

THERMOMECHANICAL APPROACH OF RUNNING  
DISCONTINUITIES

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The propagation of moving surface inside a body is analyzed within the framework of thermomechanical couplings, when the moving surface is associated with an irreversible change of mechanical properties. The moving surface is a surface of heat sources and of entropy production, intensities of which are related to particular energy release rates defined in terms of Hamiltonian gradients. We analyze successively the evolution of partial damage and the dynamical propagation of cracks.

INTRODUCTION

In the recent past, the propagation of damage has been studied in connection with fracture mechanics. Different approaches based on macroscopic or microscopic description of mechanical degradation properties have been proposed. In fracture mechanics, thermodynamical force associated with crack propagation under the assumption of isothermal evolution was obtained through energy release rate given by the crack front [6]. Within the framework of thermomechanical couplings, the same analysis defines two different energy release rate associated with heat production and entropy production [4]. This paper is concerned mostly with a description of damage involved on the evolution of a moving interface along which mechanical transformation occurs. It turns out the discontinuities of mechanical fields and the interface behavior under dynamic loading. In the following development, we consider the evolution of a moving interface along which mechanical transformation occurs. Partial damage is considered [7, 3]. The dynamical evolution is analyzed in an Hamiltonian formulation. In connection with an energetic analysis, the evolution of the surface is then characterized. The moving surface is a surface of heat sources and of entropy production, the intensities of which are defined in terms of Hamiltonian

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gradient. Connections in terms of energy release rate are then investigated.

### GENERAL FEATURES

This paper deals with partial localized damage. At each time, two materials coexist in the structure and the structure is heterogeneous.

$\Omega_t$  denotes the domain, composed of two distinct volumes  $\Omega_1$  and  $\Omega_2$  which are occupied by two materials with different mechanical characteristics. The perfect interface between them is assumed to be a regular surface and is denoted by  $\Gamma_t$ . Material 1 changes into material 2, along  $\Gamma_t$ , by an irreversible process. Hence,  $\Gamma_t$  moves with a normal celerity  $c = c\vec{\nu}$ , ( $\vec{\nu}$  normal outward to  $\Omega_2$ ,  $\phi$  positive along  $\Gamma_t$ ).

The actual state is characterized by the displacement field  $\vec{u}$ , depending on time  $t$  and position  $x$ . It is associated to the strain field  $\epsilon = \frac{1}{2}(\nabla\vec{u} + {}^T\nabla\vec{u})$ , the internal variables  $\alpha$  (which could represent plastic deformation, a volume damage parameter, etc), the temperature  $T$  and the position of the interface  $\Gamma_t$ .  $\rho$  denotes the volume mass. When the surface  $\Gamma_t$  propagates, mechanical quantities can have a jump, ( $[[f]]_{\Gamma} = f_1 - f_2$ ) and all volume integral has a rate defined by :

$$\frac{d}{dt} \int_{\Omega_t} f d\omega = \int_{\Omega_t} \dot{f} d\omega - \int_{\Gamma_t} [[f]]_{\Gamma} c \cdot \vec{N} da. \quad (1)$$

The total potential energy of the structure has the following form:

$$E^{pot}(\vec{u}, \alpha, T, \Gamma_t, \lambda) = \int_{\Omega_1} w_1 d\omega + \int_{\Omega_2} w_2 d\omega - \int_{\partial\Omega_T} \lambda \vec{T} \vec{u} da, \quad (2)$$

where  $w_i$  denotes the density of the free energy in the domain  $\Omega_i$  and is a function of  $\alpha$ ,  $\epsilon$  and  $T$ .  $\sigma$  denotes the stress tensor field that is thermodynamically associated with  $\epsilon$ :  $\sigma = \frac{\partial w_i}{\partial \epsilon}$ , while  $\mathbf{A}$  is the thermodynamic force associated with  $\alpha$ :  $\mathbf{A} = -\frac{\partial w_i}{\partial \alpha}$ .

It is important to point out that the potential energy can play the rule of the global free energy in a thermodynamic description ; in this global description, the position of the interface  $\Gamma_t$  becomes an internal variable.  $s$  denotes the volume entropy and  $e = w + sT$  the internal energy per unity of volume associated to an element of  $\Omega_t$ .

