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A SIMPLE DAMAGE MODEL FOR DUCTILE CRACK GROWTH AND TRANSFERABILITY OF R-CURVES

C.E.Turner

Mechanical Engineering Department, Imperial College, London.

ABSTRACT

A simple method is given for the transferability of R-curves for ductile growth from a datum fully plastic test piece to other sizes of pieces. The criterion for growth is a crack tip opening angle (ctoa) that is a function of geometry as well as of material. The underlying model is that crack growth by micro-void coalescence is a micro-instability process, driven by the elastic energy release rate, G, in the particular configuration of interest. This model applies to arbitrary amounts of growth and is here implemented, by iteration, using a conventional elastic plastic finite element program with no special elements or material behaviours.The work described, is for a high strength steel, HY130, in which the datum R-curve from a small deep notch bend test is transferred firstly to a geometrically similar piece, two and a half fold larger and secondly to a piece of the same thickness as the datum but the same width as the larger piece.

INTRODUCTION

In a previous paper, Turner and Kolednik [1] derived a model for ductile crack growth by micro-void coalescence, in which each step of growth was a micro-instability, driven by the elastic energy release rate, G. The reason for turning to a micro-instability model for crack growth, was that if the question were posed 'will the next step of deformation, implying an increment of work dU, cause growth or just plastic damage without growth?' the question at first appeared unanswerable in that the value of dU differed only by a second order small quanity for the two possibilities i.e. in both scenarios, dU = Qdq, irrespective of whether growth occurred or not, where Q is load and q is load point displacement.

Even in the plastic case, growth must therefore be controlled by a process that does not involve work, i.e. a process that occurs at fixed displacement, just as postulated by Griffith. The term that differs by a first order according to whether or not growth occurs, is of course the elastic energy release rate, G, again as in the Griffith model. The difference from that model is that, for real metals, throughout the whole process of initiation and crack growth, plastic damage is being done by each increment of work. Ductile fracture was therefore presented (in a strictly two dimensional model) as a two step process of damage plus separation, where the damage step occurred at constant crack length and separation at constant displacement. The increment in growth was strictly a finite step equal to the spacing, i, of the dominant inclusions, although for most purposes the conventional notations of infinitesimal calculus are still used.

NOTATION

All terms and abbreviations are defined when first used but attention is drawn to the use of
U for external work done
w for internal energy, whether recoverable or not
^ to denote an elastic component
~ to denote a plastic component.

THE MICRO-INSTABILITY MODEL FOR DUCTILE GROWTH

The combined problem of growth and plastic deformation was then set in terms of the balance between the driving energy rate, C, and the energy dissipation rate, D, as

$$C = (dU - dw^\wedge)/Bda = Qdq\sim/Bda + G(f) = D \quad (1)$$

dU being the increment in work done, dw^ the increment in elastic stored energy for thickness B and increment of crack growth, da, Q the load, dq– the change in the plastic component of displacement and G(r) the conventional energy release rate, as in lefm, but with (f) added to remind that the term is here an energy dissipated during fracture.

In [1], the deep notch bend (dnb) test configuration was seen as particularly easy to analyse for the datum case. The crack tip opening angle (ctoa) is defined in terms of the energy dissipation rate, D, and the position of the instantaneous centre of rotation r*b ahead of the crack tip, Braga and Turner [2], to give,

$$dq-/da = S\alpha(g)\sim /4r^*b \quad (2)$$

where *S* is span, b is remaining ligamant and α(g) is the ctoa, with (g) to remind that ctoa is here defined from the global field, not as a locally measured term, and marked ~ to remind that, at the onset of micro-instability, the ctod is overwhelmingly plastic. Hereafter the (g) is omitted. As shown in [1], the value of α~/r* can readily be found by plotting the plastic component of displacment, q–, versus lnb, the natural logarithm of the currently remaining ligament length. The crack mouth opening, V–, is also plotted versus q– to give r* in terms of the slope

$$r^* = \{(S/4)(dV\sim/dq\sim) - a(o)\}/b(o) \quad (3)$$

where a(o) is the initial crack length and b(o) the initial ligamant length. For a fully plastic test piece, where initiation is almost at the point of maximum load, the two plots are linear and the resulting value of *a*– is constant with growth. This is called a 'steady state' regime of growth and is essential to find the datum value of ctoa by such a simple method.

