A TRANSFERRING LAW BASED ON A SINGLE PARAMETER CHARACTERIZATION OF SLOW-STABLE CRACK EXTENSION

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ABSTRACT

Test data for two center cracks of widely different length in geometrically-dissimilar specimens made from a thin-sheet aluminium alloy are compared in an effort to demonstrate the potentialities of a new Transferring Law (TL). By "transferring" is meant here the ability to predict the onset of a steady-state crack extension in large-scale plates and shells operated under biaxial loading from data collected on small specimens of the same thickness tested under monotone uniaxial tension. Preliminary experimental results, among them those presented in this study, are extremely encouraging. There are good evidences that the proposed TL can be used as a simple semi-analytic tool for calculating the residual strength of damage-tolerant structures.

INTRODUCTION

The problem of transferring crack-extension resistance R-curves for non-proportionally scaled in-plane geometries by a single TL is rather complicated and incomprehensible. There are two principal obstacles. The first one is placed by irreversible plasticity preceding and accompanying a slow-stable crack growth. Generally, the TL is much simpler for both a very large component (globally elastic behaviour) and a very small one (rigid plasticity). Different sets of crack-driving and constraint parameters coupled with different fracture toughness quantities related to the above extremes are commonly employed. These distinctions necessarily lead to a challenging task of formulating the TL for an intermediate range of component sizes.

The second obstacle is emerging from the lack of a universally suitable one-parameter characterization of mode I crack growth. Currently available elastic-plastic continuum models like those presented in [1-4] permit to calculate relationships among tensile load, load-line displacement and crack advance with no restrictions on the extent of plastic deformation and crack advance. A computational approach to ductile crack growth [2] demonstrates convincingly by way of examples that: "no approach can be based on a single parameter resistance curve". To be precise, this statement deserves further comment: as long as one is forced to operate within the framework of the Conventional Methodology (CM) of fracture analysis. The above approaches are entirely incompatible with a phenomenon of the slow-stable crack growth in uniformly compressed brittle materials. Mode I fracture of this sort is often observed in rock masses, near free surfaces of excavations, in building structures, as well as in laboratory specimens tested under uniaxial compression. It is suffice to mention solely Bridgman's tests under high hydrostatic pressure [5].

The simplest way to demonstrate an inherent deficiency of the CM is through the analysis of a center-cracked plate (Fig.1a) subjected to uniform biaxial stresses σ and $q = k\sigma$. In the special case of uniaxial tension ($\sigma > 0$, $k = 0$), the slow-stable crack growth can be characterized in terms of the stress intensity factor K_I [1], the J_I-integral [2], crack tip opening angle ψ, crack tip stresses and strains or energy release

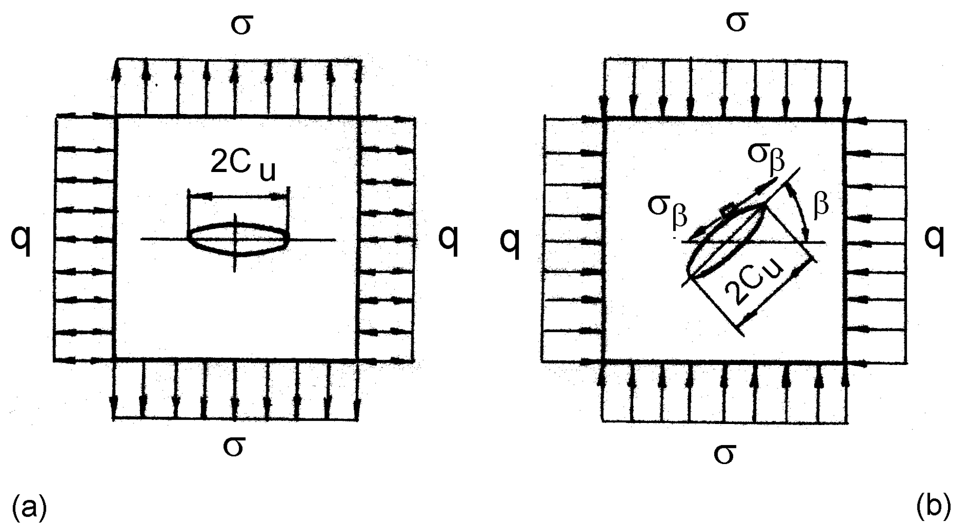

Figure 1: Through-the-thickness cracks in elastic plates under prevailing tensile (a) and compressive (b) biaxial loading.

rate [3], and the constant ψ_{SS} angle [4]. The related K_R- and, J_R-curve and ψ_{SS} values are widely used with a latent understanding that in any case of practical importance crack growth is possible only when $\sigma > 0$. And that is the case only for strain-controlled fracture processes such as stable tearing and void coalescence.

