A SIMPLE METHOD TO EVALUATE STATISTICAL CYCLIC FATIGUE STRENGTH IN CERAMICS

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ABSTRACT

A simple and convenient method to evaluate cyclic fatigue strength in ceramics was proposed using load range, ΔP-increasing procedure and applied for silicon nitride. The proposed method enabled to obtain statistical property of cyclic fatigue strength, i.e. *P-S-N* curve, by much less time and number of specimens compared with those required in the conventional procedure of statistical fatigue tests. *P-S-N* curves could be estimated by two Weibull distributions under different ΔP-increasing conditions based on the assumptions of crack growth law and of linear cumulative damage law. The estimated *P-S-N* curve agreed with the experimental data obtained under ΔP-constant condition at the high stress levels, while it was conservative under low stress levels. It would be summarized that the estimation gives a reasonable evaluation for the wide range of fatigue life.

INTRODUCTION

Figure 1(a) represents the typical loading sequence in the quasi-static experiment measuring tensile property or fracture toughness of structural materials including metals and ceramics, where P and t are applied load and time, respectively. In this case, the fracture resistance is evaluated by P value at which fracture occurs where the fracture resistance is regarded to be equal to the fracture driving force, i.e. applied load. Unlike this experiment, cyclic fatigue experiments are performed under constant value of load range, ΔP, as shown in Fig. 1(b), because the cyclic fatigue is a time dependent behavior. The fatigue limit, σ_w, is evaluated using a threshold value of ΔP below which the specimens do not fail until a certain number of cycles, typically 10^7 cycles, and thus the σ_w is an endurance limit for this number of cycles. The *S-N* curve is determined by a number of cycles to failure, N_f, and the applied stress amplitude, σ_a, in the ΔP-constant tests.

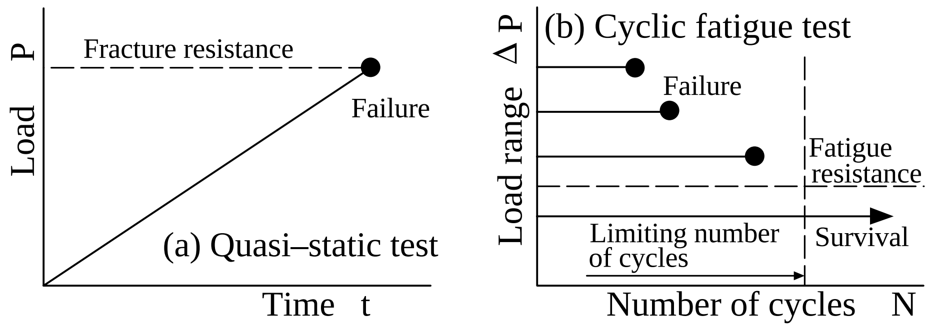

Figure 1: Loading sequences in the conventional (a) quasi-static and (b) cyclic fatigue tests.

Ceramics exhibit a time dependent fracture behavior under static loading condition, which is denoted by static fatigue. Usually, the static fatigue properties are evaluated under constant fracture driving force, i.e. *P*-constant condition, similar to Fig. 1(b). However, it can be predicted by results obtained under quasi-static *P*-increasing condition, namely dynamic fatigue experiment, based on a hypothesis of a unique crack growth property, i.e. a relationship between crack growth rate, da/dN, and stress intensity factor range, ΔK, under these loading conditions [1]. It has been reported in literature that the static fatigue strength predicted by *P*-increasing tests agrees with that obtained under *P*-constant tests [2, 3]. On the other hand, cyclic fatigue strength of ceramics is expressed by *S*-*N* curves obtained by ΔP-constant tests, as shown in Fig. 1(b). Since fatigue strength has a large scatter among the specimens in ceramics, it is preferable to be evaluated by *P*-*S*-*N* curves, which can express statistical properties of cyclic fatigue strength. However, the evaluation of *P*-*S*-*N* curve requires many specimens and is a time consuming task [4]. Depending on the test condition of ΔP, some specimens are broken during the first cycle of the cyclic loading, while some other specimens are unbroken until the limiting number of cycles selected in the tests. These specimens are regarded as a waste for the evaluation of cyclic fatigue strength [5]. These problems can be solved if the ΔP-increasing test is available in evaluating valid data of cyclic fatigue strength.