It is pointed out that ctoa, not ctod, is the criterion for growth implied by this interpretation of the experimental dnb data and that this ctoa, as here defined, is explicitly related to dU and thus to any other field term such a. dJ, so that the difficult problem of relating measured local terms, such as ctod measured by infiltration or photogrametric techniques, is side-stepped. Thus,

$$dJ(b)\sim/da = \eta\sim (Qdq\sim)/Bbda = \eta\sim L/4r^*\sigma(o)\alpha\sim \quad (4)$$

where the load, Q has been written as $L\sigma(o)Bb^2/S$, L being the plastic constraint factor on load, a value of about **1.36** at limit state in bending and σ(o) is the yield stress. The term dJ(b) is defined in terms of the η factor and the current ligament, b, without a so-called correction term of the form Jda/b since the energy Qdq– has been expended at fixed ligament size, with only the elastic component of energy being excahanged whilst b is changing at each step in growth to (b - da). As a separate issue, illustrated later but not here discussed, it has been found in all the present computations that the far field integral term, d(J(ff) - *G)*, agrees to within about 2% with the term dJ(b)~, just defined.

The computational model

The ctoa *can* also seen as the ongoing crack tip opening displacement (ctod) divided by the increment of growth. That increment would in reality be the dominant inclusion spacing, i, but in a computational model would be the element size at the crack tip, h, itself equal to the increment of growth, da. In the computations reported later, h = 0.4mm.

The 2D finite element (fe) computations have been made using conventional 8-node isoparametric elements which have no opening displacement at the crack tip. It might have been preferable to use collapsed node elements, at least for initiation, but that was not done. The ctoa is therefore represented as the ctod at one element before the tip, $\delta(t\text{-}1)$, divided by the element size, h. As already remarked, at the instant of instability this ctod is essentially plastic, whereas in the fe model an elastic component is necesarily introduced, though not relevant to the physical model. The actual ctod applied in the fe program is therefore, [1],

$$\delta(t\text{-}1) = h\alpha\sim + h\alpha\wedge \tag{5}$$

where, for a constant value of G in the steady state regime, $\alpha\wedge$ is $d(2v\wedge/da)$ evaluated at t -1, where $2v\wedge$ is the crack flank elastic displacement. i.e. $\alpha\wedge = 2\sqrt{(2G/E'h\pi)}$ where E' is the effective modulus, in plane stress or plane strain as may be.

Growth is implemented in the conventional way, by successive reductions of the crack-tip nodal force. The intention is not to model the micro-instability process per se in the fe work, but to use, or derive, ctoa values relevant to it. In [1], the micro-instability process was modelled algebraically, by considering the size of the micro-ligament remaining at any stage of growth of a spherical void, and the balance between its remaining uniaxial strength and the driving force per unit thickness, G. The result was

$$\alpha\sim\sqrt{G} = H(\alpha\sim)\, f(\text{material}) \tag{6}$$

where $H(\alpha\sim)$ was a function there evaluated to be nearly constant for a range of values of $a-$. Steady state growth occurs with both ctoa and G constant with growth, so to present a simple picture, it is describable approximately by the phrase '$\alpha\sim\sqrt{G}$ is constamt'. Even for a given material, the value of the constant is however size and configuration dependent but, as seen below, this simple model seems adequate to allow transfer of the experimentally found datum state to other sizes and configurations. It will be realised that both $a-$ and G are unknown beforehand, when attempting to transfer data from one case to another. A simple but useful approximation for G at limit laod in the dnb case was given by Merkle [4] as

$$G \approx L^2\sigma(o)^2 b/E' \tag{7}$$

where L is again the plastic constraint factor on load. In the elastic-plastic case, such a value of G may be well above the lefm value of toughness, G(Ic). Eqn.7, in conjunction with the $\alpha\sim\sqrt{G}$ rule and Eqn.4, implies that, for steady state growth, a 'wider piece, lower R-curve' effect can be expected, the curves scaling to $\Delta a/\sqrt{b}$, rather than to $\Delta a/b(o)$ reported in some literature.

