

# APPLICATION OF COHESIVE MODEL TO DYNAMIC DUCTILE FRACTURE TESTS

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## ABSTRACT

The work deals with the development of a one-dimensional model based on a cohesive zone, able to study ductile fracture propagation in pipe steels. The *essential work of fracture* ( $w_f$ ) is used to characterise the model from an energetic point of view and the CTOA is assumed as fracture parameter during the fully ductile rupture. The model is implemented in PICPRO finite element program, which is based on an explicit integration scheme. Dynamic analysis of a SEN-B specimen, broken in a TPB impact test, is performed to show the consistency of the model. Simulation is carried out by applying the condition of steady fracture propagation with constant CTOA equal to the critical CTOAc determined from laboratory test. Results are shown in terms of the evolution of impact load, cinematic quantities and the geometric estimate of CTOA. It is demonstrated that the geometric definition of such parameter can be inadequate when the net ligament or the initial crack length are too short with respect to the width of the cohesive zone. This shortcoming does not appear if a wide plate simulation is performed. High computational efficiency is preserved together with mesh insensitivity; therefore, the model is of practical usefulness in the explicit FE analysis of huge structures, like line pipes employing coarse elements.

## INTRODUCTION

In the study of longitudinal fracture propagation in gas transmission pipelines, the pipe steel is often characterised by small-scale impact test. The use of Drop Weight Tear Test is needed to evaluate fracture ductility, by shear area measurement (ASTM E436), or fracture toughness as total specific energy absorption. Other investigation on DWTT were performed by Priest et al [1], following the work of Cotterell et al [2], showing the existence of a linear type relation for total specific energy absorption with respect to ligament. According to Cotterell et al [2] and Turner et al [3], the specific energy consumption for propagation is the sum of a constant term, named *essential work of fracture*, and one that is linear with ligament. The first term accounts the energy dissipation localised at the crack tip, to form fracture surfaces, while the second is spent for the development of remote plastic field. Based on the slope of total specific energy dissipation with ligament length, it is also possible to estimate the CTOAc for a steel plate as shown in Buzzichelli et al [4].

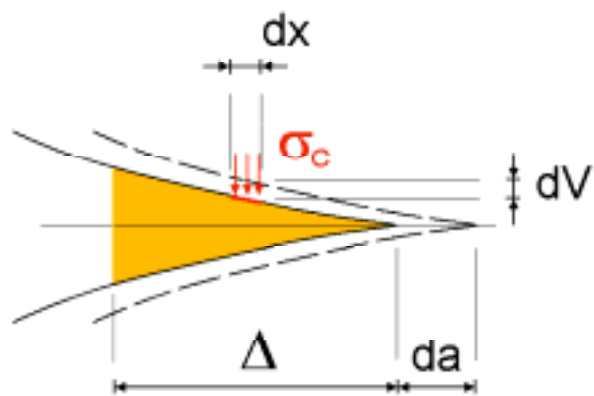
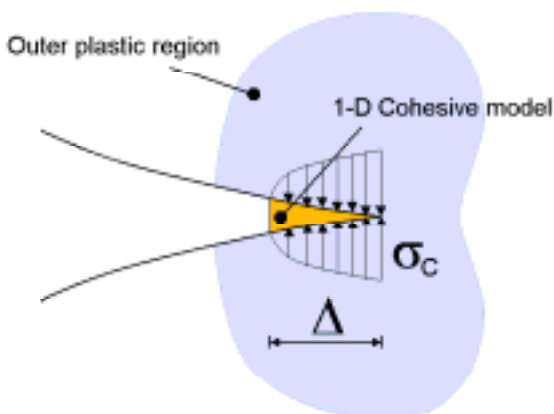
A cohesive model can account the *essential work of fracture* as dissipation produced by proper cohesive stresses applied inside a cohesive zone placed at the crack tip. This introduces a perturbation of the crack tip plastic field that can invalidate the calculation of conventional fracture parameters. However, CTOA can still be used to govern the stable fracture propagation (CTOA=CTOAc).