For metallic materials, it has been reported that the coaxing effect takes place and the σ_w value evaluated by ΔP-increasing tests becomes higher than that determined by ΔP-constant tests [6]. Thus, the value gives an inconservative evaluation and is regarded as an invalid data. It has been reported that the coaxing effect occurs due to cyclic strain hardening [6] and oxide induced crack closure [7], which do not occur in ceramics. Therefore, it can be expected that cyclic fatigue strength is evaluated correctly by the ΔP-increasing procedure.

In the present study, a simple and convenient method to evaluate cyclic fatigue strength in ceramics is proposed based on ΔP-increasing procedure and applied for silicon nitride. The proposed method enables to obtain statistical property of cyclic fatigue strength, i.e. *P*-*S*-*N* curve, by much less time and number of specimens compared with those required in the conventional procedure of statistical fatigue tests. The estimated *P*-*S*-*N* curves are compared with the experimental data obtained under ΔP-constant condition. The importance and applicability of this test method were discussed in detail.

MATERIAL AND EXPERIMENTAL PROCEDURES

Material used is a silicon nitride produced by Japan Fine Ceramics Center (JFCC), which is a standard test material, REFERCERAM (SN1). Chemical compositions and material properties from its catalogue are presented in Tables 1 and 2, respectively. Specimens are rectangular bending bar with width of 4 mm, thickness of 3 mm and length of 40 mm machining finished by #800 diamond grinder. Fatigue experiments were carried out at room temperature in air under 4 point bending with the upper span of 10 mm and the lower span of 30 mm using oil hydraulic testing machine with the capacity of 4.9 kN.

TABLE 1
CHEMICAL COMPOSITIONS OF MATERIALS (MASS%)

Si_3N_4	Mg	Ce	Sr	Al	Fe	Y	T-O
>88.5	<2.2	<3.8	<0.8	<0.02	<0.05	<0.05	<5.0

Table 2
MATERIAL PROPERTIES

Size		50x80x5 mm
Density	Archimedes	3.21 g/cm^3
Bending strength	4-point bending JIS R1601	900 MPa
Fracture toughness	SEPB, JIS1607	7.0 MPa√m
Hardness	Vickers, 980N	14000 MPa

Figure 2 illustrates loading history of ΔP-increasing test. Stress ratio, *R*, of the cyclic loading is 0.1 and the cyclic loading frequency, f, is 20 Hz. Testing parameters of the ΔP-increasing test are the initial value, $\sigma_{max,i}$, of maximum stress, σ_{max}, and increasing rate of σ_{max}, $d\sigma_{max}/dN$. Provided that the value of $\sigma_{max,i}$ is sufficiently lower than σ_{max} at failure, $\sigma_{max,f}$, all the specimens tested are broken by cyclic loading, and the $\sigma_{max,f}$ value is evaluated for the individual specimens.

ΔP-increasing tests were performed under $\sigma_{max,i}$ of 100 MPa with the $d\sigma_{max}/dN$ varied between 1.4×10^{-4} and 1.7×10^{-2} MPa/cycle. In order to determine the statistic property of $\sigma_{max,f}$, 10 specimens were selected randomly and tested both for $d\sigma_{max}/dN=2.8\times10^{-4}$ and 1.7×10^{-2} MPa/cycle. Similar test procedures were carried out for the condition of $\sigma_{max,i}$ =600 MPa with $d\sigma_{max}/dN=1.4\times10^{-4}$ MPa/cycle to investigate the influence of $\sigma_{max,i}$. P-increasing tests were performed for the conditions of initial value of σ, σ_i=100 MPa and σ-increasing rate, $d\sigma/dt$=20 MPa/hour using 10 specimens.

In order to compare the results of ΔP-increasing tests, ΔP-constant tests were performed for σ_{max}=600, 700 and 800 MPa using 5, 10 and 10 specimens, respectively, at R=0.1 and f=20 Hz.

RESULTS

Figure 3 shows the dependence of $\sigma_{max,f}$ on $d\sigma_{max}/dN$ obtained by ΔP-increasing test of $\sigma_{max,i}$ = 100 MPa. Although the results have large scatter, $\sigma_{max,f}$ decreases with decreasing $d\sigma_{max}/dN$, which indicates time-dependent characteristics of cyclic fatigue strength.

