A REFERENCE STRESS BASED COD ESTIMATION IN LBB ANALYSIS

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ABSTRACT

Modifications to existing reference stress approximations for the crack opening displacement (COD) are reported for circumferential through-wall cracked pipes under tension and under bending. Three modifications are made: (i) re-defining the reference stress using an optimised reference load to minimise the dependence of the strain hardening and the geometry, (ii) simplification of the small-scale plasticity term, and (iii) use of a power law fit to stress strain data, instead of the actual data. Comparison of the CODs, predicted using the proposed method, with published pipe test data show overall excellent agreement.

INTRODUCTION

Estimation of the crack opening displacement/area (COD/COA) is important in Leak-before-Break (LbB) assessments, and several methods are available to estimate the COD for cracked components. Being calibrated from detailed finite element (FE) computations, the GE/EPRI method [1] certainly provides accurate estimates of the COD, but suffers from several limitations, including its limited applicability in terms of geometry and loading, and its possible sensitivity to the fitting parameters of tensile data. To overcome these limitations, Langston [2] has formulated a reference stress based COD estimation approach, which agrees overall with the GE/EPRI results. However, it has been reported that both the GE/EPRI method and Langston's method generally over-predict experimental pipe test data (and are therefore non-conservative in LbB assessments) [2,3]. Often such non-conservatism is excessive. Thus, a further improvement on the COD estimation method based on the reference stress approach is very important.

This paper reports modifications to existing reference stress based COD estimation equations for circumferential through-wall cracked pipes. The proposed COD estimation equations are validated against pipe test data.

PROPOSED REFERENCE STRESS APPROXIMATIONS FOR J AND COD

Proposed COD Estimation Equations

The proposed COD estimation equations are summarised below. Two different equations, the Option 1 and 2 curves, are proposed, to cope with the quality of the material's tensile data. The Option 2 curve can be used only when full stress strain data are available, particularly with well-defined data within 0.2% plastic strain.

On the other hand, when only limited data are available, such as the yield and tensile strengths, the Option 1 curve can be used, which is always more conservative than the Option 2 curve for LbB arguments.

Option 2 Curve

When full stress strain data are available, the COD (δ) can be estimated from its elastic contribution (δ_e) and the material properties:

$$\frac{\delta}{\delta_e} = \begin{cases} \dfrac{E\varepsilon_{ref}}{\sigma_{ref}} + \dfrac{1}{2}\dfrac{L_r^2 \sigma_{ref}}{E\varepsilon_{ref}} & \text{for} \quad 0 \le L_r \le 1 \\ \left(\dfrac{\delta}{\delta_e}\right)_{L_r=1} (L_r)^{n_1 - 1} & \text{for} \quad 1 < L_r \end{cases}, \tag{1}$$

where the reference stress, σ_{ref}, and the parameter to measure proximity of plastic collapse, L_r, are related to the yield stress, σ_y, and the primary load (axial load P or bending moment M) by

$$L_r = \frac{\sigma_{ref}}{\sigma_y} = \frac{P}{P_o^*} = \frac{M}{M_o^*} \tag{2}$$

The normalising load (P_o^* or M_o^*) in eqn. (2) is related to the limit load (moment) of the cracked cylinder, P_L and M_L, by (see Fig. 1 for relevant dimensions)

$$P_o^* = \gamma(\theta) P_L \quad ; \quad M_o^* = \gamma(\theta) M_L \quad ; \quad \gamma = 0.82 + 0.75\left(\frac{\theta}{\pi}\right) + 0.42\left(\frac{\theta}{\pi}\right)^2 \quad \text{for} \quad \theta/\pi \le 0.5 \tag{3}$$

$$\begin{cases} P_L = 2R_m t\sigma_y \left[\pi - \theta - 2\sin^{-1}\left(\dfrac{1}{2}\sin\theta\right)\right] & \text{for axial tension} \\ M_L = 4R_m^2 t\sigma_y \left[\cos\left(\dfrac{\theta}{2}\right) - \dfrac{\sin\theta}{2}\right] & \text{for bending} \end{cases} \tag{4}$$