Assume that a plate (Fig.1a) in concert with a loading device is merely moved from the laboratory into a pressure vessel. As before, the crack faces are stress-free but the outer boundary loads σ and q are defined in the following manner: $\sigma = \sigma_{appl} + p$ and $q = q_{appl} + p$, where p is high hydrostatic pressure. Compressive stresses arisen from pressure may be below, equal or above the applied tensile stress σ_{appl}. In each case, namely, $K_I > 0$, $K_I = 0$ and $K_I < 0$, mode I crack extension in brittle materials is a reproducibly observable physical event. Appropriate experimental data resulting from extensive studies of slow-stable crack growth in glassy materials under condition $K_I << K_c$ and furthermore $\sigma_{appl} = 0$, $q_{appl} < 0$, $K_I = 0$ are presented partly in [6]. To be of value, any theory of fracture, if not describing these factual evidences, must at least be compatible with them.

One way around the above obstacles is emerging from the UM [7,8]. The term UM implies that the effects of the crack length, specimen geometry and size, load precracking history, boundary constraint and load biaxiality are evaluated in the context of a single conception called the ρ -theory [6]. The primary aim of the present paper is to demonstrate the potential of the new TL with respect to prediction of the resistance to the onset of a steady-state crack extension in thin-sheet ductile materials subjected to monotone and uniaxial tensile loading.

THEORETICAL BACKGROUND

We consider a 2-D problem of a center-cracked plate subjected to the preferential effect of tensile (Fig.1a) and compressive (Fig.1b) stresses. All advancements of the fracture mechanics, starting with the fundamental study [9], are associated to various degrees with the solution of the first problem. The majority of publications relating to the second problem (Fig.1b) are closely linked with a later fundamental study [10] known to a relatively small group of experts who quote this study in the context of a conventional strength approach. This place and the role of the fracture theory [10] are determined in large part by selection of the crack model in the form of an elliptic hole. According to Griffith [10], brittle fracture starts when the tensile stress σ_β in some point on the border of an ideal crack represented by an elliptic hole (Fig.1b) becomes equal to the cohesive strength of the material S_t .

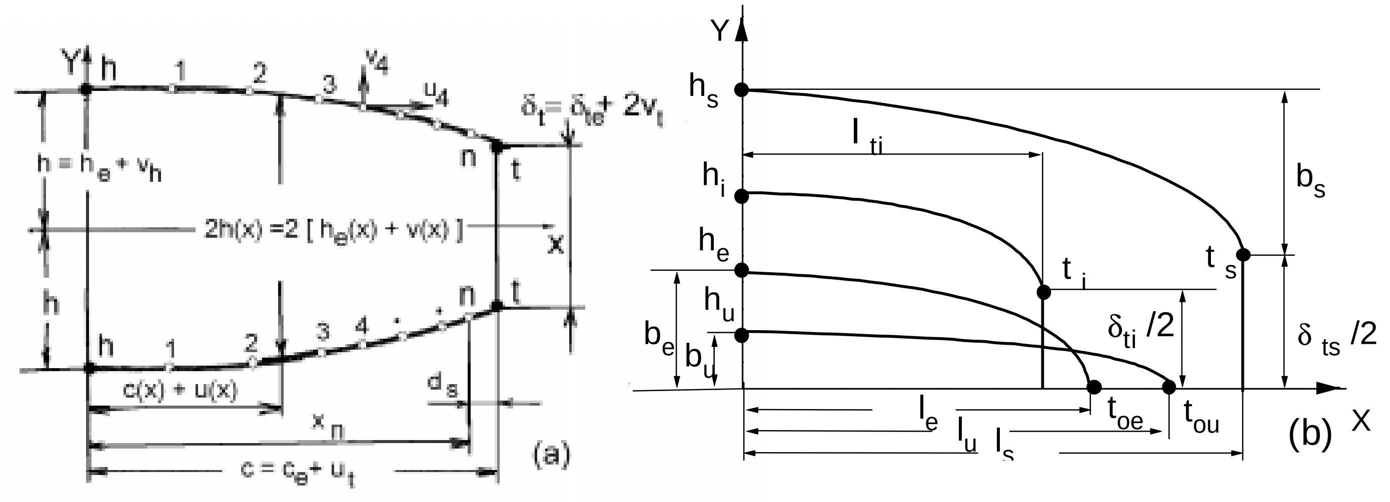

Figure 2: Schematic sketches of an actual crack profile (a) and a sequence of ideal crack profiles (b) in the course of uniaxial tensile loading