In the present work, the analysis of a DWT Test is performed by using a cohesive zone model and assuming the fracture propagation to occur with CTOA=CTOAc. Simulations are carried out with PICPRO (PI-pe C-rack PRO-pagation code) explicit finite element program dedicated to the analysis of longitudinal ductile fracture in pressurised and buried pipelines. Numerical results for the DWT Test are consistent with the assumptions introduced in the model and well reproduce the experimental behaviour of the specimen. The model is also able to tolerate coarse mesh, thus implying a great reduction in computational time. Therefore, it can be considered possible the application of the cohesive model to the study of fracture in huge structures like gas pipelines.

## COHESIVE MODEL

The ductile fracture propagation in steel plates can be studied by means of a cohesive model able to introduce a well-defined energy loss in correspondence of the crack. This energy dissipation can be related to the energy required to form the fracture surfaces and it has a prominent importance in ductile rupture. Here it is discussed a one-dimensional cohesive layer, placed at the crack tip, where internal continuity of the material is gradually removed to allow crack extension. Dissipative stresses are introduced inside the cohesive layer so as to give a certain *fracture resistance* acting in opposition to the crack flank opening. According with Cotterell et al [2], the *essential work of fracture*  $w_f$  can be defined as the component of specific energy dissipation whose absorption is localised at the crack tip. A cohesive zone can be introduced next to the crack tip, in the form of a simple layer with *cohesive distance*  $\Delta$ , surrounded by a wide spread plastic field as shown in Figure 1. Following the scheme of Figure 2, for a fracture extension  $da$ , the calculation of the energy dissipation can be done as follows.

$$dE_D = \int_0^{\Delta} B(\sigma_c dV) dx \quad (1)$$



**Figure 1:** One-dimensional cohesive zone model.

**Figure 2:** Cohesive model for  $da$  fracture extension.

Where  $B$  is the plate thickness,  $\sigma_c$  the cohesive stress,  $V$  is half the crack flank opening. The specific work of fracture, according with Cotterell et al [2], can then be deduced.

$$w_f = 2 \frac{dE_D}{Bda} = 2 \int_0^{\Delta} \left( \sigma_c \frac{dV}{da} \right) dx \quad (2)$$

Here the  $V_F$  is half the Crack Opening Displacement (COD) at the trailing edge of the cohesive layer. In what follows it is assumed that fracture propagation develops at constant crack speed and shows a self-similar crack profile. Under these two hypotheses the expression of essential work of fracture can be modified as in Eqn. 3.

$$w_f = 2 \int_0^{V_F} \sigma_c dV \quad (3)$$

Therefore,  $w_f$  can be computed both from the instantaneous energy absorption in the cohesive layer, Eqn. 2, and from the energy dissipated in correspondence of a single material point during the crossing of the whole cohesive layer (Eqn. 3).

The implementation of the cohesive model, inside the PICPRO program, follows nodal release technique in order to ensure a gradual evolution of flank nodal forces during fracture extension. Unlike other formulations (Rydholm et al [5]), the *cohesive distance*  $\Delta$  is chosen as a characteristic quantity for any steel plate independently from the mesh element size. The value  $\Delta$ , needed for each steel plate, can be obtained by a calibration procedure (Salvini et al [6]) that requires the finite element analysis of an experimental impact test. The proper value of  $\Delta$  is the one that ensures the best agreement on the load-time response between the FE prediction and experimental record.