THE ROLE OF DAMAGE

The next objective is to develop a simple model for the effect of damage on the ctoa that has to be applied to model ductile crack growth. This algebraic model is not part of the computational process, but nevertheless calls on data from the computations to make it numerate. Its role is to underscore the plausibility of the ctoa data as computed, by giving a more physical interpretation to the different patterns found and then to allow prediction of other cases, notably growth under well contained yield as would occur in a number of design cases.

In the steady state regime so far described, the damage, though unknown, is also constant with growth. It determines, of course, the actual values of ctoa measured, and is the reason why the conditions for intiation and propagation, even when expressed in similar terms, both ctoa or both ctod, are different. This varying pattern of ctoa versus growth was shown by Curr and Turner, [3], where the experimental data for two different sizes of dnb pieces were modelled in

3D, albcit using a straight crack front, based on the wotk done as the input and ctoa as output. It was seen that, for the small fully plastic piece which initiated close to maximum load, the computed ctoa (which as in the 2D work includes both plastic and elastic components) decreased to steady state after 1mm of growth, whereas for the larger piece, where initiation occurred at near elastic conditions, steady state was not reached until about 8mm growth. In this case of growth starting in well contained yield and then spreading to full plasticity, the ctoa decreased rapidly from its value at initiation to well below steady state, before rising to a steady state value appreciably less than that for the small datum piece. The ctoa values in steady state, were well related by '$\alpha \sim \sqrt{G}$ is constant' rule, after allowance for the different elastic components arising from the different values of G in steady state.

Damage is taken to be the opening of voids by local plasticity. The ctoa available in the virgin material is therefore absorbed by two components, 'damage induced' and 'applied'. The 'applied' is that necessary to create the next step of growth by a micro-instability process, as described above, after any prior damage has occurred, which will always be the case except for the first step of initiation in virgin material.

The measure of the induced damage is the apparent ctoa subtended by the ctod at successive step-distances ahead of the tip, i.e., δ/h, $\delta/2h$, $\delta/3h$ etc where δ is the ctod and h is the step size. Thus, at any one point ahead of a growing crack, there will be an accumulation of damage,

$$\Sigma \text{damage} = \delta_s/h + \delta_r/2h + \delta_q/3h + \delta_p/4h \text{ etc} \quad (8)$$

where s denotes the current tip ctod, r the previous (when the point considered was 2h ahead of the then tip), q before that and so on. This series starts immediately after initiation and will be cut off by the physical extent of the plastic zone ahead of the advancing tip, beyond which no damage is done. In general each a value will not be the same, thus **defining** the transient regime of small growth, but when, or if, in due course a steady state is reached, each a value will be the same.

In the experimental work on dnb R-curves, [5], a steady state was found in the fully plastic regime, though not necessarily starting immediately full plasticity was reached. This state, and its associated damage, is therefore taken to relate to the formation of successive slip lines (hinges, for the dnb case) which are rather sharply defined for the low hardening HY 130 steel data on which the model is based. The extent of damage ahead of steady state growth is therefore controlled by the formation of the slip bands. Equating the increment in internal plastic energy, dw–, to the dissipation in the slip hinge, suggests

$$dw\sim = (\tau\delta/2c)(Bbh\pi) \quad (9)$$

where τ is the shear yield stress with some allowance for work hardening, $(\delta/2)/c$ the shear strain induced across the width c of the damage zone, Fig.1a, and $Bbh\pi$ the volume of the slip hinge (treated as a circle). In the computations, dw– will relate to a step h = da, and dividing through by Bb, can be related conveniently to dJ~/da. Numbers cited here are for the HY 130