In eqn. (1), ε_{ref} is uniaxial true strain at stress $L_r\sigma_y$, and $(\delta/\delta_e)_{Lr=1}$ denotes the value of (δ/δ_e) at L_r=1, calculated from the first expression in eqn. (1). The strain hardening index n_1 in eqn. (1) is calculated from

$$n_1 = \frac{\ln[(\varepsilon_{u,t} - \sigma_{u,t}/E)/0.002]}{\ln[\sigma_{u,t}/\sigma_y]} \tag{5}$$

where the true ultimate tensile stress and uniform elongation at the ultimate tensile strength σ_u, $\sigma_{u,t}$ and $\varepsilon_{u,t}$, respectively, can be found from σ_u and the uniform strain at σ_u, ε_u:

$$\sigma_{u,t} = (1+\varepsilon_u)\sigma_u \quad ; \quad \varepsilon_{u,t} = \ln(1+\varepsilon_u) \tag{6}$$

Option 1 Curve

When only yield and tensile strengths are available, the following lower bound curve can be used:

$$\frac{\delta}{\delta_e} = \begin{cases} 1 + \dfrac{1}{2}L_r^2 & \text{for} \quad L_r < 1 \\ \dfrac{3}{2}(L_r)^{n_2 - 1} & \text{for} \quad L_r \ge 1 \end{cases} \tag{7}$$

with n_2 estimated from the ratio of the yield to tensile strength, σ_y/σ_u:

$$\frac{1}{n_2} = 0.629 - 1.536\left(\frac{\sigma_y}{\sigma_u}\right) + 1.723\left(\frac{\sigma_y}{\sigma_u}\right)^2 - 0.814\left(\frac{\sigma_y}{\sigma_u}\right)^3 \tag{8}$$

Note that the first equation in the Option 1 curve, eqn. (7), is obtained from the first expression in eqn. (1), assuming linear-elastic behaviour, and thus is a degenerate case of Option 2. The Option 1 curve is always more conservative than the Option 2 curve for LbB arguments. Better understanding of the proposed COD estimation equations follows three noting points, which are described below.

Minimisation of Hardening and Geometry Dependence

A typical definition of the reference stress involves the plastic limit load of the cracked component [4-6]:

$$\sigma_{ref} = \frac{P}{P_L}\sigma_y = \frac{M}{M_L}\sigma_y \tag{9}$$

where the expressions for the limit loads, P_L and M_L, are given in eqn. (4) for circumferential through-wall cracked pipes under tension and under bending. Although the choice of the limit load can be convenient due to its wide availability, it does not necessarily provide the most accurate result [4,7]. Instead, one can define a load such that the plastic component of the J integral (or the COD) in the reference stress approach is close to that from the FE solutions, when they exist. This load will be called an optimised reference load, P_o* or M_o*. In the present work, such optimised reference loads are found for circumferentially cracked pipes under axial tension and under pure bending, using the GE/EPRI FE solutions [1]. The resulting optimised loads are given in eqn. (3). The use of the proposed P_o* solutions significantly reduces the hardening dependence of the plastic J and COD influence functions, h_1 and h_2 functions in the GE/EPRI solutions, which implies that the reference stress approach using the proposed P_o* solutions will provide accurate J and COD estimates. Details of this result can be found in [8]. However, this is true only when the material's tensile data are of exactly the Ramberg-Osgood type, which is the basis of the GE/EPRI approach. Comparison of existing reference stress based COD estimates [2,3] showed consistent over-predictions of the COD, suggesting that the choice of the normalised load is not the only reason. This suggests the necessity of further modifications for COD estimation, which are discussed in the next sub-sections.

