THE LOSS OF STABILITY AND FAILURE OF PRE-STRSSED CONSTRUCTIONS UNDER TEMPERATURE EFFECT

S. Alexandrov* and N. Chikanova#

The structure consisting of an infinite plate and circular inclusion which is inserted under a forced-fit condition is considered It is assumed that the material of the plate and inclusion is not the same. Moreover, the material of the inclusion is harder than that of the plate. The structure is subjected to the homogeneous monotonically increasing temperature field. At a sufficiently large value of the temperature a plastic zone begins to form at the inner surface of the plate and moves inwards with the increasing temperature. In the plastic zone the J_2 flow theory of plasticity is used. Because the plate and inclusion are thin they were taken to be in a state of plane stress. An application of the solution obtained to the problem of instability in the inclusion is considered. Another applications of the solution to the failure of the structure are discussed.

INTRODUCTION

With the extensive application of pre-stressed techniques comes the requirement for both analytical and numerical evaluation of the stress-strain state of structures under different service conditions. These solutions are very important for more or less exact prediction of the subsequent behavior of the structure. A plate with a circular hole or inclusion is a structure which is often used in practice. The theoretical investigation of the elastic-plastic behavior of tubes, subject to various end conditions, has been treated at great length in many papers and books (for example, Hill (1) and Johnson and Mellor (2)). In these works the Tresca and von Mises yield criteria have been used. The analytical solutions using the Tresca yield condition have been obtained. For the von Mises yield condition some numerical technique based on the characteristic method has been proposed in (1) to find the stress-strain state in the expansion tube for an arbitrary but constant axial deformation. In all of these studies the stress boundary conditions at the inner radius of the tube often led to a statically determinate problem. However, in most expansion problems it is not the pressure but rather the interference between the hole and the inclusion (disk) that is known. As a result, it is necessary to define the relationship between interface pressure and level of

* Institute for Problems in Mechanics, Russian Academy of Science
(Presently: Visiting scientist, GKSS-Research Center, Germany)
Departement of Applied Mechanics, Moscow State Technical University

interference. Because of this, no statically determinate problem is possible and the joint solution of static and kinematic equations is required. The general approach to these class of problems in the frame of deformation theory of plasticity was given by Potter and Ting (3). The review and detailed analysis to the problem of interference of the plate and circular inclusion using deformation theory of plasticity were done by Ball (4). In the frame of incremental theory of plasticity there is a great number of results for the material obeying the Tresca yield condition (Guven (5), Guven (6), Lippmann (7), Mark (8), Bengeri and Mark (9) and Mark and Bengeri (10)). For this condition, as it is noted in (7), upon the assumption of plane stress the analysis becomes comparatively simple. Because of this, the analytical solutions to elasto-thermo-plastic problems were found in (7), (8), (9) and (10). In contrast, for the material obeying the von Mises yield criterion the various numerical procedures were proposed by Tuba (11) and Orr and Brown (12). As the strain-stress state in the structure is important not only by itself but rather as the basis for subsequent investigations such as stability analysis, determination of the crack and damage initiation conditions and so on then it is worth to find the distribution of the stresses and strains by an analitical way. In the paper we present some solution to the problem of interference of an infinite elastic-perfectly plastic plate with an elastic circular inclusion subject to a homogeneous changing temperature field. The solution requires only some numerical treatments of an ordinary differential equation to obtain the distribution of stress in the plate with respect to the polar radius and time as well as the dependence of stresses and strains in the inclusion on the time. The solution is applied to the problem of loss of stability in the inclusion.

STATEMENT OF THE PROBLEM

Consider an infinite plate with a circular hole in which a smooth circular disk is inserted under a forced-fit condition. The material of the plate is softer than the material of the disk and at the moment of insertion both are in elastic state. Then, the plate with the inclusion is subjected to the homogeneous temperature field which is described as monotonically increasing function of time, t. When the contact pressure reaches certain magnitude a plastic zone arises and develops around the perimeter of the hole. The solution is based on the assumption of small strains in two dimensional plane stress.We define the total strains as the sum of elastic, thermal and plastic components

$$e_r = e_r^e + e_r^T + e_r^p, \quad e_\theta = e_\theta^e + e_\theta^T + e_\theta^p, \quad e_z = e_z^e + e_z^T + e_z^p \qquad (1)$$