From what has been said, it follows that different assumptions of initial crack border spacings can lead to qualitatively different fracture criteria. Therefore, the problem of mode I crack growth under tension and uniform compression is solved from different positions and without reference to each other. The sources of this gap are in a tacit assumption on the existence of a qualitative differences in the physical essence of crack extension under tensile and compressive loads. However, in terms of physical processes, a crack-tip extension presumably must involve separation of the material under local tensile stress, even if only compressive loads are applied. Erdogan and Sih [11] had every reason to conclude that the so-called "sliding" and "tearing" modes of crack growth do not take place and the mode of fracture always seems to be a crack opening.

The UM postulates that the dominant mechanism of mode I crack-tip fracture is independent of the sign and value of the load biaxiality ratio k i.e. it could be treated as basically the same for any combination of tensile and compressive loads. Crack growth is modeled by omnidirectional extension of some originally elliptic hole (ideal crack). The experimental procedure used for evaluating the parameters of an actual crack (Fig.2a) and subsequent converting them into the parameters of an ideal crack (Fig.2b) is outlined earlier [7]. It should be emphasized that there are three qualitatively different relationships among displacements of the extreme points h and t on an ideal crack profile (Fig.2b). During elastic blunting of a stationary crack an outwardly-directed displacement v_h is accompanied by an inward displacement u_t. Plastic blunting superimposes transverse displacement 0.5 δ_t over that process. And finally, in the course of a transient crack growth all crack-border points are displaced outwardly.

The minimum radius of an ideal crack, $\rho_e = b_e^2 / l_e$ relates to the profile of an actual crack with a zero crack-tip stress σ_l. This event is to be defined from the experimental diagram the net-section stress, σ_N (q_N) versus the crack-tip opening spacing, δ_t (Fig.2a). Fracture analysis is based on the employment of the so-called crack volume ratio V. This is the ratio between an increment of the volume in response to loading M=A·B enclosed by ideal crack surfaces, and the volume, $M_e = A_e B_e$, of the same crack under condition $\sigma = \sigma_e$ or $q = q_e$ ($\sigma_l = 0$). Here A and A_e are in-plane areas of an ideal crack with a stressed and stress-free crack tip, respectively, B and B_e are the averaged thicknesses of the plate in the same states. For a center elliptic hole in a rectangular plate subjected to uniform biaxial loading the V ratio can be represented by a single, rational function of transverse and longitudinal displacements of the extreme points h and t (Fig.2b). A special convenience of the V parameter is that its elastic and plastic components may be readily determined for both stationary and growing cracks across the whole range between small-scale and large-scale yielding. In all cases V has a common physical meaning mentioned above.

For simplicity assume that the equality $B = B_e$ is met at every instant of a slow-stable crack growth. In that case the onset of a steady-state crack growth under plane stress state is governed by the following condition:

$$V = \left(1+C_l\frac{\sigma^*}{E}\right)\left(1+C_b\frac{\sigma^*}{E}\right)+\frac{2\,l\,\delta_t}{\pi\,b_e l_e}-(1+V_u) = V_{RS}, \tag{1}$$

where $\sigma^* = \sigma - \sigma_e$, E is the elastic modulus, V_{RS} is the resistance to the onset of a steady-state crack growth in a plate of a given material and thickness. Here V_u reflects an initial value of the crack volume ratio caused by load precracking history effects:

$$V_u = \left(1-C_l\frac{\sigma_e}{E}\right)\left(1-C_b\frac{\sigma_e}{E}\right)-1\,. \tag{2}$$

The elastic stress concentration factors C_l and C_b at the extreme points take the form:

$$C_l = F_V\left[1+2(l/\rho_e)^{0.5}-k\right] \quad \text{and} \quad C_b = F_u\left[k+2k(\rho_e/l)^{0.5}-1\right], \tag{3}$$

where F_V and F_u are the non-dimensional elastic compliances of the hole border related to the extreme points, k is the load biaxiality ratio. The half-length l of an ideal crack is defined by the following condition:

$$v_h = F_V\left[2l+(1-k)\,(\rho_e l)^{0.5}\right]\frac{\sigma}{E}\,, \tag{4}$$

where v_h is the displacement taken from an experimental diagram, σ (q) vesus $2v_h$.