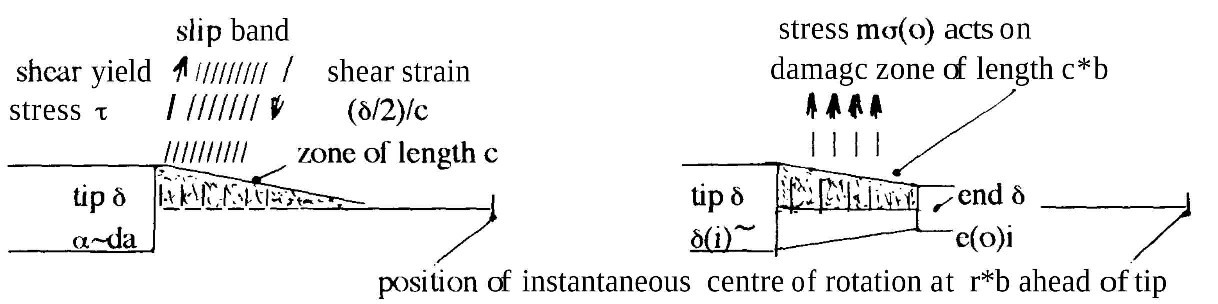

Fig.1a Damage zone for full plasticity

Fig.1b Damage zone for initiation

dnb piece that is of the same thickness, B = 20mm, as the datum test, but two and a half times wider, W = 95mm, S/W = 4, a(o)/W = 0.54. The computed data show c ≈ 0.7mm, i.e. between one and two step lengths, here taken as two steps for the element size used, 0.4mm. From Eqn.8, the available plastic ctoa in steady state, α(ss)available, therefore becomes

$$\alpha_{ssavailable} = \alpha_{ssapplied}(1+ 1/2 + 1/3), \text{ i.e. } 1.833\ \alpha_{ssapplied} \qquad (10)$$

with α(ss,applied) = 0.086 in the present case.

Initiation is regarded as a steady state in that there is no pre-existing damage so that, using the steady state rule 'α~√G is constant' the value at initiation (suffix i) is taken to be:-

$$\alpha\sim_i = 1.833\ \alpha_{ssapplied}(\surd(G_{ss}/G_i)) \qquad (11)$$

with G(ss) = ***0.280,*** G(i) = 0.144MN/m in the present case, giving α~(i) = 0.220. Since there is no prior damage, this value is the applied term to cause initiation. The more conventional argument would be that the ctod at initiation is the same for different pieces with the same constraint. That ignores the effect of the different driving force, G at initiation, in the two cases.

In between initiation and the onset of steady state, there is the transient regime for which the available ctoa at any step, m, prior to local instability, is taken as

$$\alpha\sim_m = \alpha_{ss}\surd(G_{ss}/G_m) \qquad (12)$$

initiation being just **a** particular case of this relationship. But after initiation, the value α~(m) cannot be directly applied, since the prior damage will reduce the capacity for ctoa that remains. For the first step after initiation,

$$\alpha\sim_{applied,1} = \alpha\sim_1 - \alpha\sim_i\ (1/2) \qquad (13a)$$

For the second step,

$$\alpha\sim_{applied,2} = \alpha\sim_2 - \{\alpha\sim_i\ (1/3)\ + \alpha\sim_{applied,1}\ (1/2)\} \qquad (13b)$$

This pattern is repeated for subsequent steps, with successive terms being added onto the accumulating damage,

$$\alpha\sim_{applied,m} = \alpha\sim_m - \{\alpha\sim_i\ (1/(m+1)) + \alpha\sim_{applied,1}\ (1/m) + ...+\ \alpha\sim_{applied,m-1}(1/2)\} \qquad (13c)$$