In the finite element program it can be assumed the essential work of fracture to be derived from the work done by the nodal cohesive forces ( $F_c$ ) during a given crack extension ( $\delta$ ). In the case of only one node inside the cohesive zone the definition in Eqn. 4 can be applied.

$$w_f = \frac{2}{B\delta} \int_0^{\delta} F_c \frac{dV}{da} dx \quad (4)$$

Since this last definition accounts the evolution of a single material point, it can be considered valid only for self-similar propagation with constant speed, as seen for Eqn. 3. By introducing the evolution parameter  $\alpha$ , as normalised cohesive force with respect to its maximum ( $F_0$ ), it is assumed that the crack opening displacement fits a power type function with the following expression:

$$V = V_F (1 - \alpha^k) \quad (5)$$

Substituting and solving the integral in Eqn. 4, a more explicit form for  $w_f$  can be obtained:

$$w_f = 2 \frac{F_0 V_F}{B\delta} \frac{k}{k+1} (1 - \alpha^{k+1}) \quad (6)$$

Since steady propagation is considered, a constant  $w_f$  must be imposed during the nodal release (for each  $\alpha$ ); therefore, one can write:

$$w_f(\delta) = 2 \frac{F_0 V_F}{B\delta} \frac{k}{k+1} (1 - \alpha^{k+1}) = 2 \frac{F_0 V_F}{B\Delta} \frac{k}{k+1} = w_f(\Delta) \quad (7)$$

Leading to a general description of the evolution law for nodal forces as a function of the distance from the crack tip and the *cohesive distance*.

$$\frac{F_c}{F_0} = \alpha = \left(1 - \frac{\delta}{\Delta}\right)^{\frac{1}{k+1}} \quad (8)$$

The value of  $k=1$  is then assumed in accordance with the work of Rydholm et al [5].

This method requires that at least one node must fall inside the cohesive layer and then a maximum size  $\Delta$  can be established for fracture elements. In the case of finer meshes it can be assumed the Eqn. 8 to remain valid for each node ( $i$ ) and thus substituting  $\delta$  with the node actual position  $\delta_i$  relative to the crack tip.

$$w_f = \frac{2}{B} \sum_{NODES} \frac{1}{\delta_i} \int_0^{\delta_i} F_i \frac{dV_i}{da} dx \quad (9)$$

In this case, named  $n$  the number of cohesive nodes, the nodal maximum forces ( $F_{0,i}$ ) are expected to reduce by a scale factor  $n$  with respect to the one given for a single node ( $F_0$ ). However the summation of all  $n$  nodal contributes to  $w_f$  would restore the same value obtained for the single node case. Solving the integral, the expression for  $w_f$  can be obtained in the form used by PICPRO.

$$w_f = \frac{2}{B} \frac{k}{k+1} \sum_{i=1,n} \frac{F_{0,i} V_F}{\delta_i} [1 - \alpha(\delta_i)^{k+1}] \quad (10)$$

## ANALYSIS OF DWTT SPECIMEN

The Drop Weight Tear Test (DWTT) has a relevant importance in the characterisation of fracture propagation in steel pipes employed for gas transportation lines. The test is conducted on a SEN specimen in a Three Point Bending (TPB) impact machine. According to ASTM (E436) and API (API 5L) requirements the ligament is 76 mm, the effective span 256 mm and the initial crack length  $a_0$  is 5 mm. This test can be used primarily for the observation of shear fracture surface at a given temperature, useful to guarantee fully ductile operating conditions. Furthermore it is possible to include a basic instrumentation to measure the energy absorption, the load-time record and impact speed during the test. The recorded data can then be used to the development and verification of numerical models for ductile fracture propagation.

In a TPB test of a SEN specimen, that is to say in DWTT specimen, a cinematic relation can be found to relate the fracture extension to global cinematic quantities and the Crack Tip Opening Angle (CTOA). It can be assumed that a plastic hinge develops in the ligament of the specimen so that the two halves, with respect to the crack line, can be fairly considered as rigid. In accordance to analyses made by Turner et al [7] and Martinelli et al [8], it is possible to express a necessary condition for stable fracture propagation in such specimen.