Modification of Fully Plastic Term

Existing comparison of the GE/EPRI method and Langston's method with the test data [2,3] showed that both methods overall provide non-conservative results for LbB assessments (over-prediction of the experimental COD results) in the fully plastic regime. One fundamental reason why the use of the actual stress strain data (in the reference stress approach) or its best fit (in the GE/EPRI approach) can provide poor results for the COD estimation is as follows. The global parameters, such as the load, the load-line displacement and the J integral, of the defective component tend to follow the actual stress strain behaviour, if those parameters are normalised properly. Thus, when the actual stress strain data (or its best fit) is used, the estimated J integral will be satisfactory (or at least conservative). However, the COD is measured near the crack tip, and thus is a local parameter. The local strain level can be very different from the global strain level that roughly matches the global stress level. Therefore, the use of actual stress strain data can provide satisfactory results for J estimation, but poor results for COD estimation. Typical materials show a lower hardening rate for a lower strain range, which in turn is a possible source of non-conservatism for COD estimation, when either actual stress strain data or their best fit are used.

In this context, the proposed COD estimation equations assume that the plastic portion of the true stress-strain data is described by

$$\frac{\varepsilon}{\varepsilon_{p0.2}} = \left(\frac{\sigma}{\sigma_y}\right)^n \tag{10}$$

for $\sigma \geq \sigma_y$, where $\varepsilon_{p0.2}$ and σ_y denote the 0.2% proof strain and stress, respectively. Equation (5) is based on the fit between the 0.2% proof stress and the tensile strength [5]. Equation (8) is based on an empirical relationship between n and σ_y/σ_u [9].

Modification of Plasticity Correction Term

The GE/EPRI method employs the plasticity corrected crack length (a_e) to evaluate the COD under contained yielding. Although such a correction is important to cope with plasticity contributions below the widespread plasticity level, it involves cumbersome calculations, which hamper easy use of the GE/EPRI COD estimation equations. In this context, it would be practically useful to replace the plasticity–correction term with a simple term.

In the R6 Option 2 J estimation curve, the plasticity effect in the contained yielding regime is conservatively estimated by the simple term [5]:

$$\Omega = \frac{1}{2} \frac{L_r^2 \sigma_{ref}}{E \varepsilon_{ref}} \tag{11}$$

The constant of 2 in eqn. (11) comes from the plane stress condition, which provides conservative J estimates. Noting that pipes with circumferential part-through defects can be in between plane stress and plane strain conditions, the plane strain constant of 6, instead of 2, can be used for conservatism in COD estimation. However the value of 6 is valid only when the plasticity is confined within a dominantly elastically behaving body. The plasticity effect can be much more significant even below the widespread plasticity level. Moreover, the COD experiences a much enhanced plasticity effect, due to its local nature, as discussed in the above. Therefore, the plane strain assumption can be unduly conservative and eqn. (11) is retained for the proposed COD estimation equation, see the first equation in eqn. (1).

In the literature, two models have been proposed to estimate the influence of plastic deformation before widespread plasticity, the Irwin-type correction model due to Kastner *et al.* [10], and the Dugdale-type correction model due to Wüthrich [11]. The proposed equation, eqn. (11), agrees well with both the Irwin and Dugdale-type correction models for typical materials. The proposed plasticity correction, eqn. (11), is also supported by experimental pipe test data, as will be shown in the next section.

A final note is that continuously hardening materials show a smooth transition from elastic to plastic hardening behaviour. For such materials, the plastic behaviour within the 0.2% plastic strain also contributes (often significantly) to the plasticity effect in the contained yielding regime.

EXPERIMENTAL VALIDATION OF COD ESTIMATION EQUATIONS

The pipe test data are taken from two sources [3, 12], and are summarised in Table 1. The calculated results are compared with pipe test data in Figs. 2 to 8, each of which shows COD as a function of applied moment M, axial load N, or pressure P. In each case, two predictions are shown. One prediction uses full stress strain data, based on eqn. (1), which is termed the reference Option 2 curve ("Ref. Opt 2"). The other prediction is according to eqn. (7), assuming that only yield and tensile strengths are available, and is termed the reference Option 1 curve ("Ref. Opt. 1"). Some figures also include the predictions according to the GE/EPRI approach, which are simply reproduced from [3]. Note that the GE/EPRI results are based on the best Ramberg-Osgood fit based on full stress strain data.

The results in Figs. 2-8 firstly show the expected order for the two different levels of present predictions: the prediction based on limited tensile data (Ref. Opt 1) is always more conservative for LbB arguments, than that based on full stress strain data (Ref. Opt 2). It can be clearly seen that the present predictions are much better than or at least similar to those based on the GE/EPRI method, and are overall in excellent agreement.