The **damage** fraction for α~(i) continues up to 1/n where n will be determined by the quite extensive damage caused by the large applied ctoa at initiation. **A** simple estimate is made from the force opening the damage zone and the plastic displacement through which it moves:

$$w\sim(i) = m\sigma(o)Bc^*b\{\delta+e(o)i\}/2 \qquad (14)$$

where w~(i) is the plastic component of **work** up to initiation, c*b is the extent of the damage ahead of the tip, δ is the applied ctod, here the initiation value, 0.088mm, (i.e., (h)ctoa(i) = 0.4(0.220)mm) again envisaged as a plastic term. The force opening the crack is taken as mσ(o)Bc*b, and the ctod, δ, at initiation, taken from the Dugdale model as G/mσ(o). The average movement of that force is taken as the mean of the tip opening, δ, and the very small plastic displacement at the far end of the zone, notionally the yield strain, e(o), acting over the inclusion spacing, i, Fig.1b. Writing J– and G in the form ηw/Bb gives c* = (2/η~)(J~/G) ≈ (J–/G) for dnb. For the present *case,* c*b = **4.**1mm, so that damage at initiation extends about

10 stcps ahead of the tip.
For small growth, just after initiation, the applied ctoa reduces so that the extent of the further damage ahead, also reduces. The above simple model becomes **less** secure; for example the instantaneous centre of rotation moves 'below' the centreline **so** that the (1- c*/r*) expression is not correct for the movement at c*b ahead of the tip. In the present work, the estimate of 4mm (10 stcps) has been retained for all the transient growth, because another effect soon over-rides it. This effect is the onset of full plasticity at maximum load, at about $\Delta a = 3.6mm$ **(9** steps of growth) for the present data, at which point the steady state estimate of damage for only 2 steps ahead of the tip, cuts in.

Thus four regimes arc seen in the present examplc:
i) initiation, where there is no prior damage;
ii) a reducing ctoa controlled by the prior damage caused by initiation and then, at each subsequent step, by the accumulation of the **(Icsscr)** damage further ahead of the original tip plus the **(lesser)** damage ahead of each successive smaller ctoa.
iii) **as** full plasticity (maximum load) is reached and slip hinges form, the extent of damage ahead is reduced to the width of the slip zone, **so** that each immediately prior step has a much smaller effect than in the contained yield regime where the damage from initiation and just after, still dominated. The summation of these two effects allows ctoa to increase, giving a rising transient after maximum load but before steady state.
iv) steady state is reached when growth has passed beyond the effect of damage at initiation and soon thereafter, so that each applied CTOA induces some damage for just (here) two steps ahead of the advancing tip, both applied and induced values soon becoming constant.

TWO EXAMPLES

An example exercise for three HY 130 dnb pieces arc shown, all S/W = 4, a(o)/W = 0.54.
A) the datum small piece, B = 20mm, W = 95mm; (b(o)/B ≈ 0.9)
B) a geometrically similar piece, B = 50mm, W = 95mm, (b(o)/B ≈ 0.9) two and a half fold larger, for which the results are to be predicted and
C) a thin, wide piece, B = 20mm, W = 95mm, (b(o)/B ≈ 2.2) for which the results are to be predicted.
For both cases B) and C) the remote body is modelled in plane stress. For case B), where the fracture, [5], is substantially flat, all the plastic zones throughout growth are modelled in plane strain. For case C), for which the fracture, [5], is slant, except at its start, only the estimated initial Irwin plastic zone (about 5mm) is modelled in plane strain.
Fig 2 gives the experimental load displacement diagrams for all three pieces and the computed results for pieces B) and C) after several iterations, starting from the experimental ctoa data for the small datum test as the only known data, other than the stress-strain curve for the material.
Fig.3 shows the J-R-curves for the three cases. It will be **seen** that curve B) is well below the datum curve A), the so called 'larger-lower' size effect. Curve C) is only marginally lower than curve A), the much lesser degree of plane strain offsetting most of the effect of larger width.
Fig.4, in which some of the computed values, for example, G and dw–(i) and dw–(ss), have been used to make the foregoing damage relationships numerate, shows three values of the ctoa for piece C), together with the ctoa for the datum piece A), this last from [3]:
i) the input to the computations (derived **as** just said by iteration from the datum test data),
ii) the 'output' **as** calculated from Eqn.4 where dJ~ is taken **as** the far-field integral value in the computations and a– is the term here called a–(applied),
iii) the present estimate based on the damage model **as** described above.
The experimental steady state lines for ctoa, from [5], are also shown, for the pieces **A)** and C).