In the cylindrical coordinate system there are only the components σ_r and σ_θ that are not equal to zero, therefore, the von Mises yield criterion has the form

$$s_r^2 + \sigma^2 - \sigma s_r = k^2 \qquad (2)$$

From thermo-elasticity

$$\xi_r^e = s_r/2\mu, \quad \xi_\theta^e = s_\theta/2\mu, \quad \xi_z^e = s_z/2\mu \quad e = \sigma/3K, \quad e^T = \alpha(T - T_0) \qquad (3)$$

where s_r, s_θ are the components of stress deviator, σ is the hydrostatic stress, k is the shear yield stress, ξ_r, ξ_θ, ξ_z are the components of strain deviator, $3e$ is the volume expansion, μ is the shear modulus of elasticity , K is the bulk modulus, α is the linear coefficient of thermal expansion, T is the current temperature, T_0 is the initial temperature.The associated flow rule with the von Mises yield condition gives

$$\dot{\xi}_r^p = \dot{\lambda} s_r, \quad \dot{\xi}_\theta^p = \dot{\lambda} s_\theta, \quad \dot{\xi}_z^p = \dot{\lambda} s_z, \quad \dot{\lambda} \ge 0 \qquad (4)$$

where the dots denote the differentiation with respect to time. The non-trivial equilibrium equation can be written in the form

$$\partial\sigma_r/\partial r + (\sigma_r - \sigma_\theta)/r = 0 \quad \text{.........(5)}$$

The solution of set (1)-(5) must satisfy the following boundary conditions

$u_r = 0$ at $r = 0$,

$$|u_r| < \infty, \quad |\sigma_r| < \infty, \quad |\sigma_\theta| < \infty \quad \text{at} \quad r \to \infty \quad \text{.........(6)}$$

$$u_r - \hat{u}_r = \Delta, \quad \sigma_r - \hat{\sigma}_r = 0 \quad \text{at} \quad r = R$$

$$[u_r] = 0, \quad [\sigma_r] = 0 \quad \text{at} \quad r = \gamma$$

Here and in what follows "∧" denotes the magnitudes corresponding to the material of the disk and "[]" denotes the jump of the corresponding quantities and γ is the radius of elastic-plastic boundary and R is the radius of the contact surface.

THE THERMO-ELASTIC SOLUTION

There is a homogeneous strain-stress state in the disk.

$$\hat{e}_r = \hat{e}_\theta = u_0/R, \quad \hat{e}_z = 2u_0(2\hat{\mu} - 3\hat{K}) + 9\hat{K}\hat{\alpha}R(T - T_0)/(4\hat{\mu} + 3\hat{K}) \quad \text{.........(7)}$$

$$\hat{\sigma}_r = \hat{\sigma}_\theta = 18\hat{K}\hat{\mu}(u_0/R - \alpha(T - T_0))/(4\hat{\mu} + 3\hat{K})$$

where $u_0 = u_0(t)$ is the radial displacement of the contact surface. The solution in the elastic zone of the plate has the form

$$e_r^e = -e_\theta^e = -B/r^2, \quad e_z^e = 9K\alpha(T - T_0)/(3K + 4\mu) \quad \text{.........(8)}$$

$$\sigma_r^e = -2\mu(B/r^2 + 9K\alpha(T - T_0)/(3K + 4\mu)), \sigma_\theta^e = 2\mu(B/r^2 - 9K\alpha(T - T_0)/(3K + 4\mu))$$

where B is an arbitrary function of time.

THE STRESSES IN THE PLASTIC ZONE

In oder to satisfy yield condition (2) let us introduce some function $\varphi(r, t)$ as follows

$$s_r = 2k\sin\varphi/\sqrt{3}, \quad \sigma = k(\cos\varphi + \sin\varphi/\sqrt{3}) \quad \text{.........(9)}$$

Then, from equation (5) we find

$$r/R = \sqrt{(\sqrt{3}\sin\varphi_R - \cos\varphi_R)/(\sqrt{3}\sin\varphi - \cos\varphi)e^{\sqrt{3}(\varphi - \varphi_R)/2}} \quad \text{.........(10)}$$

where φ_R is the value of the function φ at r=R. Let φ_γ be the value of the function φ at $r=\gamma$ then the radius of elastic-plastic boundary is

$$\gamma/R = \sqrt{(\sqrt{3}\sin\varphi_R - \cos\varphi_R)/(\sqrt{3}\sin\varphi_\gamma - \cos\varphi_\gamma)e^{\sqrt{3}(\varphi_\gamma - \varphi_R)/2}} \quad \text{.........(11)}$$

