THE PHYSICAL EVALUATION OF THE T^* INTEGRAL

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Until now, the evaluation of T^* has been a matter for computation. It has recently become possible to contemplate the measurement of T^* experimentally from an analysis of the crack-tip strain fields. A strategy is proposed for complying with these limitations, to yield an accurate assessment of T^*. This strategy consisted of measuring T^*, under conditions of limited crack-growth, from the strain field components around the crack, using moire interferometry to visualise the deformation field. The experimentally derived value of T^* was validated by comparison with values of J which were also derived from the strain field. Under the conditions of the test, the values of T^* and J were found to be equal, within the overall accuracy of the measurements, in accordance with theoretical prediction.

INTRODUCTION

This investigation concerns the experimental measurement of T^*, and the difficulties that are encountered in the process. The origins of these difficulties may be shown to lie in the nature of T^* itself.

The Basic Definition of T*

Now, when these inadequacies of J as a parameter became clear, a new family of similar integrals was proposed. Of these, T^* [Nishioka etal,1983,Brust etal,1985] was designed to cope with general loading conditions, where

$$T^* = \int_O^t T^* \, dt = \Sigma \Delta T^* \tag{1}$$

with

$$\Delta T^* = \int_{\Gamma_\varepsilon} \left((\Delta W + \Delta K)\delta_{1j} - (\sigma_{ij} + \Delta U_{i,1} - \Delta\sigma_{ij} U_{i,1}) n_j ds \right) \tag{2}$$

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Σ denotes the summation along the loading history;

$$\Delta W = \int_{\varepsilon_{ij}}^{\varepsilon_{ij}+\Delta\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \sim \left(\sigma_{ij} + \tfrac{1}{2}\Delta\sigma_{ij}\right)\Delta\varepsilon_{ij} \tag{3}$$

is the increment of stress working density, and ΔK is the increment of kinetic energy density; the σ_{ij} are the components of stress.

The Measurement of T*

If one is to measure T^*, the basic formulation may be recast into a form containing only field parameters, which may be measured or calculated from a known material response,viz.:-

$$\Delta T^* = \int((\Delta W + \Delta K)\delta_{ij} - (\sigma_{ij} + \Delta\sigma_{ij})\Delta u_{i,1} - \Delta\sigma_{ij} u_{i,1})n_j ds$$

$$-\int\int_{v-v_\varepsilon}\left(\Delta K_{,1} + \Delta\varepsilon_{ij}\left(\sigma_{ij} + \frac{1}{2}\Delta\sigma_{ij}\right)_{,1} - \Delta\sigma_{ij}\left(\varepsilon_{ij} + \frac{1}{2}\Delta\varepsilon_{ij}\right)_{,1}\right)dv \tag{4}$$

This expression may be calculated from the strain fields only, as long as the material constitutive relations are known. (Fig. 1)

Basically, then, one may discern the beginning of a strategy for measuring T^*, on the basis that one may measure the surface strains around the crack at each increment of loading, and if the appropriate material response is known

Measurement Strategy

In the first instance, under conditions of modest crack growth, the values of J and T^* should be the same (Nishioka etal,1983).

Experimental Techniques

The material and specimen configuration used for the evaluation of T^* was the same as those used for the evaluation of the J-integral by MacKenzie et al[1986]. This choice was made in order to facilitate exact comparisons with the previous data.

The material chosen was Aluminium alloy 2618:(see Fig 2). After fatigue pre-cracking, a 500 line mm^{-1}, bi-directional grating was formed on the side of the specimen in the area around the crack tip.[Walker,1988] The specimen was then loaded, stagewise, through the elastic regime into elastic-plastic loading. At each stage, the loading ramp was halted, and a series of three interferometric moiré deformation fields were recorded , one each of the 0^o-, 45^o-, and 90^o fields, relative to the plane of the crack (fringe sensitivity 1.05 x 10^{-3} mm). By recording the three fields, the complete strain tensor field may be reconstructed over the area of interest . One typical interferogram is shown in Figure 3.

PROCEDURE

For the computation of T^*, the path described in Figure 4 was analysed. At each load point, the data consisted of three pictures: one for the 0°-field, one for the 90°-field, and the last one for the 45°-field. The far-field path was the same of each set of pictures, but the small path around the crack tip was allowed to grow at the same rate as the crack itself.. The results are presented in Table 1:

All results in (KN/m)	Load = 2.9kN (1st load point)	Load = 2.9kN (2nd load point)	Load = 2.9kN (3rd load point)
T*	6.36	7.83	10.12

Table 1

Experimental Measurements of T* as a Function of Load

Assessment of the measurement of T^*

As was discussed earlier, these measurements may be regarded as an accurate estimation of T^*, since the routines are based upon those developed for the measurement of J. In order to validate further these measurements, the experimental values were compared with values of J derived from the same pictures.

First of all, the validity of the $\mathbf{T}^*$ analysis program was checked. In order to achieve this, the data already entered was analysed with the program written by MacKenzie

[6] for the evaluation of J, with the contribution of the path to T^* in the total form (and not incremental form) and without taking any plasticity into account.