BRIEF EXPERIMENTAL ANALYSIS

Fracture resistance tests were made on 1.05 mm thick sheets of an aero-skin aluminium alloy 1163 similar to Al 2024-T351. Its tensile properties under ambient conditions are: E = 73 GPa, 0.2% offset yield stress σ_Y = 334 MPa and ultimate tensile strength σ_u = 446 MPa. We consider two geometries (Fig.3) as consisting of a rectangular Problem Domain (PD) of width 2W and height 2H, which is embedded into a Load Surroundings (LS). The first type of LS provides rigid clamping along horizontal boundaries AB and DC of the PD,

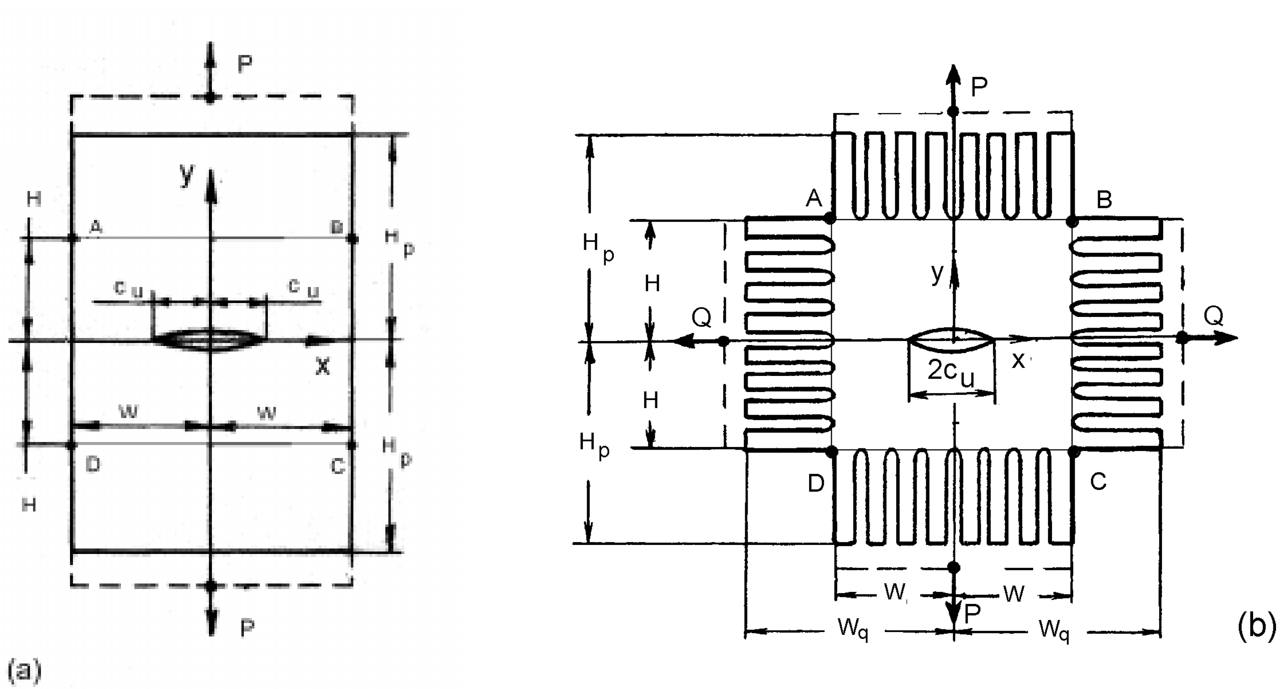

Figure 3: Rectangular problem domains ABCD embedded into different load surroundings.