CONCLUSIONS

Modification to the reference stress approximation for the COD are reported in this paper, for circumferential through-wall cracked pipes under tension and under bending. Three modifications include:

- re-defining the reference stress using an optimised reference load to minimise the dependence of the strain hardening and the geometry;
- simplification of the small-scale plasticity term; and
- use of a power law fit to stress strain data, instead of the actual data.

A lower bound COD estimation equation is also given, similar to the R6 Option 1 J estimation curve, which is suitable when only limited tensile properties are available. The resulting estimation equations are simple to use. Comparisons with experimental pipe test data show that the proposed COD estimation equations provide overall good agreement, which gives confidence in applying them to LbB analyses.

Being based on the reference stress approach, the proposed COD estimation methodology can be easily extended to more general classes of problems. Possible extensions include COD estimation under combined bending and tension; in welds (strength mismatch effect); for components operating in high temperature (creep); for axial cracks. Further development will be reported later.

ACKNOWLEDGEMENT
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Table 1. Summary of pipe test data.

Loading	Test #	Material	$D_o=2R_o$	t	R_m/t	θ/π	Temp (^{o}C)
Pure Bend	GE/1/B	304SS	4.5 (in)	0.34 (in)	6.12	0.25	20
	GE/3/B	304SS	4.5 (in)	0.34 (in)	6.12	0.5	20
	NRC/4111/1	A333Gr6	4.5 (in)	0.35 (in)	5.93	0.37	288
	4.3-1*	STS-49	763.5(mm)	38.2(mm)	9.5	0.166	300
	3.3-1*	STS410	166.0(mm)	14.5(mm)	5.22	0.166	300
Tension	GE/3/90/T	304SS	4.5 (in)	0.34 (in)	6.12	0.25	20
Pressure**	4121-1*	304SS	168.1(mm)	12.9(mm)	6.02	0.386	288

* These data are extracted from [12]. All other data are from [3].
** The pressure loading is transformed into the equivalent tension loading.

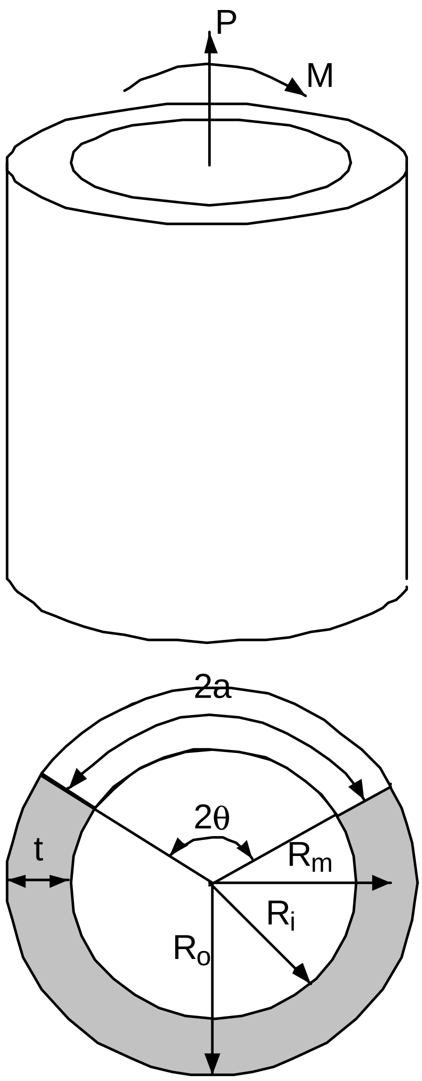

Fig. 1. Circumferential through-wall cracked pipes under axial tension and under bending.