### DISSIPATION due to PARTIAL LOCALIZED DAMAGE

The mass conservation leads to  $m = \rho c \cdot \vec{\nu} = \rho \dot{\phi}$  which is continuous along  $\Gamma_t$ . The first law and of the second law of thermodynamics give rise to local equations inside the volume and along the moving surface  $\Gamma_t$  :

$$\text{In } \Omega_t, \rho \dot{e} = \sigma : \dot{\epsilon} - \text{div} \vec{q}; \text{ on } \Gamma_t, m[[e + \frac{1}{2}\vec{v}^2]]_{\Gamma} + [[\vec{\nu} \cdot \sigma \cdot \vec{\nu}]]_{\Gamma} - [[\vec{q} \cdot \vec{\nu}]]_{\Gamma} = 0. \quad (3)$$

The entropy production is given by :

$$\int_{\Omega_t} \{(\rho \dot{s} + \frac{1}{T} \text{div} \vec{q}) - \vec{q} \cdot \frac{\nabla T}{T^2}\} d\omega + \int_{\Gamma} (-m[[s]]_{\Gamma} + [[\frac{\vec{q}}{T} \cdot \vec{\nu}]]_{\Gamma}) da \geq 0. \quad (4)$$

In the previous formula, only real evolutions are considered and  $-\vec{q} \cdot \vec{n}$  represents heat flow received by  $\Omega_t$  on  $\partial\Omega_t$ .

Assuming that contact is perfect through  $\Gamma_t$ ,  $[[\vec{u}]]_{\Gamma_t} = \vec{0}$  and  $[[T]]_{\Gamma_t} = 0$ . Under the assumption of the separability of the two terms, one gets the following expressions for the volume dissipation and the dissipation along  $\Gamma_t$  :

$$\mathcal{D}_{\Omega_t} = \frac{\mathbf{A} \cdot \dot{\alpha}}{T} - \frac{1}{T^2} \vec{q} \cdot \nabla T, \quad \forall x \in \Omega_t, \quad (5)$$

$$\mathcal{D}_{\Gamma_t} = \frac{1}{T} (\rho [[w]]_{\Gamma_t} c \vec{v} - \vec{v} \cdot \vec{\sigma} \cdot [[\nabla \vec{u}]]_{\Gamma_t} \cdot c) = \frac{G_s}{T} \phi, \quad \forall x \in \Gamma_t. \quad (6)$$

In the previous formula,  $\sigma = \frac{1}{2}(\sigma_1 + \sigma_2)$ .

#### HAMILTONIAN FORMALISM

The total Hamiltonian of the structure is :

$$H = \int_{\Omega_t} \frac{1}{2} \rho \vec{v} \cdot \vec{v} d\omega + E^{pot}(\vec{u}, \alpha, T, \Gamma_t, \lambda) + \int_{\Omega_t} \rho s T d\omega.$$

Momentum conservation : Dynamic equations are obtained thanks to :

$$\frac{\partial H}{\partial \vec{p}} \delta \vec{p} = \int_{\Omega_t} \dot{\vec{u}} \cdot \delta \vec{p} d\omega \quad \text{and} \quad \frac{\partial H}{\partial \vec{u}} \delta \vec{u} = - \frac{d}{dt} \left( \int_{\Omega_t} \vec{p} \cdot \delta \vec{u} d\omega \right), \quad (7)$$

where  $\vec{p} = \rho \vec{u}$  is the impulsion,  $\delta \vec{p}$  is continuous on  $\Omega_1$  and  $\Omega_2$ ,  $\delta \vec{u}$  is continuous on  $\Omega_t$  and differentiable on  $\Omega_1$  and  $\Omega_2$ . These equations lead to the classical equations in  $\Omega_t$  and on the moving surface  $\Gamma_t$ .