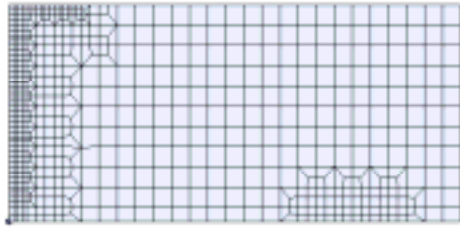
$$\frac{da}{d\theta} = r^* \frac{W - a}{\tan\left(\frac{CTOA}{2}\right)} \quad (11)$$

Here  $CTOA$  is assumed as dominant fracture parameter,  $a$  the crack length,  $(W-a)$  the actual net ligament,  $\theta$  the rotation of one half of the specimen and  $r^*$  is the plastic hinge geometric factor. This last factor has a conventional meaning as the non-dimensional distance of the instantaneous centre of rotation from the crack tip with respect to the actual ligament value. Many authors gave an estimation for  $r^*$  that generally is assumed to be about 0.45 (ASTM E1737, Matsoukas [9], Turner [7]). Two literature expression are found respectively in [8] and [9]:

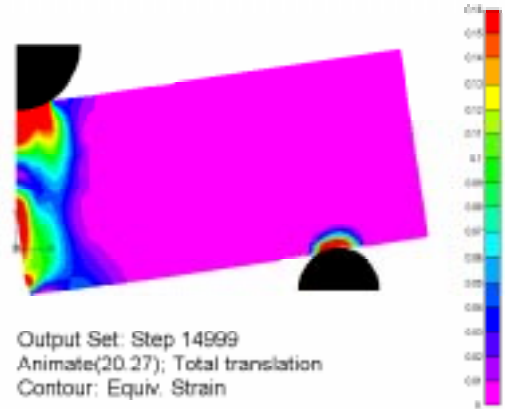
$$r^* = 3.8 \left(1 - \frac{a}{W}\right)^{30} + 1.27 \left(1 - \frac{a}{W}\right)^3 + 0.436 \quad (12)$$

$$r^* = 0.463 - 0.04 \left(\frac{a}{W}\right) \quad (13)$$

In order to perform a finite element analysis of a DWT Test the Eqn. 9 is implemented in PICPRO to give the fracture speed in the hypothesis of stable crack propagation. Under this condition the CTOA is replaced with the constant CTOAc value obtained from experimental procedure discussed in Buzzichelli et al [4]. Half specimen is studied for symmetry reason (Figure 3). PICPRO computes  $r^*$  by assuming that one specimen half is a rigid body that rotates around a yielded region and then characterised by the existence of the instantaneous centre of rotation  $C_R$ . If the reference coordinate system is located on the impact hammer, it can be found that  $r^*$  factor is the ratio between the distance, from crack tip, of the  $C_R$  projection on the symmetry axis and net ligament ( $W-a$ ).



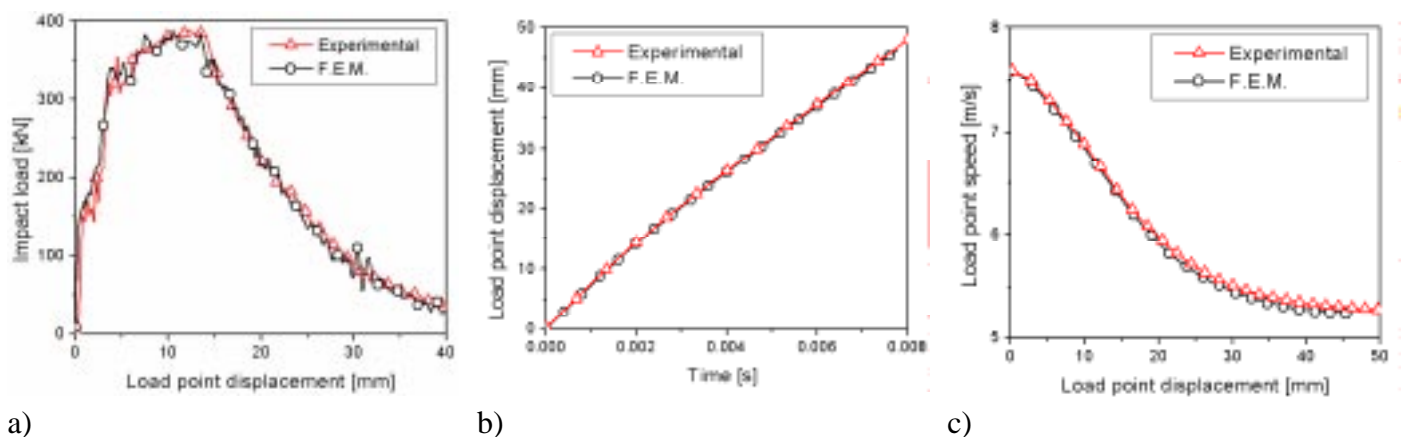
**Figure 3:** 2D mesh used for DWT analysis in PICPRO. Fracture elements are 1.58 mm.