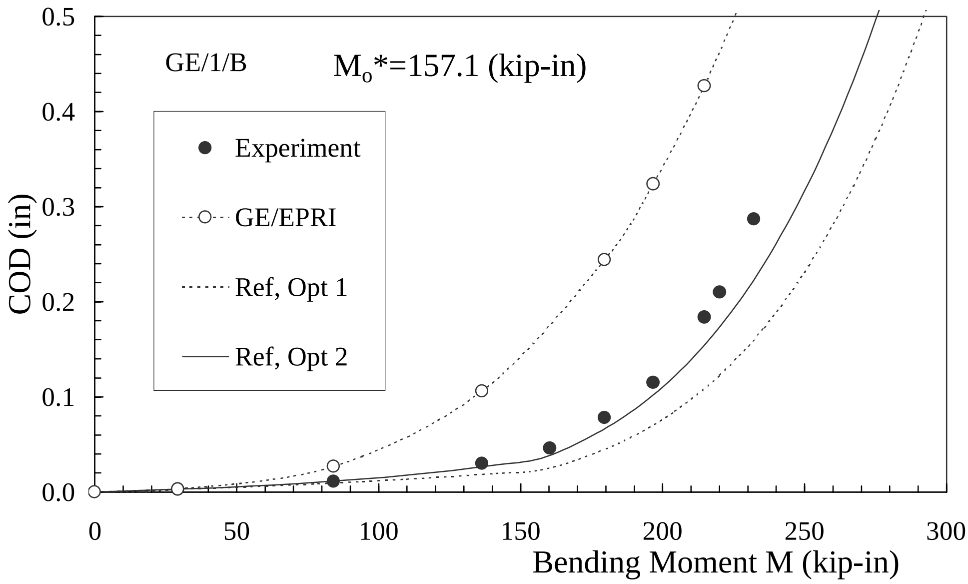

Fig. 2. Comparison of the COD predictions with pipe test data, GE/1/B (see Table 1).

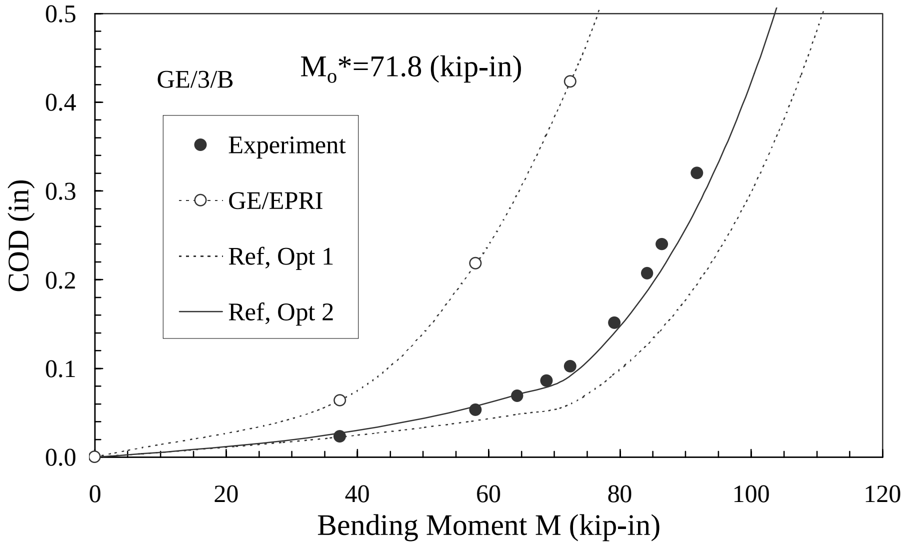

Fig. 3. Comparison of the COD predictions with pipe test data, GE/3/B (see Table 1).