The estimates ii) and iii) are in effect self-consistency checks on the whole micro-instability model and the interpretation of it via ctoa and its use in the fe computations, [1], in that they make use of algebraic relationships not themselves **part** of the fe model. They are also of use in assessing when the converged solution has been reached.

The reasons for the differnt shapes of the two curves in Fig.4, fully plastic datum A), and the wide piece C). For the datum case where initiation is very close to maximum load, the initiation and steady state damage processes practically swamp any transient regime other than the first rapid fall with small growth (as seen not only in the computations of [3] but also in the original infiltration studies of Garwood, [6]), whereas in piece C), all the above features play a **part.**

DISCUSSION AND CONCLUSIONS

Eqn.4 implies there will be a term dJ~/da and thus a rising J-type R curve (with the elastic component also added) whenever α~ exists. That will always be the *case* for ductile growth in real metals where at least some plasticity will always exist. Thus, even in plane strain and near lefm conditions, there will always be a rising R-curve in real materials.

For quasi-lefm cases, where plasticity is always well contained, the slope of the conventional J-R-curve may be quite shallow. It will only become appreciable as, or if, the ctoa increases with growth (although other factors such as the normalised load, L, and the elastic component will also be increasing). A quite marked increase in ctoa will occur only when, or if, full plasticity spreads across the remaining ligament, an event not always seen in design or operating cases.

For lower strength higher hardening metals, for which no relevant ctoa data are known, it is anticipated that the transient regime will be more extensive so that the steady state regime may not occur until well after maximum load, probably beyond much of the small growth test data in the literature. Size effects were reported for moderate growth by Hutchinson [7] in his computations on A533B or similar type of materials. That probably implies that the ctoa would follow the present trends although whether a steady state was reached, is not clear.

For small growth, if initiation and thus the damage ahead, were identical in two pieces and if the term here called the available α~ were invariant with growth, then the resulting α~(applied) and thus dJ~ would be the same for the two pieces. Such a pattern would no doubt be the regime called J-controlled growth. In fact, for the micro-instability model, the ctoa for initiation, **as well as** steady state, depends on the driving force G since G is f(configuration, *h*). That is unlikely to be identical in the two cases. *So* the values of α~(applied), and probably dJ~ from Eqn.4, will in principle, be different. However, it is not clear whether the other terms in Eqn.4 (such as L in dJ~) and of G to add in the elastic component, would increase **or** reduce the differences just stated and to within experimental uncertainty there may well be sizes or configurations for which these arguments leave J-R-curves for small growth, not very specimen size dependent.

The previously reported scaling of certain R-curves to an abscissa of Δa/b(o) now appears to be a consequence of the pattern of ctoa v *h*. The scaling factor varies between the transient and steady state regimes and a specific scale factor of Δa/b(o) has not been deduced algebraically.