From boundary conditions (6) and the condition that the plastic portions of the strains are equal to zero at $r=\gamma$ we have

$$u_0/R = k(4\hat{\mu} + 3\hat{K})(\sqrt{3}\sin\varphi_R + \cos\varphi_R)/(18\hat{\mu}\hat{K}) + \hat{\alpha}(T - T_0) \quad \text{.........(12)}$$

$$\alpha(T - T_0) = -k(4\mu + 3K)(\sin\varphi_\gamma + \sqrt{3}\cos\varphi_\gamma)/(12\sqrt{3}\mu K) \quad \text{.........(13)}$$

$$B/R^2 = \gamma^2 k(\cos\varphi_\gamma - \sqrt{3}\sin\varphi_\gamma)/(4\mu) \quad \text{.........(14)}$$

Now, to determine the stresses and strains from (7) and (8) and stresses from (9) due to (10), (12), (13) and (14) it is necessary to find φ_R as the function of φ_γ from the differential equation

$$d\varphi_R/d\varphi_\gamma = \Lambda(\varphi_R, \varphi_\gamma) \quad (15)$$

where $\Lambda(\varphi_R, \varphi_\gamma)$ is the known function which may be expressed by virtue of (10)-(14).The boundary condition for this equation is $\varphi_\gamma=\varphi_0$ and $\varphi_R=\varphi_0$ where φ_0 must be determined from (7) and (8) when no plastic zone is available. Thus, φ_0 depends on the thermo-elastic properties of both materials and the value of fit-tolerance and corresponds to the temperature at which the elastic stresses in the plate satisfy condition (2) at r=R.

EXAMPLE AND DISCUSSIONS

Consider the structure consisting of the steel disk and aluminum plate $\left(\alpha / \hat{\alpha} = 0,6; \mu / \hat{\mu} = 2.9; K / \hat{K} = 0.258; k / K = 0.002\right)$. Figure 1 shows the variation of the temperature at which the plastic zone begins to form, T^*, with the value of fit-tolerance. Thus, the maximum value of fit-tolerance at which the insortion is available without plastic strains approximately equals 0.0034. The dependence of elastic-plastic boundary radius on the temperature at $\Delta = 0$ is plotted in Figure 2. The distribution of the stress σ_r for the different values of temperature at $\Delta = 0$ is shown in Figure 3. σ_r in the disk is plotted in Figure 4 as function of the temperature. These calculations assist to investigation of the stability loss in the disk with subsequent failure of the structure. In the Figure 4 it is shown the dependence of the critical thickness of the disk at which the loss of stability takes place, H, on the temperature. The solution presented may be applied for the analysis of different phenomena leading to the failure of the structure such as stability loss of the plate, damage initiation and so on. In some cases it requiers the knowledge of the plastic strains which may be determined from the equation $du_r/d\varphi_R = \Phi(\varphi, \varphi_R, \varphi_\gamma)$ which must be integrated along the characteristics $\left(\sqrt{3}\sin\varphi - \cos\varphi\right)/\left(\sqrt{3}\sin\varphi_R - \cos\varphi_R\right) = Ce^{\sqrt{3}(\varphi-\varphi_R)}$ where C is a constant along any charachteristic and $\Phi(\varphi, \varphi_R, \varphi_\gamma)$ is the known function of its arguments and φ_γ is the known function of φ_R due to (15).

SYMBOLS USED

α = coefficient of linear thermal expansion (mm/mm/^{0}C)

Δ = fit-tolerance (m)

e_r, e_θ, e_z = total normal components of the strain tensor

γ = radius of elastic-plastic boundary (m)

H = thickness of the plate and disk (m)

K = bulk modulus (MPa)

k = shear yield stress (MPa)

ξ_r, ξ_θ, ξ_z = the deviatoric components of the strain tensor

r = current radius (m)

R = radius of the contact surface (m)

σ = hydrostatic stress (MPa)

$\sigma_r, \sigma_\theta, \sigma_z$ = normal components of the stress tensor (MPa)

s_r, s_θ, s_z = deviatoric components of the stress tensor (MPa)

T = current temperature (^{0}C)

t = time (s)

u_r = radial displacement (m)

REFERENCES

(1) Hill, R., "The Mathematical Theory of Plasticity", Oxford, England, 1950.

(2) Johnson, W.and Mellor, P.B., "Plasticity for Mechanical Engineers", D.Van Norstrand, London, England, 1962.

(3) Potter, R.M. and Ting, T.W., Arch. Rat. Mech. and Analysis, Vol. 58, No.1, 1975, pp. 77-94.

(4) Ball, D.L., Fat. Fract. Engng Mater. Struct., Vol.18, No. 1, 1995, pp. 47-63.