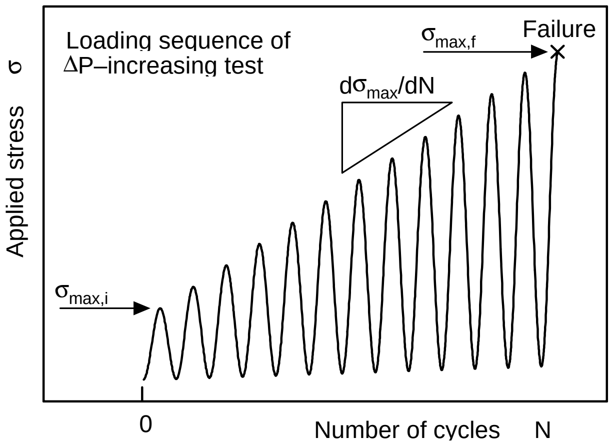

Figure 2: Loading sequence in the proposed ΔP-increasing test, in which the initial value of maximum stress, $\sigma_{max,i}$, and the increasing rate of maximum stress, $d\sigma_{max}/dN$, are the test parameters.

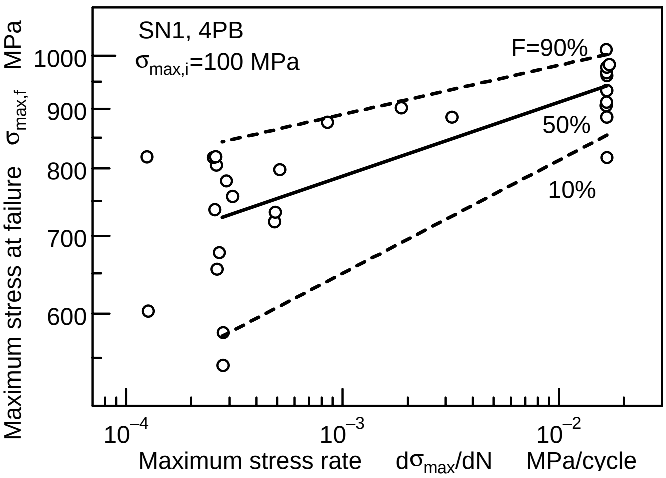

Figure 3: Values of maximum stress at failure, $\sigma_{max,f}$, evaluated in the ΔP-increasing tests under various $d\sigma_{max}/dN$ conditions.

Ten specimens randomly selected are tested under the same cyclic loading condition of ΔP-increasing test, and the results of $\sigma_{max,f}$ are plotted on the Weibull statistic diagram shown in Fig. 4. The $F(=(i\text{-}0.5)/n)$ is a cumulative probability, where i is an order of $\sigma_{max,f}$ and n is a number of specimens tested under the same loading condition. Figure 4 also shows flexural strength of the same material measured under monotonic loading at the crosshead speed of 0.5 mm/min for comparison [8]. This strength distribution agrees with that of $\sigma_{max,f}$ values for $d\sigma_{max}/dN = 1.7\times10^{-2}$ MPa/cycle, in which ΔP-increasing rate is maximum. However, for the conditions of $d\sigma_{max}/dN=2.8\times10^{-4}$ and 1.4×10^{-4} MPa/cycle, the strength distributions of $\sigma_{max,f}$ degrade because of cyclic fatigue behavior. For these conditions, the effect of $\sigma_{max,i}$ on the fatigue strength is remarkable for the statistically lower strength specimens, but not for the higher strength specimens.

σ_f values obtained under quasi-static P-increasing tests, i.e. dynamic fatigue test, are plotted on the Weibull statistic diagram, as shown in Fig. 5, where the cyclic fatigue data by the ΔP-increasing tests of $d\sigma_{max}/dN = 2.8\times10^{-4}$ MPa/cycle are plotted again for comparison. The increasing rate of σ_{max} for a unit time of the cyclic fatigue test, $d\sigma_{max}/dt$ = 20 MPa/hour, is the same with the increasing rate of σ of the dynamic fatigue test, $d\sigma/dt$. Flexural strength [8] is also presented in the figure. The σ_f of the dynamic fatigue test is lower than that of flexural strength. The difference between them indicates the degradation in strength due to static fatigue behavior. The $\sigma_{max,f}$ of the cyclic fatigue test is lower than the σ_f of the dynamic fatigue test. The difference between them indicates the degradation in strength due to cyclic fatigue behavior. It is interesting to note that the extent of the cyclic fatigue degradation in strength is more pronounced in the lower strength specimens.