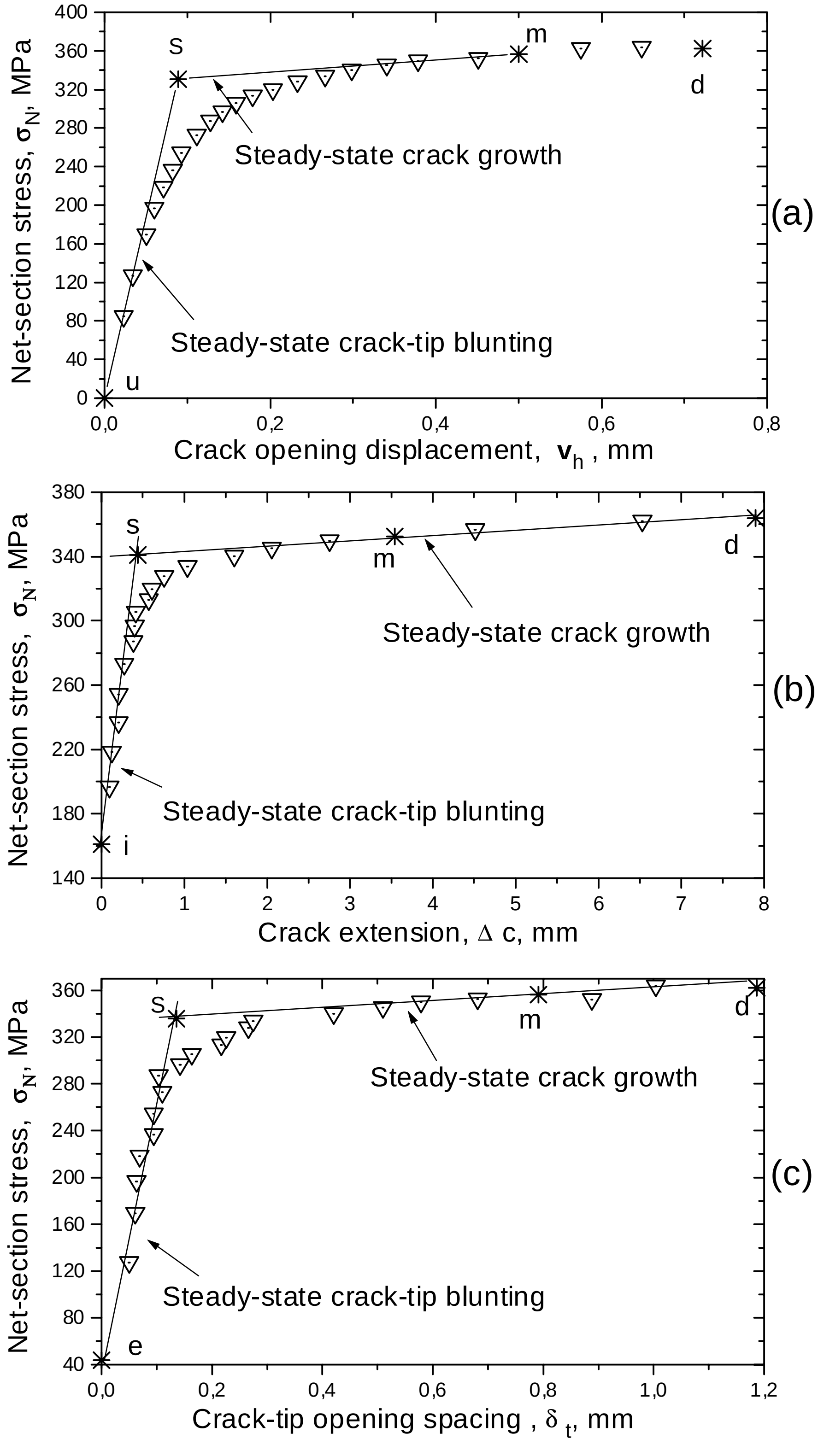

Figure 4: Test records and related pseudo-steady diagrams expressed in terms of the specific extreme-point displacements and crack extension in the MM(T-TC) specimen during monotone tensile loading.

usually referred to as M(T) specimen. The latter has 2W = 1200 mm and 2H = 2760 mm. For the second geometry nearly uniform stresses **σ** and **q** can be applied, respectively, along horizontal and vertical boundaries of the PD [12]. A cruciform specimen of sizes 2W = 2H = 240 mm is designated as MM(T-TC).