(5) Guven, U., Acta Mech., Vol. 89, 1991, pp. 65-72.

(6) Guven, U., Acta Mech., Vol. 92, 1992, pp. 29-34.

(7) Lippmann, H., Int. J. Plasticity, Vol. 8, 1992, pp. 567-582.

(8) Mack, W., Arch. Appl. Mech., Vol. 63, 1993, pp. 42-50.

(9) Bengeri, m. and Mack, W., Acta Mech., Vol. 103, 1994, pp. 243-257.

(10) Mack, W. and Bengeri, M., Int. J. Mech. Sci., Vol. 36, No. 8, 1994, pp.699-705.

(11) Tuba, I.S., Appl. Sci. Res., Vol. 16, No. 4, 1966, pp.241-255.

(12) Orr, J. and Brown, D.K., Eng. Fract. Mech., Vol. 6, 1974, pp. 261-274.

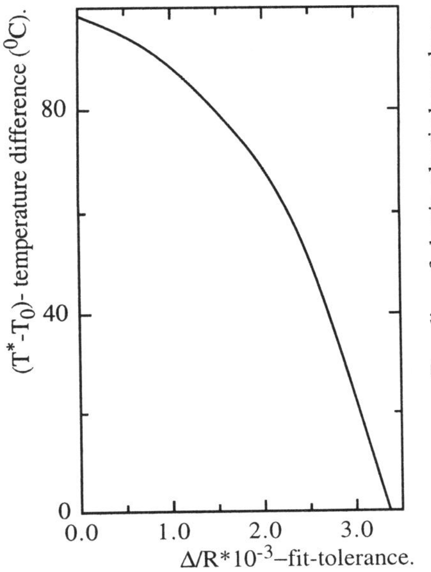

Figure 1 Variation of the temperature with the fit-tolerance

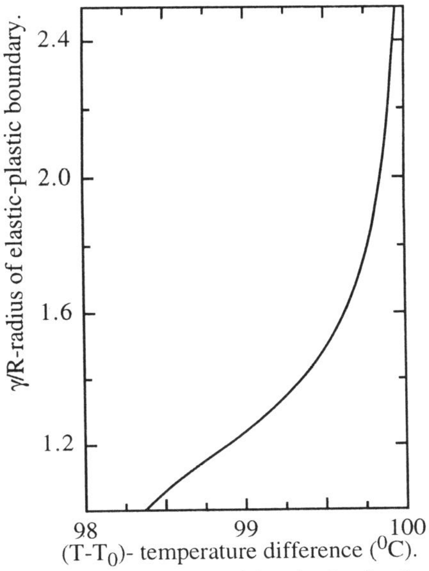

Figure 2 Dependence of the elastic-plastic boundary radius γ on the temperature

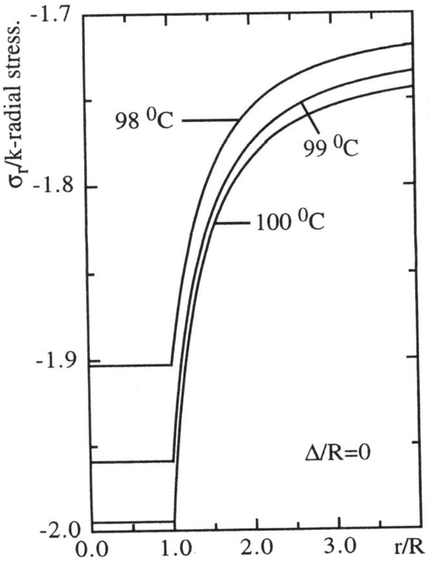

Figure 3 Distribution of stress σ_r for the different values of temperature

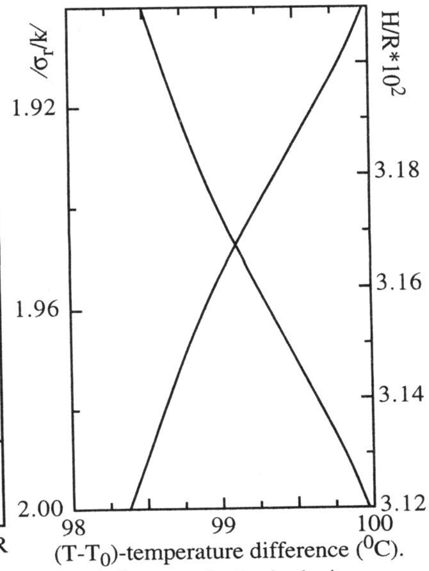

Figure 4 Stress σ_r in the inclusion and the critical thickness