First and second principles : Using results from [9], they can be written as :

$$\frac{dH}{dt} - \frac{\partial H}{\partial \lambda} \dot{\lambda} = \mathcal{P}_{cal} = \int_{\partial\Omega_t} (-\vec{q} \cdot \vec{n}) da \quad \text{and} \quad \frac{d}{dt} \int_{\Omega_t} \rho s da + \int_{\partial\Omega_t} \frac{\vec{q} \cdot \vec{n}}{T} ds \geq 0. \quad (8)$$

So, the power heat  $\mathcal{P}_{cal}$  is constituted by two terms. The first one equals  $\frac{\partial H}{\partial \alpha} \dot{\alpha} = - \int_{\Omega_t} \mathbf{A} \dot{\alpha} d\omega$  is associated to the dissipation of internal variables all over the domain. The second one is  $\frac{\partial H}{\partial \Gamma_t} \dot{\Gamma}_t = - \int_{\Gamma_t} G_{th} \phi ds$ .  $G_{th}$  corresponds to an energy release rate including the dynamical effects.

$$G_{th} = \rho [[\epsilon]]_{\Gamma_t} - \vec{v} \cdot \vec{\sigma} \cdot [[\nabla \vec{u}]]_{\Gamma_t} \cdot \vec{v}. \quad (9)$$

The second principle gives the evolution of entropy and under the assumption of the separability of the different contributions, one gets :

$$- \int_{\Gamma_t} \left\{ \rho [[s]]_{\Gamma_t} \phi + [[\frac{\vec{q} \cdot \vec{v}}{T}]]_{\Gamma_t} \right\} ds = \int_{\Gamma_t} \frac{G_{th}}{T} \phi ds, \quad (10)$$

ISOTHERMAL EVOLUTION

When the temperature is assumed to be constant, a pseudo-potential is introduced and denoted by  $\hat{H}$ .

$$\hat{H} = E^e(\dot{u}, \Gamma_t) + E^{pot}(\vec{u}, \alpha, \Gamma_t, \lambda). \quad (11)$$

As previously, the total dissipation is :

$$\frac{d\hat{H}}{dt} - \frac{\partial \hat{H}}{\partial \lambda} \dot{\lambda} = \frac{\partial \hat{H}}{\partial \alpha} \dot{\alpha} - \int_V \hat{G}_{dyn} \phi ds \text{ with } \hat{G}_{dyn} \phi = \rho [W]_{\Gamma} \phi - [[\vec{v} \cdot \sigma \cdot \vec{u}]]_{\Gamma}. \quad (12)$$

The dynamic energy release rate equals the derivative of the Hamiltonian with respect to the evolution of  $\Gamma_t$ . The result is equivalent to what has been obtained by [1].

CONCLUSION

The Hamiltonian approach shows the link between the evolution of the moving interface and the variation of global energy and entropy. It is to be underlined that the heat source due to the propagation is different from the production of entropy. If we consider isothermal evolution, this result is the dynamical expression of the well-known quasistatic version : the quasistatic potential energy is replaced by the isothermal Hamiltonian.

In the same spirit, the analysis of the propagation of crack is investigated thanks to the transport of the singularity hypothesis of [6]. Classical results on crack propagation [2] of the propagation of cracks are then recovered.

This description of phenomena can also be applied to the evolution of delaminated structures modeled by an assemblage of beams or plates [8].

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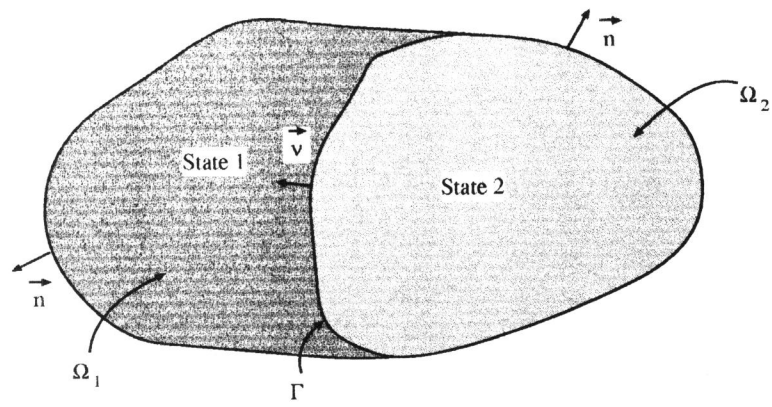


Figure 1: Localized partial damage