**Figure 4:** Equivalent strain at 3 ms after impact.

In what follows it is taken into account a DWT Test of a steel pipe piece with 19 mm thickness and  $CTOAc=8.4^\circ$ . The impact test has been made with a dropping hammer of 550 kg reaching 7.58 m/s at the instant of first impact. The value of  $\Delta$  for the plate investigated is 12 mm with the procedure from Salvini et al [6]. The mesh used in the calculation is shown in Figure 3 while Figure 4 shows the post-processed equivalent strains 3 ms after impact. This frame reveals the interference of the fracture plastic field with the one originated by the indentation from the impact hammer.

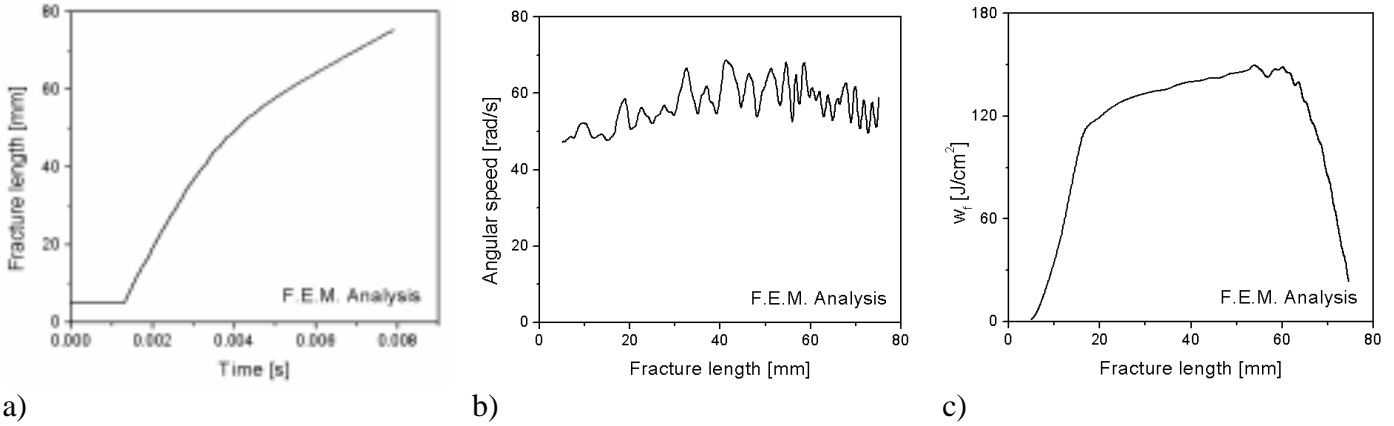
As shown in the Figure 5a, the choice on  $\Delta$  makes possible to follow the complete propagation event after crack initiation (imposed after 10 mm of load-point displacement). As a matter of fact the descent slope of the load-displacement diagram can be related with the propagation energy spent both as *essential work of fracture* and global plasticity ([1-2-3]). Still other cinematic quantities reveal good agreement between the PICPRO simulation and experimental results (Figures 5b-c).