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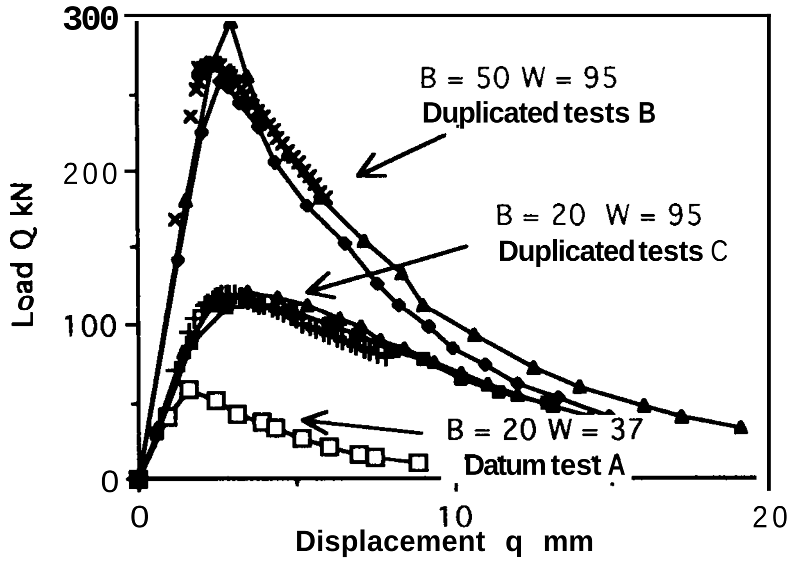

Fig.2 Load-displacement diagrams: lines. experimental: crosses. Predicted.

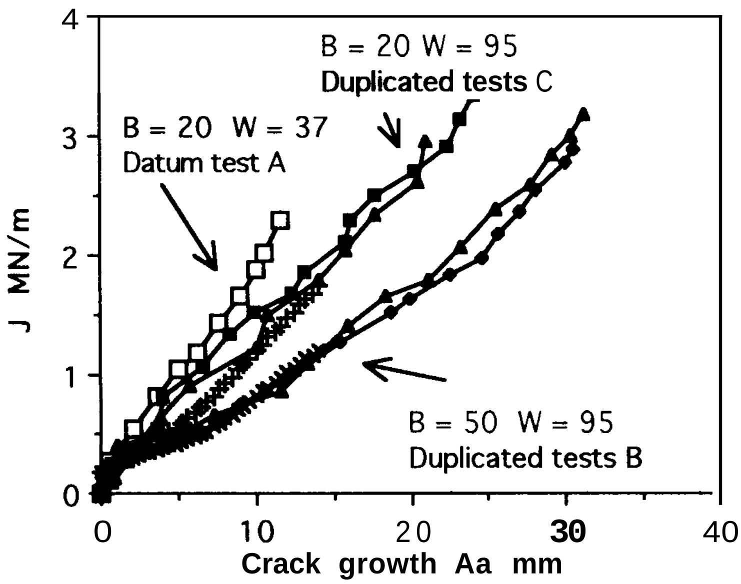

Fig.3 J-Aa curves: lines experimental: crosses. Predicted.

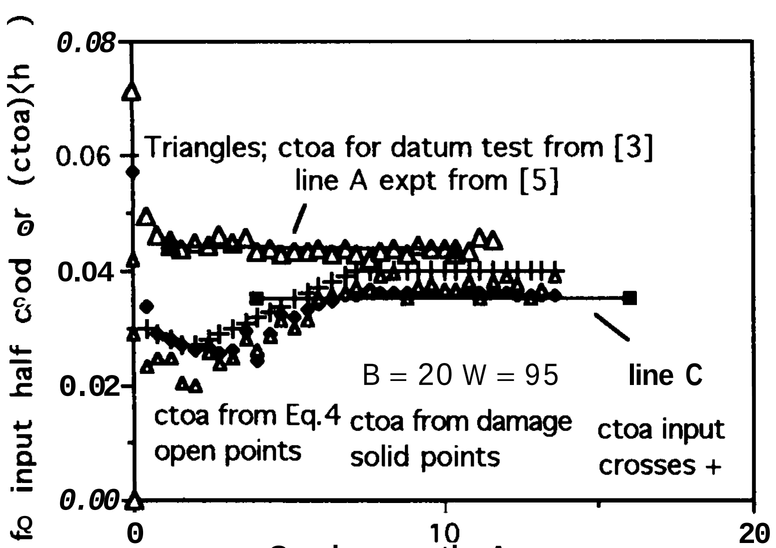

Fig 4. Variation of *ctoa* with growth: line A for the datum. line C for this experiment.