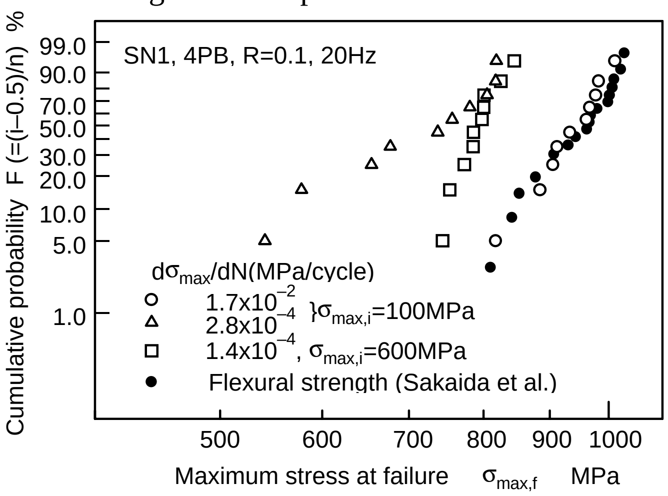

Figure 4: Weibull plots of $\sigma_{max,f}$ evaluated in the ΔP-increasing tests.

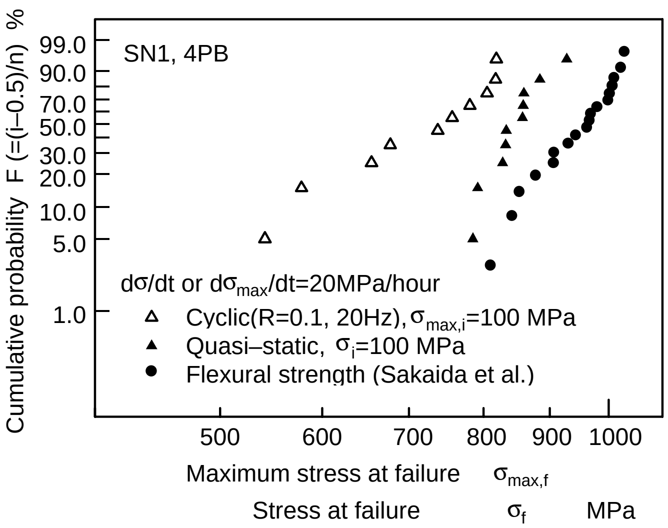

Figure 5: Weibull plots of $\sigma_{max,f}$ in the ΔP-increasing (cyclic fatigue) tests and stress at failure, σ_f , in the P-increasing (dynamic fatigue) tests, showing the contribution of cyclic stress on the fatigue strength.

Ten specimens tested for the individual test conditions, which is not enough for detailed investigation of Weibull distribution. However, two parameter Weibull distributions are assumed to approximate the test results.

$$F(\sigma_{max,f}) = 1 - \exp\left\{1 - \left(\frac{\sigma_{max,f}}{\alpha}\right)^m\right\}, \tag{1}$$

where m and α are shape and scale parameters, respectively. Table 3 shows the values of m and α determined by least squire method of the results.

TABLE 3
WEIBULL PARAMETERS

Cyclic fatigue (ΔP-inc.)		m	α
$\sigma_{max,i}$ (MPa)	$d\sigma_{max}/dN$ (MPa/cycle)		(MPa)
100	2.8×10^{-4}	8.02	760
100	1.7×10^{-2}	19.3	960
600	1.4×10^{-4}	30.7	805
Dynamic fatigue (P-inc.)			
σ_i (MPa)	$d\sigma/dt$ (MPa/hour)		
100	20	23.3	865

$\sigma_{max,f}$ values for F = 10, 50 and 90% are determined by the Weibull distributions of $d\sigma_{max}/dN = 2.8\times10^{-4}$ and 1.7×10^{-2} MPa/cycle for $\sigma_{max,i}$ = 100 MPa, and the results for each F values are drawn by straight lines in Fig. 3. The data in Fig. 3 show large scatter, but the straight lines may approximate the tendency of the test results. Therefore, $\sigma_{max,f}$ for each F values is expressed by the following equation in terms of $d\sigma_{max}/dN$,

$$\frac{\sigma_{max,f}^{n+1}}{d\sigma_{max}/dN} = C, \tag{2}$$

where n and C values are empirical constants, e.g. n = 14.6 and $C = 2.0\times10^{48}$ for F = 50%.