**Figure 5:** Comparison between the PICPRO and experimental data from DWT Test.

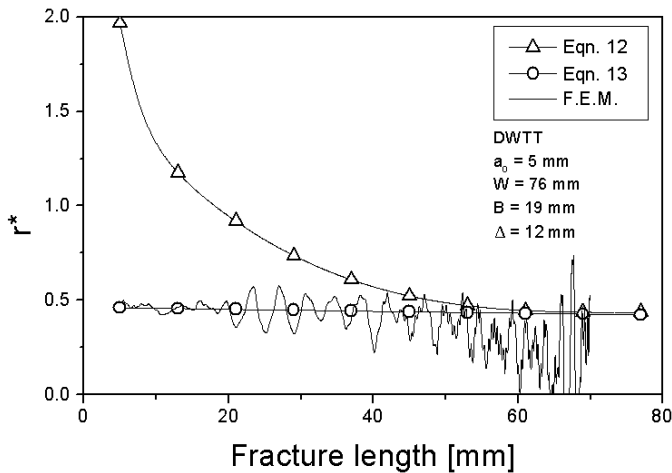
Other quantities can not be measured directly during the experiment but PICPRO can estimate them as reported in Figure 6. The crack length evolution can be studied (Figure 6a): here crack speed is found to decrease from initial value of 22.5 m/s to about 5 m/s at the end of the ligament. However the angular speed of the half specimen has a small rise from 47,4 rad/s to 68.5 rad/s (Figure 6b). Among these data  $w_f$  is computed (Figure 6c) and its mean value, regardless from end effects, spans between 112 J/cm<sup>2</sup> and

147 J/cm<sup>2</sup>. This increase is probably related with the hardening induced ahead the crack tip by hammer impact (Figure 4) and the predicted reduction of crack speed.

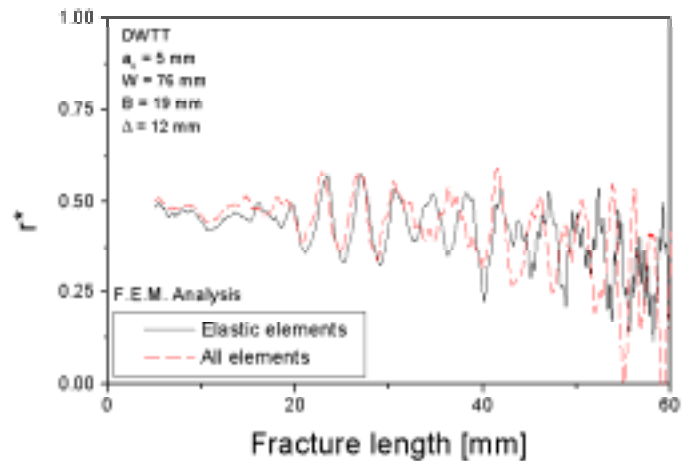


**Figure 6:** PICPRO analysis of fracture propagation.

Verification of  $r^*$  values is also studied with respect to estimations from Eqn.s 12 and 13. Results are shown in Figure 7 where a very good agreement appears with the Eqn. 13 even if a progressive reduction of  $r^*$  is predicted by PICPRO after 50 mm of fracture length. A further check is shown in Figure 8 where  $r^*$  is computed by removing, or not, plastically deformed elements. Since the results on a more magnified scale are more or less the same it can be concluded that the schematisation of rigid body motion that led to Eqn. 11 is reasonable.