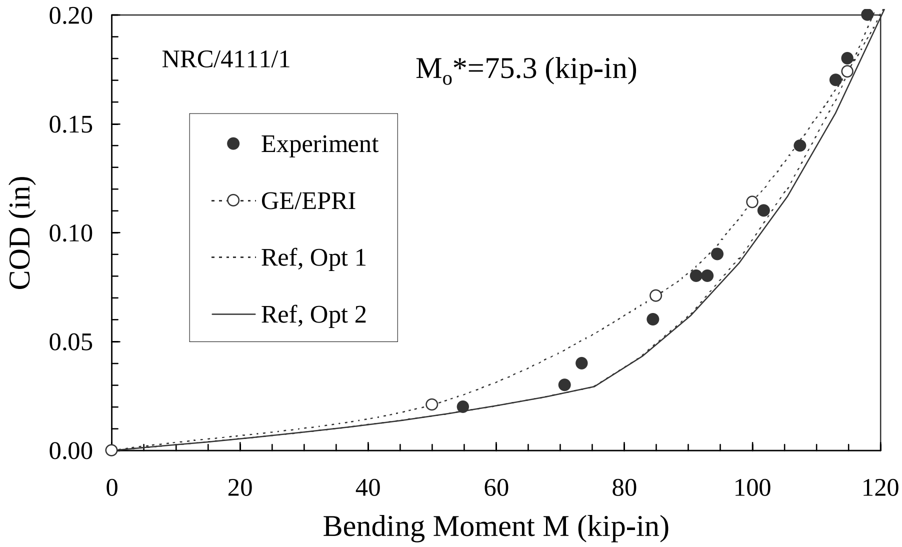

Fig. 4. Comparison of the COD predictions with pipe test data, NRC/4111/1 (see Table 1).

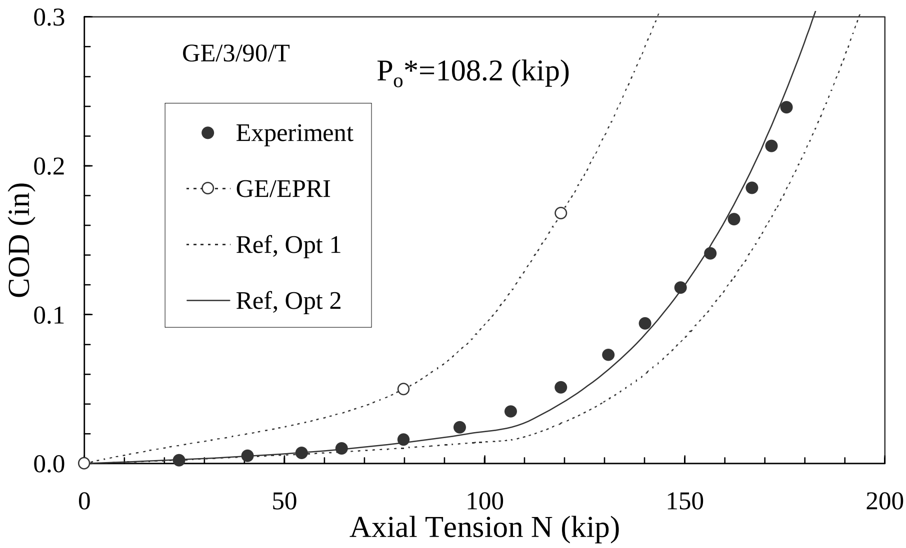

Fig. 5. Comparison of the COD predictions with pipe test data, GE/3/90/T (see Table 1).

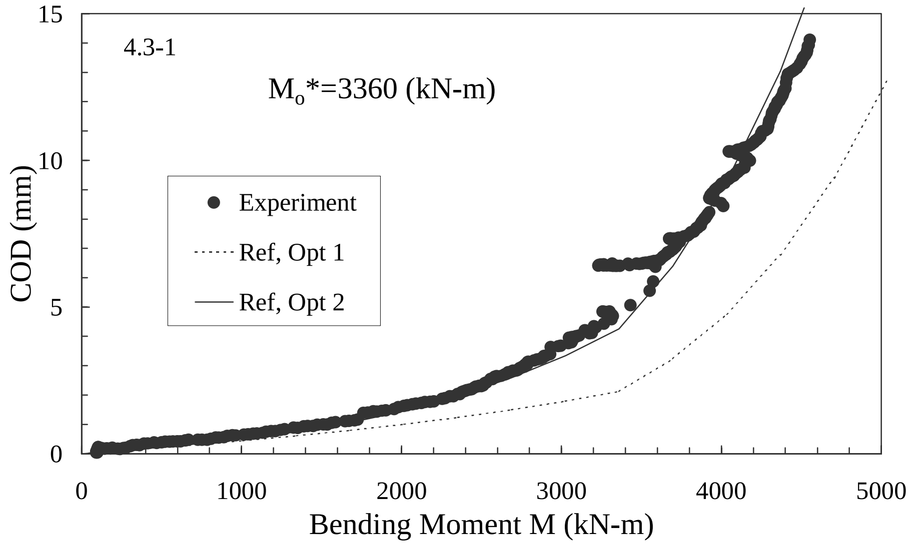

Fig. 6. Comparison of the COD predictions with pipe test data, 4.3-1 (see Table 1).