Figure 6 shows the results of ΔP-constant tests. For the case of σ_{max} = 600 MPa, all five specimens ran out at the number of cycles, $N = 10^7$. Numbers of the run out specimens were presented in the figure. Four specimens ran out and six specimens were broken at σ_{max} =700 MPa. All the specimens were broken at σ_{max} = 800 MPa. The N_f values evaluated exhibits large scatter over three decades.

DISCUSSION

Estimation of S-N Curves

For the dynamic fatigue tests, if the expression,

$$\frac{\sigma_f^{n'+1}}{d\sigma/dt} = C' \tag{3}$$

is prevailed, static fatigue life, t_f, is predicted by the following equation, assuming double logarithmic linear relationship between crack growth rate, da/dt, and stress intensity factor, K [1], where n' and C' values are empirical constants.

$$\sigma^{n'} \cdot t_{\mathrm{f}} = \frac{C'}{n'+1} \tag{4}$$

When a similar method is applied for cyclic fatigue tests, N_{f} can be obtained from Eqn. 2 by the following equation.

$$\sigma_{\max}^{n} \cdot N_{\mathrm{f}} = \frac{C}{n+1} \tag{5}$$

P-S-N curve can be estimated by Eqn. 5 using *n* and *C* values of Eqn. 2 for the specific *F* values obtained in the ΔP-increasing tests .

P-S-N curve can be estimated also by the assumption of linear cumulative damage rule, i.e. Miner rule. Cumulative damage, *D*, is expressed by the following equation assuming the *S-N* curves are linear relationship on the double logarithmic plot.

$$D = \sum_{k=1}^{N_{\mathrm{f,inc}}} \frac{1}{N_{\mathrm{f},k}} = \sum_{k=1}^{N_{\mathrm{f,inc}}} \frac{(n+1)\sigma_{\max,k}^{n}}{C}, \tag{6}$$

where $N_{\mathrm{f,inc}}$ is a number of cycles to failure in the ΔP-increasing tests. $\sigma_{\max,k}$ and $N_{\mathrm{f},k}$ are $\sigma_{\max}$ and N_{f} at the *k*th cycle. $\sigma_{\max,k}$ is given by the following equation.

$$\sigma_{\max,k} = \sigma_{\max,i} + \left(\frac{d\sigma_{\max}}{dN}\right) \cdot k \tag{7}$$

P-S-N curve is estimated by the relationship between $\sigma_{\max}$ and N_{f} of Eqn. 5. The values of *n* and *C* are determined for specific *F* values so that the *D* value in Eqn. 6 becomes unity for the conditions of ΔP-increasing tests.

Comparison of Experimental Results and Estimation

P-S-N curve is expressed by Eqn. 5 using the data of $d\sigma_{\max}/dN = 2.8 \times 10^{-4}$ and 1.7×10^{-2} MPa/cycle for $\sigma_{\max,i} = 100$ MPa shown in Fig. 4, where *n* and *C* values are determined based on both Eqn. 2 and Eqn. 6. Although these equations are based on the different hypothesis, but the predicted N_{f} values well agree within 1%.

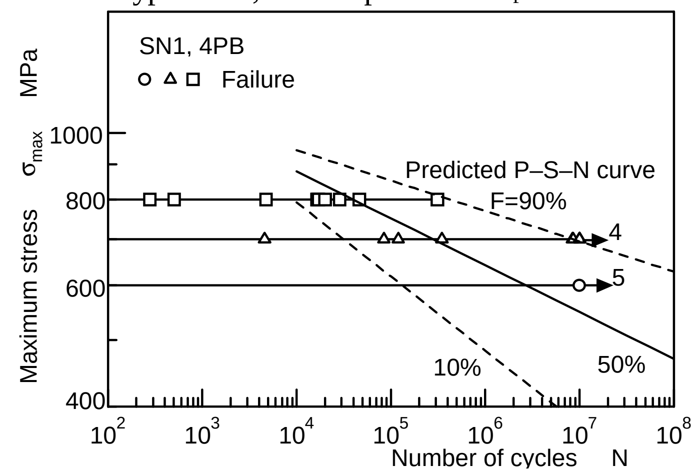

Figure 6: Fatigue life under ΔP-constant tests compared with the predicted *P-S-N* curve by ΔP-increasing tests.