**Figure 7:** Comparison by literature and PICPRO estimates of  $r^*$  factor.



**Figure 8:** PICPRO calculation of  $r^*$  factor using all elements or removing plastically deformed ones.

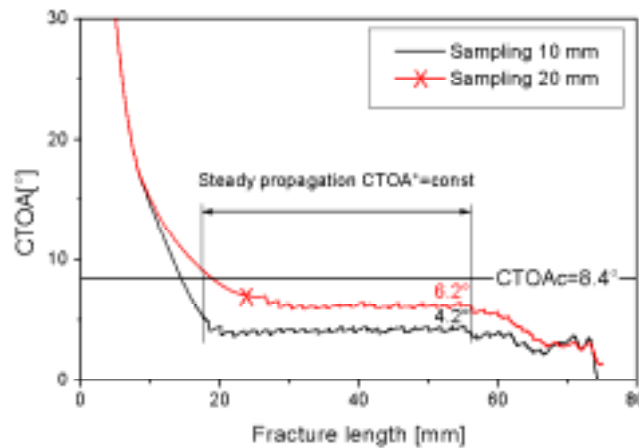
## CTOA PARAMETER

The CTOA parameter is the angle formed by crack flanks at the crack tip. Assuming a sharp profile, neglecting blunting effect, the following definition would be appropriate:

$$CTOA_a = \lim_{x \rightarrow 0^-} \left( 2 \cdot \arctg \left( \frac{1}{2} \frac{d}{da} COD(x) \right) \right) \quad (14)$$

The use of this expression, with the cohesive model here developed, is possible if one could establish a point, to be considered as effective crack tip, to which the limit must be calculated. As a convention, here it is assumed that the computational crack tip is coincident with the leading edge of the cohesive zone. By this position the crack profile is approximated with a polynomial whose slope, at the crack tip, is calculated as a measure of CTOA. Unfortunately there will be a non-unique value of this angle because it is dependent on the sampling width (i.e. number of nodes) and the order of polynomial used in the approximation.

After some investigation a second order polynomial is used for the interpolation, while the optimal sampling width depends on the mesh size, cohesive distance  $\Delta$ , ligament and fracture length. As an example, the post-processing of CTOA values for the DWT Test examined in this work is presented in Figure 9 with two sampling widths of 10 mm and 20 mm. Therefore, since  $\Delta=12$  mm most, or all, sampling nodes are inside the cohesive layer. Results show that crack initiation occurs with CTOA of about  $30^\circ$  but it suddenly reduces after crack initiation. The transient effect, during first stage of propagation, is due to abrupt change in the profile shape and it persists for a longer time if sampling length is higher. Anyway after a short amount of propagation a fairly constant CTOA value is reached depending on sampling width ( $4.2^\circ$  with s.w. 10 mm and  $6.2^\circ$  with s.w. 20 mm). As a result the values obtained (Figure 9) are not consistent with the one applied in PICPRO through Eqn. 11 ( $CTOAc=8.4^\circ$ ).



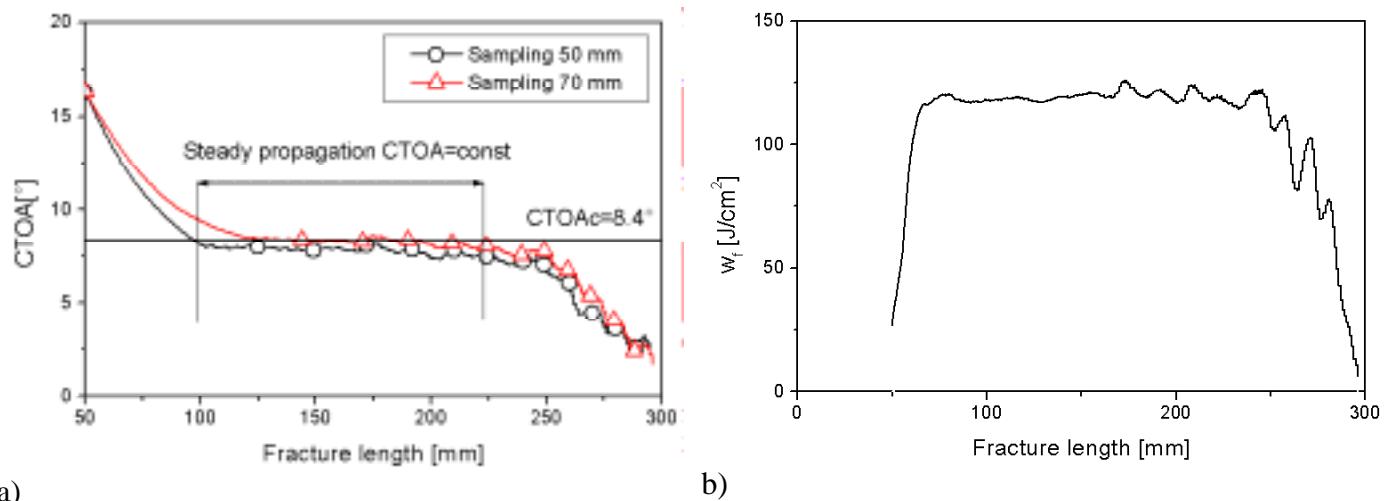
**Figure 9:** CTOA evolution in the DWT Test here studied with PICPRO.

