
C^*_{1212}/C^M_{1212}	3,01	1,45
γ^*_1/r_F^2	0	5,99
γ^*_2/r_F^2	0	8,84

The numerical results for the effective section γ^* of the scattering are given in the Table for propagation along an axes r_1 the longitudinal (γ^*_1) or transversal (γ^*_2) waves in the isotropic plane $r_1\,Or_2$ of the unidirectional fibre composite. The value γ^* was calculated by equation $\gamma^*=E^*/I^*$ where I^* is intensitie of incident wave, scattered energy for a time unit is

$$E^*=\frac{i\omega R}{4}\int_0^{2\pi}\left(\bar{\sigma}^{\bullet}_{(a)\xi\xi}\bar{u}_{(a)\xi}+\bar{\sigma}^{\bullet}_{(a)\xi\theta}\bar{u}_{(a)\theta}-\bar{\sigma}_{(a)\xi\xi}\bar{u}^{\bullet}_{(a)\xi}-\bar{\sigma}_{(a)\xi\theta}\bar{u}^{\bullet}_{(a)\theta}\right)\Big|_{\xi_o=R}d\theta\,,$$

where volume $R >> r_F$, the symbol $\bullet$ designates a completely conjugate value.

Conclusions

Developed is a self-consistent statistical mechanics approach for determining the effective dynamic properties of the composites with random structures. The problem is reduced to the average boundary – value problem of the generalized self-consistent method. The calculated scheme of this average problem is the diffraction of the incident longitudinal and transversal waves on the single fibre with inhomogeneous elastic neighbourhood in a homogeneous medium with the effective dynamic elastic properties (Fig.2). The numerical results for the tensor $\mathbf{C}^*$ of the effective dynamic elastic properties and for the effective section γ^* of the scattering of the unidirectional fibre composite in isotropy plane $r_1 Or_2$ are represented.

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