The predicted *S-N* curves for *F*=10, 50 and 90% are presented in Fig. 7 together with the loading history of $\sigma_{\max}$ with *N* of the ΔP-increasing tests. Because of larger scatter in $\sigma_{\max,f}$ of $d\sigma_{\max}/dN = 2.8 \times 10^{-4}$ MPa/cycle, the predicted *P-S-N* curves exhibit larger scatter in the long life region. The values of *n* are 9.33, 14.6 and 22.8 for

F=10, 50 and 90%, respectively.

The estimated *P-S-N* curves are compared with those obtained by ΔP-constant tests presented in Fig. 6. Figure 8 shows relationships between F and N_f for the σ_{max} tested under ΔP-constant tests. For the case of σ_{max}=800 MPa, although the estimation exhibits a little longer fatigue life than that of experimental data, the fatigue life was estimated fairly well. For the case of σ_{max}=700 MPa, the estimation stands when F<40%, however, it becomes conservative when F>40%. Five specimens ran out at σ_{max}=600 MPa, which indicated that the estimation is conservative for the experimental data. It would be summarized that the estimation gives a reasonable evaluation for the wide range of fatigue life.

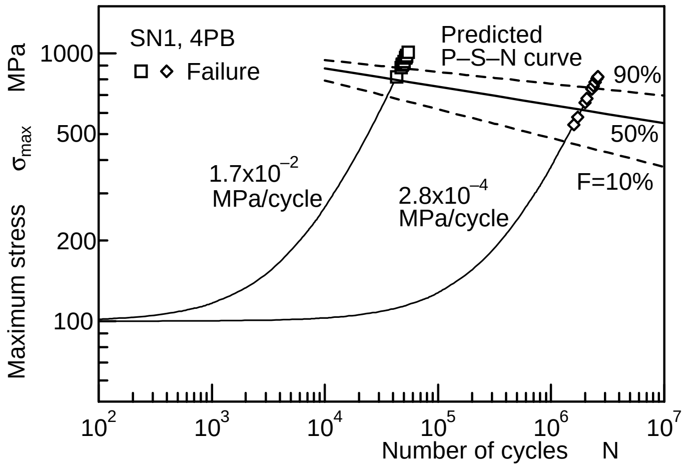

Figure 7: Loading sequence of ΔP-increasing tests and the estimated *P-S-N* curve by these tests.

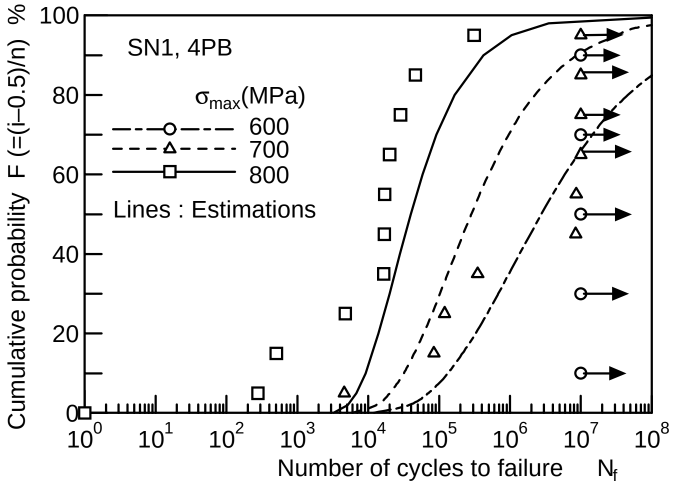

Figure 8: Fatigue life distribution under ΔP-constant tests compared with the prediction by ΔP-increasing tests.

CONCLUSIONS

A simple and convenient method to evaluate cyclic fatigue strength in ceramics was proposed using load range, ΔP-increasing procedure and applied for silicon nitride.

1. The proposed method enabled to obtain statistical property of cyclic fatigue strength, i.e. *P-S-N* curve, by much less number of specimens compared with that required in the conventional procedure of statistical fatigue tests.
2. *P-S-N* curves could be estimated by two Weibull distributions under different ΔP-increasing conditions based on the assumptions of crack growth law and of linear cumulative damage law. The estimated *P-S-N* curves

were independent of the above assumptions.

3. The estimated *P-S-N* curves agreed with the experimental data obtained under Δ*P*-constant condition at the high stress levels, while it was conservative under low stress levels.

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