Fatigue precracks of length $2c_u$ = 19.4 and 508 mm, respectively, in the MM(T-TC) and M(T) specimens were grown under constant amplitude loading at the stress ratio R = 0.4 and the maximum net-section stress

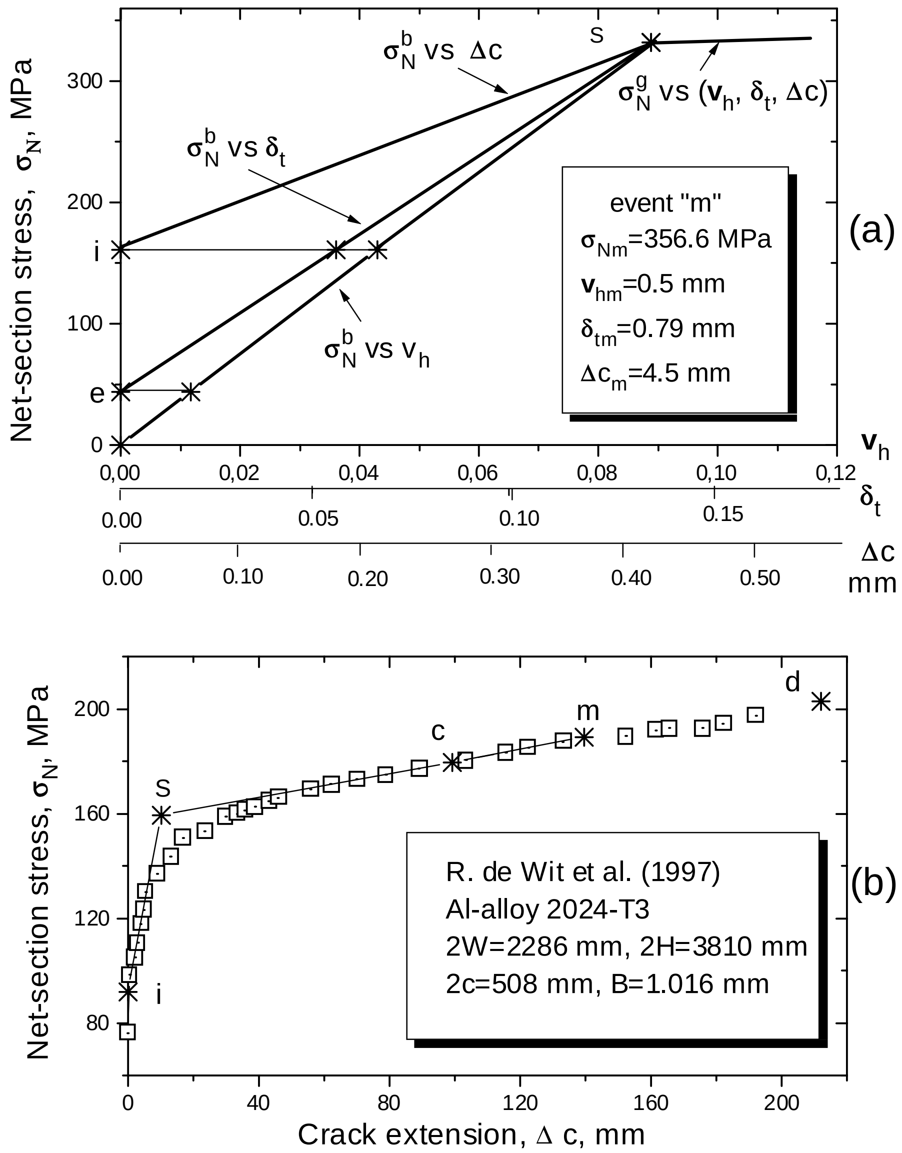

Figure 5: Generalized pseudo-steady diagram of omnidirectional extension of the crack surfaces in the MM(T-TC) specimen (a) and a particular pseudo-steady diagram for the M(T) specimen of Al 2024-T3 (b).

σ_N about 100 and 55 MPa, respectively. It should be mentioned that prior to precracking, the M(T) specimen had been subjected sequentially to four crack-extension resistance tests. An extended description of the experimental procedure and crack-border characterization is presented in [13]. Both specimens were tested by applying a monotone tensile load P (Fig.3) up to point "d" on test records (see Fig.4) followed by complete unloading.

To simplify matters as far as possible we consider only the so-called pseudo-steady diagrams (see Figs 4, 5) and the following crack-tip events (states): "u" - the onset of loading; "e" - the zero crack-tip stress σ_l; "i" - the initiation of steady-state crack-tip blunting; "s" - the imaginary start of steady-state crack growth; "m" - the completion of the steady-state crack growth. The corresponding crack-profile changes are conveniently illustrated by a generalized pseudo-steady diagram. Here again (Fig.5) the particular pseudo-steady diagram is presented. It has been derived from crack-extension data [14] for the largest M(T) specimen that has ever been tested. Doing so provides good evidence that expression (1) could be used as a simple TL for calculating residual strength (σ_S stress) at least under monotone uniaxial tension.