The results obtained were as follows:

All results in KN/m (error about +/-5%)	Load = 2.9kN (first load point)	Load = 2.9kN (second load point)	Load = 2.9kN (third load point)
J evaluation (program in ref 6)	6.91 +/- 0.34	8.70 +/- 0.43	11.31 +/- 0.56
T^* evaluation in total form (no plasticity)	6.54 +/- 0.33	7.99 +/- 0.40	10.32 +/- 0.52

Table 2

Comparison of the Experimental values of the J-integral and T*

It can be seen that those results are within the estimated measurement errors. The slight differences observed may be imputed, first of all, to an imprecision in the digitisation of distances; secondly to the interpolation used for T^* evaluation.The similarity of those values is a cogent proof of the validity of the T^* analysis program,and indicates that one may proceed with a degree of confidence.

CONCLUSION

Values of T^* have been derived from experimental measurement of the crack-tip strain fields using moiré interferometry. The values of T*, under conditions of low levels of plasticity and limited crack growth, were found to be the same as the values of J as demanded by the basic definitions of both integrals.The nature of the T* integral, has been discussed, and from this, a strategy for measurement and validation has been proposed and justified by the experimental measurements.It will be readily appreciated that this basic methodology may be applied to conditions of extended plasticity, crack growth, time-dependent plasticity and post-yield elastic unloading. While each of these will require an element of separate validation, the first step has been taken towards a generalised method for the experimental measurement of crack-tip integrals, whether they be T* or any other integral which depends upon the strain field components around the crack tip.

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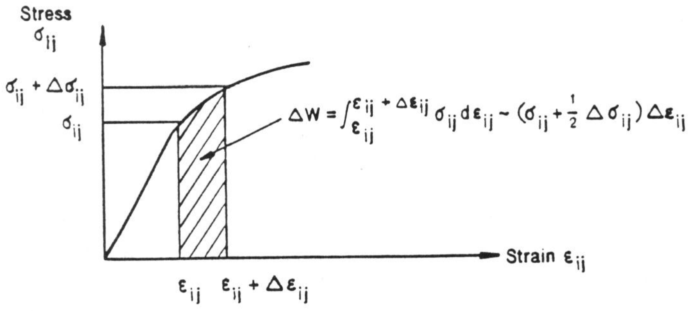
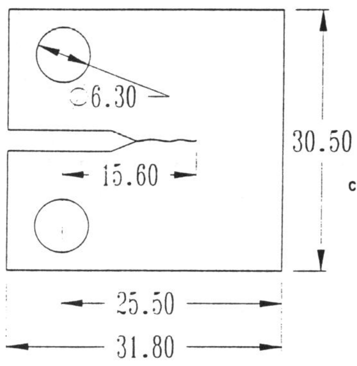
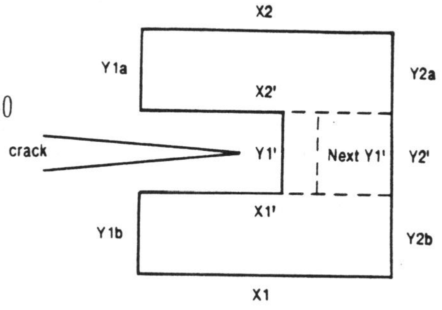


Figure 1 : Variation of W with Progress Along the Stress-Strain Curve

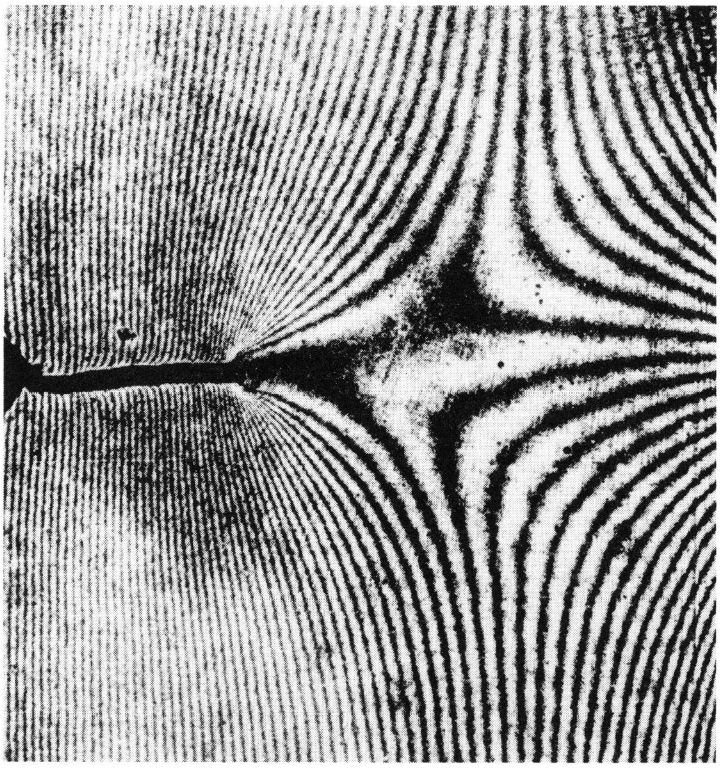




Aluminium Alloy 2618 Thickness 12.6mm

Figure 2 : Specimen Dimensions

Figure 4 : Schematic Path actually used for the T* measurements



U_y
$\Uparrow \Rightarrow U_x$

Figure 3 : Interferogram of the U_y-Field Deformation Component (sensitivity -1.05x10^{-3} mm per fringe ; magnification x 8)