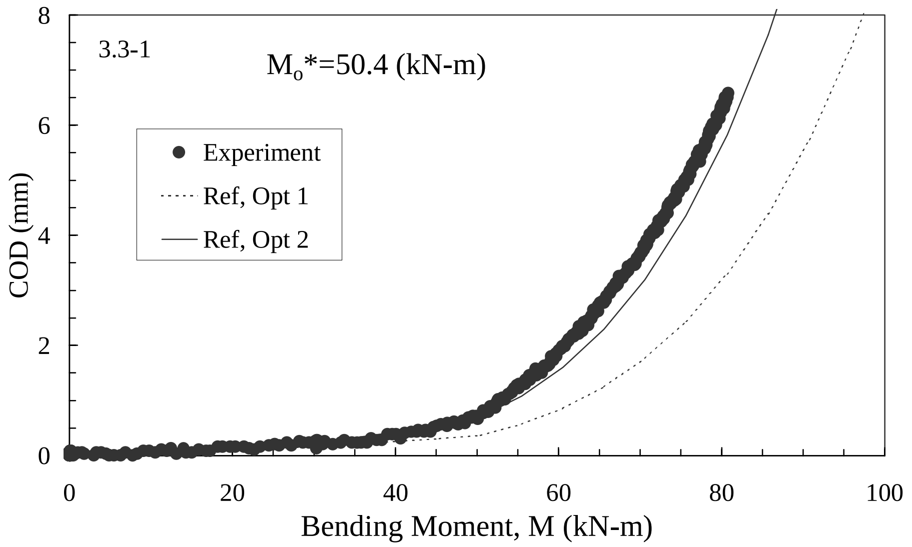

Fig. 7. Comparison of the COD predictions with pipe test data, 3.3-1 (see Table 1).

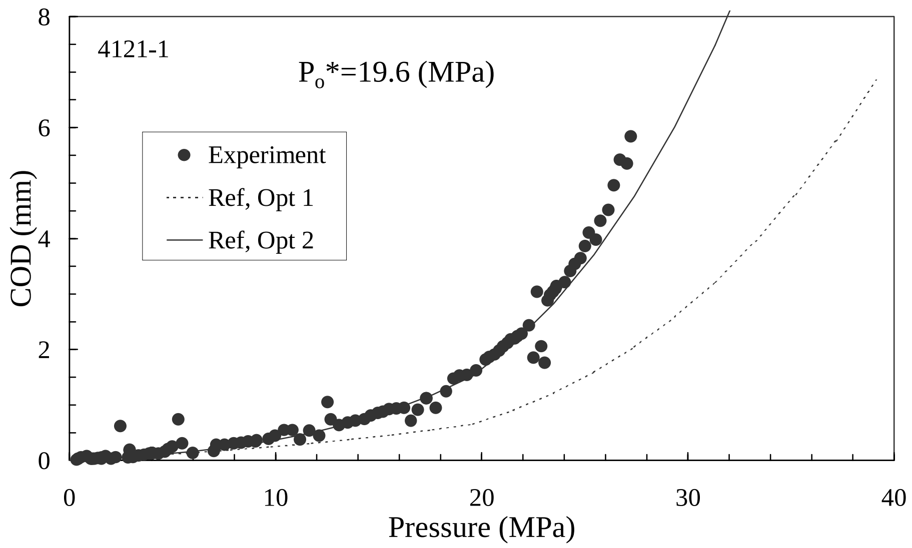

Fig. 8. Comparison of the COD predictions with pipe test data, 4121-1 (see Table 1).