The error can be ascribed to the geometric estimation of CTOA from nodal position inside the cohesive layer, where displacements are perturbed from their effective values. As a matter of fact, the cohesive model accounts only the energy dissipation for the new fracture surfaces but it is not able to follow all processes that lead to final rupture. From this consideration, it appears that geometrical CTOA must be calculated accounting a sampling region outside the cohesive layer. Anyway this region must be far enough from the boundary so that it can be considered "*at the crack tip*". Unfortunately this seems not possible in a standard DWTT specimen.

To validate this concept a DWTT type specimen with increased dimensions is simulated (ligament 300 mm, span 500 mm, initial crack length 50 mm) with the same material previously studied and then again with  $CTOAc=8.4^\circ$  and  $\Delta=12$  mm. Here the CTOA is computed with two different sampling widths (50 mm and 70 mm) and some results are plotted in Figure 10a-b. In this case steady CTOA values, computed from profile geometry, are observed to have minor differences and reproduce well the  $CTOAc$  value ( $8.4^\circ$ ) imposed. A further computation has been made with a hypothetical  $CTOAc=15^\circ$  and consistency is still maintained (i.e. the steady condition  $CTOA=CTOAc$  is reached whatever the value of  $CTOAc$  is).

From these considerations, it follows that Eqn. 11 can be used to govern fracture extension, in a DWT Test, by imposing a constant value of  $CTOAc$ . Specimen geometry and cohesive distance  $\Delta$  can affect the post-processed CTOA value while the constancy of CTOA is reproduced during steady propagation. Anyway, for each case a confirmation is needed because, as a principle, it can also happen that CTOA never reach a constant value for some  $\Delta$ , crack length,  $CTOAc$ , steel tensile strength and toughness. Differences between the imposed  $CTOAc$  value and the CTOA attained during steady propagation can be related to crack profile approximation and can not be considered the effective fracture behaviour.

Finally  $w_f$  is computed still in the virtual wide-plate experiment, showing a constant value of about  $118 \text{ J/cm}^2$ , which is similar to the one obtained in the DWTT specimen. This is a promising point for an indirect computation of CTOA from  $w_f$  actual value. However this last position requires further investigation and will be object of future work.



**Figure 10:** PICPRO simulation of a wide SEN plate.

## CONCLUSIONS

A one-dimensional cohesive model is introduced to simulate steady fracture propagation under ductile conditions dominated by CTOA parameter. The *cohesive distance*  $\Delta$  and CTOAc come out to be the only model quantities needed from laboratory experiments. The model is implemented in PICPRO program in order to simulate fracture in DWTT specimens. Analysis of one test reveals the cohesive model to give consistent results both from cinematics and dynamics of the whole test. Special care is devoted to the computation of CTOA from a simple crack profile approximation. This definition, based on some functional approximation, is found to be effective only when ligament and crack length are much greater than  $\Delta$  (say a ligament of 20-30 times and an initial crack of 5-10 times) as in the case of whole pipes or wide plates. However in small DWTT specimens the *essential work of fracture* calculation is independent from the profile approximation and it is suited a CTOA indirect estimation through this parameter.

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