The conventional critical point "c" is left out of the UM analysis [8] since this event could not be defined immediately from a test record. In [14], the instability point "c" is determined as tangency of a crack-extension force line with an R-curve. In the framework of the UM, the preference is given to the use of the event "m" defined as the point of departure between the test record and the related line of steady-state crack growth (Figs 4 and 5b). Further crack advances are accompanied by progressively decreasing increments in the net-section stress σ_N . An acceptable procedure for identifying the completion of the steady-state crack growth must involve a close examination of all test records taken together which are shown in Fig.4.

TABLE 1
COMPARISON OF FRACTURE RESISTANCE QUANTITIES

Specimen and Material	Fracture resistance quantities						
	V_u	V_{Ri}	V_{Rs}	Δc_s / c_u	ψ_s deg.	ψ_m deg.	ψ_{ss} deg
MM(T-TC) Al 1163 AT	-0.0069	0.0486	0.1069	0.041	≈25.0	5.3	-
M(T) Al 1163 AT	-0.0452	0.0581	0.1018	0.007	5.5	-	3.9
M(T) Al 2024-T3	-	0.0660	0.1155	0.042	-	-	5.5 (3.4)[a]

[a] The average values presented in [4] are based on evidences taken from a test program [14]: 5.5 deg. is derived from experimental measurements, 3.4 deg. from analytical simulations.

Assume that the value of $\rho_e = 0.262$ mm taken from [7] could be used in the calculation of the V ratios for a crack in the M(T) specimen made of Al 2024-T3 (Fig.5b). Assuming again: l = c, $\sigma_e = 0$ we obtain the V_R data in Table 1 for Al 2024-T3. As to the events "i" and "s" they are in reasonable agreement with the data for both specimens made of Al 1163 AT. The pronounced differences related to the "u" and "i" events are believed to be caused mainly by the differences in load precracking histories. A simulated crack in the Al 2024-T3 specimen was a saw cut, having a final tip radius of 0.076 mm [14]. This suggestion is strongly support by findings [15] of unexplained distinctions between near-tip stress fields for saw-cuts and fatigue precracks in Al 2024-T3 specimens. Our experimental results on loading history effects also support the above statement. Some of them are presented in [13]. When considered in terms of ψ_R resistance, these results show that the excellent agreement between the ψ_m and ψ_{ss} values in Table 1, determined directly from closed-up photographs, is accidental. The most surprising thing is that non-proportional variations of an in-plane geometry can have a dramatic effect on ψ_s resistance.

GENERAL OBSERVATION

A need for a universal mode I crack-extension law based on a single-parameter characterization of brittle and ductile fracture had long been felt and was continuously becoming more urgent. It appears that the UM may be thought of as a step forward in this direction. Available experimental data, among them those presented in this study, give grounds for considering the UM as a usable alternative to the CM approach. Clearly it has advantages of an appropriate engineering approximation. The UM obviates the need for harmonizing qualitatively distinct descriptions of purely elastic and purely plastic behaviours of a cracked body. It keeps derivation of the desired TL from becoming too involved since it accounts for the plasticity effects immediately from test records. An important point is that the UM gives a more detailed information on the crack-tip events of practical significance.

The TL in question permits to correlate slow-stable crack growth data in ductile sheet materials by using a purely mechanistic and elastic analysis. The applied loads, the crack geometry parametersand the V_{Rs} resistance bear the same relationship (1) to each other in the elastoplastic range as they do in the elastic attention on the whole crack border instead of the near-crack-tip region, as the subject of much concentrated range.This ability of the UM could appear strange to a greater extent than realistic. It arises from focusing attention in the CM. The proposed global approach is compatible with analysis of the crack-tip stress and strains: some results are presented in [13]. Obviously the UM could be coupled with the currently available local approaches of the Beremin type. The prospects of such coupling are exceptionally intriguing in the light of close agreement between the V_{RS} ratio for Al 2024-T3 specimen in Table 1 and the ratio $[(R_c - R_o) / R_o)] = 0.12$ for M(T) specimen made of a thin-sheet Al 2024-T3 [16]. The latter value is generated within a framework of Rice-Tracey cavity growth model, where R_o and R_c are, respectively, the initial and critical radii of a spherical cavity.

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